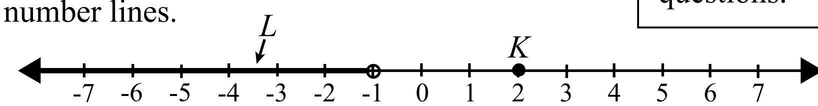
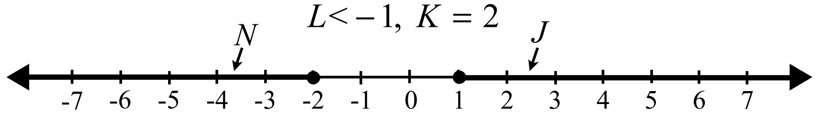
✧ *Solve One-variable Inequalities and Graph the Solution on a Number Line* ✧

In grade school, you may have learned about inequalities as those symbols that look like alligator mouths (>, <). Since the alligator is hungry, the mouth always opens toward the larger number, so 9 > 3 and 6 < 23. Technically, inequalities are used to show that two values are not equal, but the alligator mouth definition is much easier to remember. **Greater than** (>) and **less than** (<) signs still look like alligator mouths on the ***Algebra I*** **EOCT**, only now they include the **greater than or equal to** (≥) sign and the **less than or equal to** (≤) sign as well. Understanding how all these inequalities are represented on a number line is the critical concept for this standard. Look at the following inequalities and equation, and see how they are graphed on the two number lines.

> Questions under this standard fall into the Type 3 category discussed earlier on page 35. Translating algebra into a visual element is the common theme on most of these questions.



$L < -1,\ K = 2$



$N \le -2,\ J \ge 1$

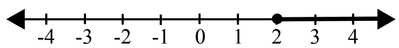
In the first number line, Point K ($K = 2$) is marked by a filled-in circle. There is nothing else to do, since $K = 2$ and nothing else. In contrast, the inequality $L < -1$ is marked by an empty circle starting at –1 and a **ray**, or arrow, heading to the left from this point. On a number line, all numbers to the left of –1 are less than –1, and since $L < -1$, that is where the arrow points.

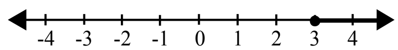
The second number line shows how the "or equal to" part of an inequality is represented by a solid, or filled-in, circle on the starting point. For $J \ge 1$, the circle is dark, and the ray extends to the right. Looking back at point $K = 2$, you can see how $J \ge 1$ is just a combination of the graphs $J = 1$ (a solid circle) and $J > 1$ (a ray extending to the right). Since N is equal to or less than –2, it also has a solid circle on point –2 before extending to the left and covering all numbers less than –2. The open circle indicates all values *excluding* the circled number, and the solid filled-in circle indicates all values *including* the circled number.

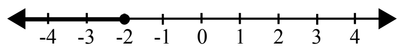
Do not be confused if you see a negative sign in front of the variable. It is important, however, to treat that situation differently. For instance, if you are asked to graph $-M \ge 2$, consider the values for M that would make the inequality a true statement. Is 1 part of the solution? No, because –1 is not greater than 2. Is –4 part of the solution? Yes, because –(–4) is greater than 2. Actually, rather than selecting a bunch of numbers, divide each term by –1. Be sure to switch the direction of the inequality when you divide by –1. $\frac{-M}{-1} \ge \frac{2}{-1}$ is equivalent to $M \le -2$. Since $N \le -2$ is graphed above, you can see how $M \le -2$ would look.

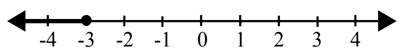
Once you understand this, you'll be able to solve an inequality like $4d + 5 \geq 13$.

Which graph represents the inequality $4d + 5 \geq 13$?

A 

B 

C 

D 

First, solve the inequality:

$$\begin{aligned} 4d + 5 &\geq 13 \\ 4d + 5 - 5 &\geq 13 - 5 \\ 4d &\geq 8 \\ \frac{4d}{4} &\geq \frac{8}{4} \\ d &\geq 2 \end{aligned}$$

Choice A has a solid circle at 2, showing $d = 2$, and then a ray extending to the right, indicating the values greater than 2.

> Once you know how the four inequality symbols ($<$, $>$, $\leq$, $\geq$) appear on a number line, you solve the inequality and then graph it.

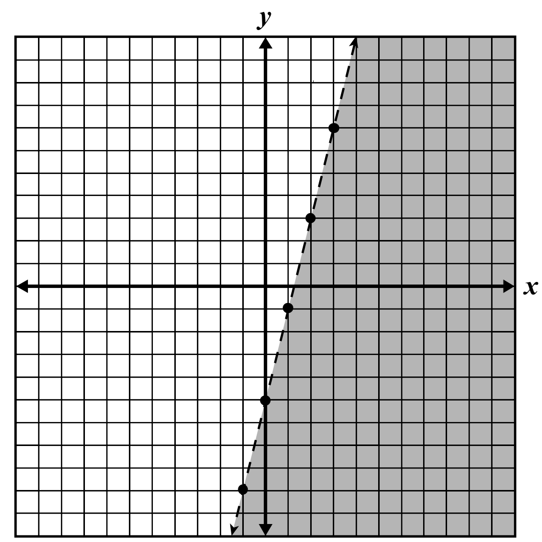
✧ *Solve Linear Inequalities and Graph the Solution on a Coordinate Plane* ✧

The Type 3 problems found here contain several elements from previous standards. In essence, you will have to do the same thing you just did, only this time you will be working on a coordinate plane instead of a number line. The key to the last concept was understanding how an inequality appears on a number line. Here, the key concept is understanding how an inequality appears on a coordinate plane.

One linear inequality is $4x - y > 5$. To graph this inequality, you must first graph the line $4x - y = 5$. On a coordinate plane, draw a dotted line showing $4x - y = 5$. (Graphing lines is addressed more extensively in Content Domain IV, beginning on page 55.)

Drawing a chart helps to find some points on the line

x	y
–1	–9
0	–5
1	–1
2	3
3	7



Why a dotted line? Recall that the inequality is $4x - y > 5$, so you know that $4x - y$ does *not* equal 5. If the inequality had a ≥ sign instead of just a > sign, you would draw a solid line.

For linear inequalities, a dotted line has the same function as the hollow circle on a number line. You can only fill in the circle or solidify the line when the equal sign is included with a "greater than" or "less than" sign.

You must now determine which side of the dotted line needs to be shaded. A good way to do this is to pick a point on one side of the line and find out if it satisfies the inequality $4x - y > 5$. If it does, you know that the point should be shaded, and therefore all the points on that side of the line should be shaded as well.

A good point to use for this test is the origin (0, 0). Substituting 0 for x and y will make this step simpler.

$$4x - y > 5$$
$$4(0) - 0 > 5$$
$$0 > 5$$ (This is an incorrect statement.)

Since 0 is not greater than 5, the origin is not part of the inequality $4x - y > 5$. This means everything on the other side of the dotted line should be shaded instead. This has been done in the graph of $4x - y > 5$ shown above.

STRATEGY BOX – What's My Line?

You should treat a linear inequality like an equation in order to find the line that needs to be graphed. If the inequality has a < or > sign, the solution will be a dotted line. If the inequality has a ≤ or ≥ sign, the solution will be a solid line. Next, select a point (like the origin) and test it in the original inequality. If the inequality is true, then the region should be shaded. If the inequality is false, then the region on the other side of the line should be shaded.

✧ *Solve Simple Rational Equations* ✧

Mathematicians typically prefer variables in the numerator (top of a fraction). It's more efficient and less complicated. There are other advanced reasons to move variables out of the denominator—a process sometimes called *rationalizing the denominator*.

The goal is to isolate the variable on one side and a numerical value on the other. This will usually involve a step where you must multiply both sides of the equation with the variable in order to eliminate it from the denominator. This might seem strange at first, but after some practice, it shouldn't seem so strange.

What is the value for *z* in the equation $-\frac{3}{z}+16=7$?

A -9

B -3

C $\frac{1}{3}$

D 3

Take the "usual" steps to solve for z:

$$\frac{-3}{z}+16=7$$

$$\frac{-3}{z}+16-16=7-16$$

$$\frac{-3}{z}=-9$$

$$\frac{-3}{z}\times z=-9\times z$$ (Multiply both sides by z to remove z from the denominator.)

$$-3=-9z$$

$$\frac{-3}{-9}=\frac{-9z}{-9}$$

$\frac{1}{3}=z$, which is choice C.

It's always a good idea to check your work. Here's the work when this solution is checked:

$$\frac{-3}{\frac{1}{3}}+16=7$$

This can be re-written as: $\left(-3\times\frac{3}{1}\right)+16=7$

$$-9+16=7$$

$$7=7$$

✧ *Solve Simple Radical Equations and Problems Involving Radical Equations* ✧

When an equation has a radical, you first need to isolate the radical on one side of the equation. You can then square both sides, thus eliminating the radical. After that, it's another round of the "Solve the Linear Equation" game.

What is the value of q in the equation $\sqrt{7q+8}-5=3$?

A 0

B $\frac{3}{7}$

C $3\frac{5}{7}$

D 8

Here are the steps to find the correct answer:

$\sqrt{7q+8}-5=3$

$\sqrt{7q+8}-5+5=3+5$ (Add 5 to both sides of the equation.)

$\sqrt{7q+8}=8$ (The radical is isolated, so you can now square both sides.)

$\left(\sqrt{7q+8}\right)^2=8^2$

$7q+8=64$

$7q+8-8=64-8$

$7q=56$

$\frac{7q}{7}=\frac{56}{7}$

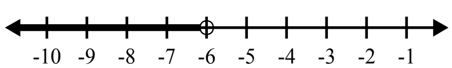
$q=8$

Answer choice D is correct.

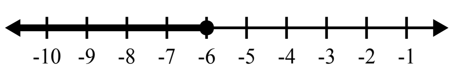
Sample Questions for Content Domain III

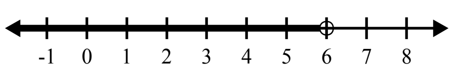
This section has some sample questions for you to try. After you have answered all of the questions, check your answers in the "Answers to the Content Domain III Sample Questions" section that follows. This section will give you the correct answer to each question and show you the appropriate steps for solving the problem.

1 Which graph represents the solution to the inequality $3x + 7 \le -11$?

A 

B 

C 

D 

2 If $\frac{x+7}{3x}=-2$, what is the value of x?

A -1
B 1
C $\frac{7}{5}$
D $\frac{9}{2}$

3 What is the value of y in the following system of equations?

$2x + 3y = 4$
$3x + 4y = 5$

A $y=-3$
B $y=-1$
C $y=\ 1$
D $y=\ 2$

4 Erika earned a grade of 80 or 90 on 5 tests. If her average test score was 86, on how many tests did Erika receive a score of 90?

A 2
B 3
C 4
D 5

5 If $\sqrt{2x} - 3 = 1$, what is the value of x?

A 2
B 4
C 8
D 16

Answers to the Content Domain III Sample Questions

1. Answer: **C** Standard: *Solve one-variable inequalities and graph the solution on a number line*

Remember that this a two-part process: first solve the linear inequality, and then use your knowledge of how the different inequality symbols appear on a number line to find the right answer. (Using your knowledge of these symbols, you can even eliminate choices **A** and **D** right away, since the inequality uses a ≤, meaning there must be a solid circle used. Choices **A** and **D** have hollow circles at the origin of the ray.)

$$3x + 7 \le -11$$
$$3x + 7 - 7 \le -11 - 7$$
$$3x \le -18$$
$$\frac{3x}{3} \le \frac{-18}{3}$$
$$x \le -6$$

Since the ≤ symbol stands for "less than or equal to," there should be solid circle on –6 (the "equal to" part) and an arrow pointing towards the left (containing all numbers "less than" –6). The answer is **C**.

2. Answer: **A** Standard: *Solve simple rational equations*

Rationalizing, or eliminating the $3x$ in the denominator, is the first thing that needs to be done. After that, the equation is solved using standard methods.

$$\frac{x+7}{3x} = -2$$
$$\frac{x+7}{3x} \times 3x = -2 \times 3x$$
$$x + 7 = -6x$$
$$x - x + 7 = -6x - x$$
$$7 = -7x$$
$$\frac{7}{-7} = \frac{-7x}{-7}$$

$-1 = x$, choice **A**.

3. Answer: **D** Standard: *Solve a system of linear equations*

Since there are no graphs in the answer choices, the system of linear equations should be solved by one of the first two methods that begin on page 41. To add or subtract the equations, you need to have the coefficients of one of the variables in both equations equal to one another. Often this is done by multiplying one equation by the coefficient in the other variable, and vice versa. The original equations are shown below.

$2x + 3y = 4$
$3x + 4y = 5$

Since the question asks for the value of y, eliminating the x variables is a good idea. To do this, multiply the first equation by 3 (the coefficient of x in the second equation) and the second equation by 2 (the coefficient of x in the first equation.)

This gives you:

$3(2x + 3y) = (3)4 \quad = \quad 6x + 9y = 12$
$2(3x + 4y) = (2)5 \quad = \quad 6x + 8y = 10$

Subtracting these equations gives you:

$6x + 9y = 12$
$-\underline{(6x + 8y = 10)}$

$9y - 8y = 12 - 10$
$y = 2$, answer **D**.

4. Answer: **B** Standard: *Solve problems involving linear equations*

You need to create two equations that use the same two variables in order to answer this question. There are 5 tests, and Erika scored either an 80 or a 90. This can be written:

No. of tests in which she scored an 80 + No. of tests with a 90 = 5

Making these into the variables e (for eighty) and n (for ninety) gives you $e + n = 5$.

One equation down, one to go. The average of the 5 tests was 86. Average is found by adding up all the tests to find the total number of points scored and then dividing by the total number of tests.

$$\text{Average} = \frac{\text{total number of points scored}}{\text{total number of tests}}$$

$$86 = \frac{\text{total number of points scored}}{5}$$

The total number of points scored can be found by adding $80e + 90n$. This is because e is the number of times Erika scored an 80, while n is the number of times Erika scored a 90.

$$86 = \frac{80e + 90n}{5}$$

You now have two equations with the same two variables. Using the substitution method, you can take the first equation $e + n = 5$ and make it $e = 5 - n$.

$$
\begin{aligned}
86 &= \frac{80e + 90n}{5} \\
86 \times 5 &= \frac{80e + 90n}{5} \times 5 \\
430 &= 80e + 90n \qquad \text{(Now substitute } e = 5 - n.\text{)} \\
430 &= 80(5 - n) + 90n \\
430 &= 400 - 80n + 90n \\
430 &= 400 + 10n \\
430 - 400 &= 400 - 400 + 10n \\
30 &= 10n \\
\frac{30}{10} &= \frac{10n}{10} \\
3 &= n
\end{aligned}
$$

Erika scored a 90 three times, choice **B**.

5. Answer: **C** Standard: *Solve simple radical equations and problems involving radical equations*

After isolating the radical on one side, square both sides of the equation. After that, solve for x.

$$
\begin{aligned}
\sqrt{2x} - 3 &= 1 \\
\sqrt{2x} - 3 + 3 &= 1 + 3 \\
\sqrt{2x} &= 4 \\
\left(\sqrt{2x}\right)^2 &= (4)^2 \\
2x &= 16 \\
x &= 8, \text{ choice } \mathbf{C}.
\end{aligned}
$$

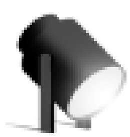
Content Domain IV: Functions and Their Graphs

A LOOK AT CONTENT DOMAIN IV

Test questions in this content domain will measure your understanding of applying properties of relations and functions with an emphasis on linear functions and their graphs. Your answers to the questions will help show how well you can perform on the following standards:

- ◆ Understand the basic use of functions
- ◆ Recognize functions and related terms
- ◆ Graph points on a coordinate plane
- ◆ Define and calculate the slope of a line
- ◆ Use data displayed on a graph
- ◆ Identify the slope and intercepts of a linear equation
- ◆ Understand and use information on a linear equation graph
- ◆ Identify specific lines on a graph
- ◆ Identify the equation of a line from its various properties

Many of the problems in this content domain will involve a coordinate graph, so be sure to review the basic set-up of coordinate systems on page 43 in the previous domain. Thinking in large, strategic terms, your goal on these problems will be to translate the numbers and variables of an algebraic equation into a visual form that appears on a coordinate graph, or you might be asked to do the opposite: translate the visual information of a coordinate graph into the numbers and variables of an algebraic equation. Either way, the key to success lies in your ability to move the same information back and forth between the visual (graph) and the written (equation).



Spotlight on the Standards

◆ *Understand the Basic Use of Functions* ◆

A **function** shows you how to match one group of numbers with another. In a function, one set of numbers is used to generate a second set of numbers. Look at the two sets of numbers in the table below, and see if you can figure out how they are related.

x	y
6	9
8	11
24	27
31	34

For every value of x in the table, the value of y is three units greater. This is how the two sets of numbers match up, and it can be written as the function $y = x + 3$.

Functions are often written in the form $y = f(x)$. In other words, the equations $y = x + 3$ and $f(x) = x + 3$ would mean the same thing.

Thinking about a function as a machine is a good way to understand the relationship between x and y. You place an x value into a "function machine," it does some mathematical "work" on it, and then the function machine spits out a y value. The function $y = x + 3$ always does the exact same mathematical work—regardless of what the value of x is—adding 3 to every x value placed in the function to create a matching y value.

Of course, functions can be much more difficult than just $y = x + 3$. Even so, the function $y = \dfrac{345.54x + 87{,}394}{22{,}306.76}$ still does the same work as the simple function $y = x + 3$.
It takes a single value of x and creates a single matching value for y after some computation.

Which of the following values of x and y correspond to the function $y = 4x – 2$?

A $x = 4$ and $y = –2$
B $x = –2$ and $y = 0$
C $x = 3$ and $y = 14$
D $x = –1$ and $y = – 6$

Going down to the answer choices, you must take each value of x and place it into the function to find the corresponding value of y. For choice A, if $x = 4$ then

$y = 4x – 2$
$y = 4(4) – 2$
$y = 16 – 2$
$y = 14$

This does not agree with choice A's "$y = –2$," so A must be incorrect. However, when you place $x = –1$ (choice D) into the equation, you get

$y = 4x – 2$
$y = 4(–1) – 2$
$y = – 4 – 2$
$y = – 6$

Choice D has $x = –1$ and $y = – 6$, so the two values correspond to the function $y = 4x – 2$.

◆ *Recognize Functions and Related Terms* ◆

Understanding the terms *domain* and *range* is easier if you again think about a function as a machine that "works" on one set of numbers to produce a matching set of new numbers. All of the numbers that go into the function are called the **domain**, while all the numbers that come out of the function are its **range**. If you are familiar with computers, you can think of domain as the *input* of a function and the range as the *output*. Using the established format for writing functions as $y = f(x)$, all the x values make up the domain, while the y values comprise a function's range.

If the domain of the function $y = 2x + 9$ is {5, 7}, what is its range?

A {19, 23}
B {7, 5}
C {5, 7}
D {2, 9}

Although two numbers, 5 and 7, are given for the domain, you need only use one of them to find the right answer in this problem. Take the first domain value, 5, and place it into the function to find the corresponding range value.

$y = 2x + 9$
$y = 2(5) + 9$
$y = 10 + 9 = 19$

The notation for the set of values in a domain or range look similar to the notation for coordinate points, but they mean very different things. The domain of a function that includes the x values of 5 and 7 are represented as {5, 7}. (Remember, both the 5 and the 7 are x values since they're both domain values.) However, the notation (5, 7) refers to the point located 5 units to the right of the origin and 7 units up.

The number 19 is the first range value, and since choice A is the only answer that has 19 as a value, it must be correct. If you were to place the domain value 7 into the function, you would undoubtedly come up with the range value of 23. It's good testing practice to do this step in order to verify that your response is the correct answer.

In addition to knowing the correct function terms, other questions in this standard will ask you to tell the difference between a function and a relation. With a **function**, keep in mind that there is only a single value of y for every single value of x. Every y value is matched to one x value. This is not the case with a relation.

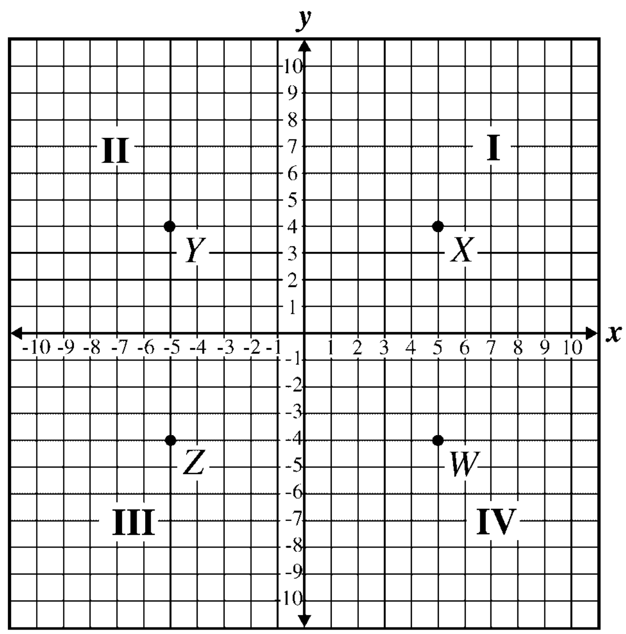
A **relation** may not have a one-for-one match. This means in a relation, x may have *two* corresponding values of y, or three, or four, or as many as you want.

STRATEGY BOX – Test With a Vertical Line

The first two standards focus on the definition of function as some related terms. Be sure to understand how functions operate on paper, since the bulk of this domain will now focus on how functions look when graphed. For a function, if you drew a vertical line at any location on the graph, the line would **never** go through two points. If your vertical line did go through more than one point, then the x value would have more than one y value—therefore, the graph would be a relation, but not a function. Review the definition of a function in the paragraph above.

◆ Graph Points on a Coordinate Plane ◆

The basics of a coordinate graph were first discussed on page 43. The horizontal *x*-axis is perpendicular to the vertical *y*-axis. These lines meet at the origin. All values of *x* right of the *y*-axis are positive, while all those left of it are negative. All *y* values above the *x*-axis are positive, and all *y* values below the *x*-axis are negative. This can be seen in the illustration below.



A coordinate graph is numbered so that any point on the graph can be described by using an ordered pair of values in the form (x, y). The x component gives the horizontal placement, while the y component gives the vertical placement. Since these are all the dimensions a two-dimensional coordinate graph has, you can use the ordered pair $(5, -4)$ to mark the location of a point like *W*. The positive value of x, 5, puts it five spaces to the *right* of the *y*-axis, while the value for y as -4 moves it four spaces *below* the *x*-axis. This is where Point *W* is located on the illustration.

Each point can be described as an ordered pair. As long you remember that the ordered pair is in the form (x, y), you should be able to graph any point or locate a point using only its coordinates.

Use the previous illustration to solve the following problem.

Which point can be found at (–5, 4)?

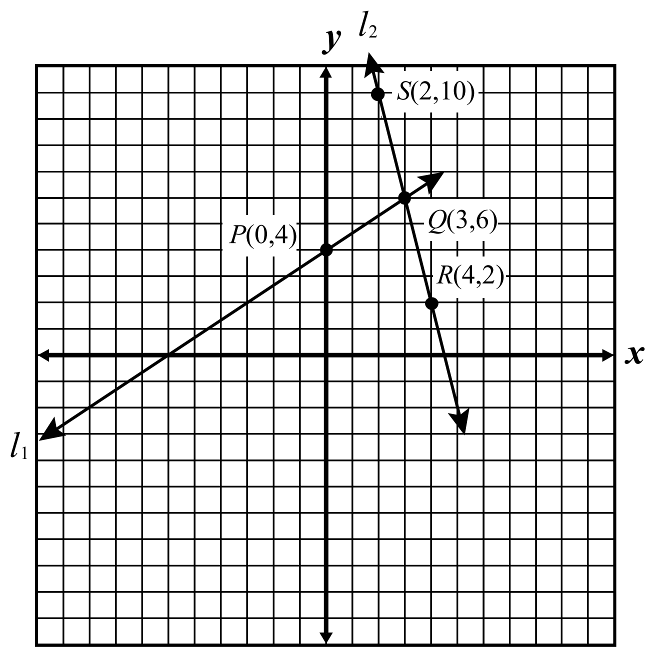
A Point *W*
B Point *X*
C Point *Y*
D Point *Z*

The intersection of the x- and y-axes creates four different regions of a coordinate graph. These are called **quadrants**, and they are marked using the Roman numerals I, II, III, and IV on the illustration. All (x, y) values in a particular quadrant share the same positive/negative values. For example, Quadrant I contains only (positive x, positive y) values, while all values in Quadrant III are (negative x, negative y).

To locate the point (–5, 4), first go five spaces to the *left* on the x-axis since the x value is negative. From there, move up four spaces to find Point Y, answer option C.

◆ *Define and Calculate the Slope of a Line* ◆

When graphing systems of linear equations on page 43, you saw that a line could be graphed once you determined any two points along the line. However, this is not the only way to figure out the graph of a line. You can also use the mathematical idea of slope.



The **slope** of a line measures the rate of change in the vertical component of a line relative to the change in the horizontal component. Algebraically, the slope of a line is defined as:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

The slope represents the change in y divided by the change in x. The variable m is commonly used to refer to slope.

To find the slope of line l_1 on page 59, first take two points along this line, such as $P(0, 4)$ and $Q(3, 6)$. Make P's coordinates equal to (x_1, y_1), and have Point Q's coordinates at (x_2, y_2). Placing them into the equation for slope results in the following:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(6-4)}{(3-0)}$$

$$m = \frac{2}{3}$$

Therefore, the slope of line l_1 is $\frac{2}{3}$. Slope is often called "rise over run," and you can use this definition to get from P to Q using the slope $\frac{2}{3}$. Starting at P, "rise" up two spaces, since the rise part of $\frac{2}{3}$ is the number 2. Now, "run" over 3 spaces. This will get you to Point Q.

It does not matter which point you call (x_1, y_1) or (x_2, y_2). Reversing the order in the previous problem would give you the same slope after the negatives are cancelled out:

$$m = \frac{(4-6)}{(0-3)}$$

$$m = \frac{-2}{-3} = \frac{2}{3}$$

A line passes through points (3, 6) and (4, 2). What is its slope?

A -1

B $-\frac{1}{4}$

C -2

D -4

Placing these two points into the slope equation will give you the slope:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(6-2)}{(3-4)}$$

$$m = \frac{4}{-1} \text{ or } -4$$

Choice D is the correct response.

On page 43, you needed to know at least two points before you could graph a line. However, if you know the slope of a line, you only need *one* point. Suppose you knew that line l_2 had a slope of – 4 and passed through Point *Q*(3, 6) on page 59. Since this slope is negative, you can use two different approaches.

1. Since – 4 can be $-4 = \frac{-4}{1}$, you could go 4 spaces down (a rise of – 4) and one space to the right. This puts you on Point *R*.

2. Since $-4 = \frac{4}{-1}$, you could also go four spaces up (a rise of + 4) and one space to the left (a run of –1). This lands you on Point *S*.

HINT! Look at the figure on page 59 and imaging it is a mountain. If this were true, the summit (or top) would be at Point Q. Lines l_1 and l_2 would be the slopes of the mountain. Viewed from left to right, uphill slopes are positive, while downhill slopes are negative.

STRATEGY BOX – Rise Over Run

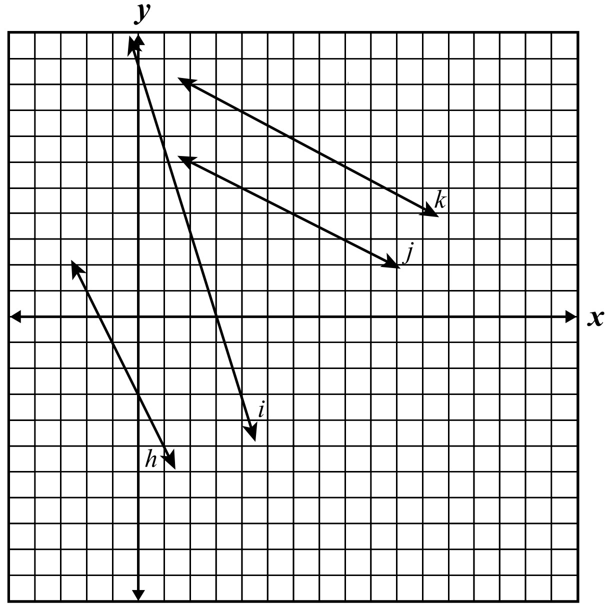
If you know any two points of a line, you can find the slope. If you have the slope of a line and one point on that line, you can graph that line. A common mistake with slope is to switch the order to the *x* and *y* values, so remember that slope is "rise over run" (or $\frac{\text{rise}}{\text{run}}$).

◆ *Use Data Displayed on a Graph* ◆

Questions in this standard will combine a visual element and an understanding of algebraic terms like *slope*, *domain*, *range*, and *intercepts*. You should already be familiar with the definitions of these terms; if not, take the time to get familiar with them. Pay special attention to how these terms look on a graph, since that is how they will appear on the ***Algebra I*** **EOCT**. For example, you know what the range of a function is from page 57. Since range is the set of *y* values of a function, on a graph the range would appear as the *y* values of any ordered pairs. (Since the domain of a function is its input, or *x* values, the domain would consist of the *x* values on a graph.)

For the most part, then, you will be given a graph and you will be asked a question with some terminology in it. Understanding how the term appears on a graph will lead you to the right answer. These problems may look difficult at first, but finding the right answer should just involve the act of translating a term into its visual form.

If the slopes of lines *j* and *k* are the same, which of the following lines has the *x*-intercept of the greatest value?



A Line *h*
B Line *i*
C Line *j*
D Line *k*

The *x*-intercept of a line is where a line crosses the *x*-axis and its *y* value is zero. You can see that the *x*-intercept of line *h* is a negative number between –2 and –1, while the *x*-intercept of line *i* is at (3, 0). With this knowledge alone, you can cross out choice A, since 3 is greater than line *h*'s *x*-intercept value. This leaves lines *j* and *k*. The question states that the slopes are the same, meaning line *j* isn't going to pull some tricky maneuver and end up farther out than line *k*. Extending these lines until they cross the *x*-axis, you will see the line *k* is farther to the right. Therefore, its *x*-intercept value is the greatest, and choice D is correct.

For this question, you needed to take your knowledge of the term "*x*-intercept" and apply it to a graph. This approach will work well for all questions in this standard, which combine terminology and a visual element.

◆ *Identify the Slope and Intercepts of a Linear Equation* ◆

In the previous domain you worked with many linear equations, usually using a form like $-2x + 3y = 12$. This form—with the two variables and their coefficients on one side equation and a real number on the other side—was useful when solving systems of linear equations by addition or subtraction, for instance. When graphing, however, it is often better to take a linear equation and change it so that the y-variable is isolated on one side of the equation and the x-variable and a constant term are on the other side. For example:

$$-2x + 3y = 12$$
$$-2x + 2x + 3y = 12 + 2x$$
$$3y = 2x + 12$$
$$\frac{3y}{3} = \frac{2x}{3} + \frac{12}{3}$$
$$y = \frac{2}{3}x + 4$$

The equation, also called point-slope form or slope-intercept form, now takes the form $y = mx + b$. This form is very useful when graphing because m, the coefficient of x, denotes the slope of the line, in this case $\frac{2}{3}$. Also, the numerical constant b is the y-intercept of the line, showing that the y-intercept occurs at point (0, 4). This makes sense when you consider the equation $y = mx + b$.

Since x must equal zero at the y-intercept, then mx term will always drop out of the equation because anything multiplied by zero is zero. This gives you $y = b$, which is why the numerical constant b always denotes the value of the y-intercept, or where the line crosses the y-axis.

> The line $y = \frac{2}{3}x + 4$ has already been seen several pages back, since it is line l_1 of the graph on page 59. Point P on that line marks the y-intercept at (0, 4).

This standard assesses your ability to identify the slope and intercepts of a linear equation. You could rephrase this by saying that this standard assesses your ability to manipulate linear equations into the form $y = mx + b$. Once you get the linear equation into this form, you have its slope (the coefficient of x) and its y-intercept value (b).

What is the slope of the line $12x + 3y = 54$?

A –4
B 4
C 12
D 18

The simplest way to find the slope of this linear equation is to change the equation into the form $y = mx + b$.

$$
\begin{aligned}
12x + 3y &= 54 \\
12x - 12x + 3y &= 54 - 12x \\
3y &= -12x + 54 \\
\frac{3y}{3} &= \frac{-12x}{3} + \frac{54}{3} \\
y &= -4x + 18
\end{aligned}
$$

The slope is the coefficient of x, in this case – 4. Choice A is the correct answer. If this slope seems familiar to you, it is because the equation $y = -4x + 18$ describes line l_2 of the graph on page 59.

◆ *Understand and Use Information on a Linear Equation Graph* ◆

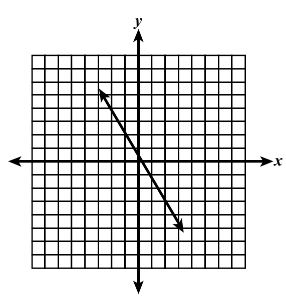
Many of the questions in this standard will have graphs as the answer choices. Your task will be to select the correct graph when given a piece of information like any of the following:

- the x-intercept
- the y-intercept
- the slope of the line
- points on the line
- a linear equation

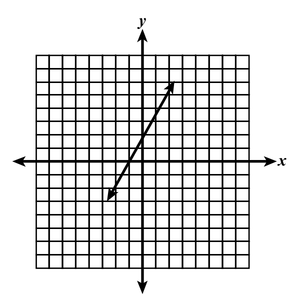
A problem might feature some combination of these elements—both the x- and y-intercepts, for instance—but the main idea behind these questions will stay the same: You are given some information about a line, and you need to select the correct line out of the answer choices.

Which of these graphs shows a line with a negative slope and a negative *y*-intercept?

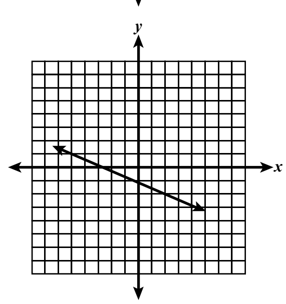
A



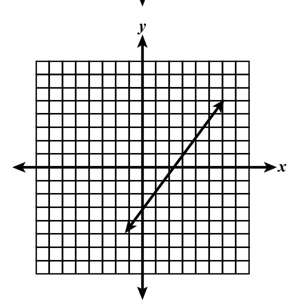
C



B



D



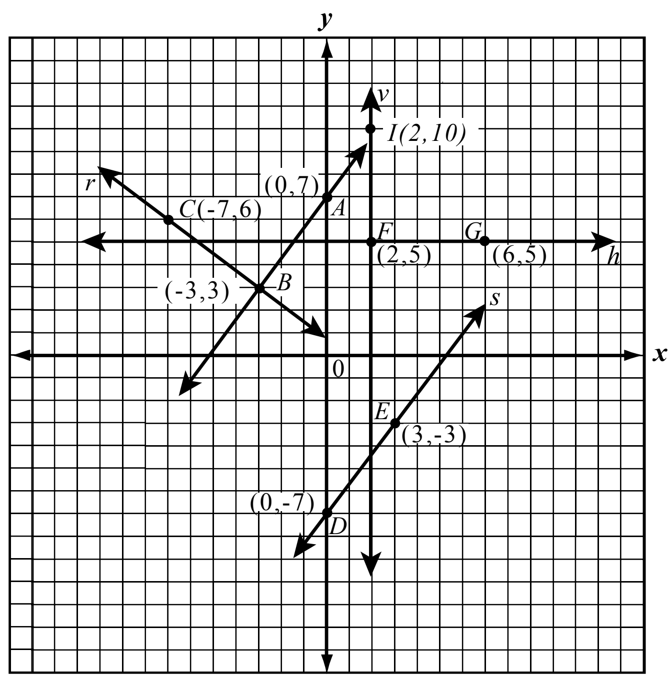
You are given two facts about this line: its slope and its *y*-intercept. Starting with the slope, you know from page 59 that a negative slope looks "downhill" when viewed from left to right. Both choices C and D are "uphill," so you can eliminate those choices. This leaves only A and B. A line with a negative *y*-intercept would cross the *y*-axis *below* the *x*-axis, so this eliminates A. Option B is the correct choice, since it satisfies both conditions of the question.

STRATEGY BOX – Learn the Terms

Doing well on these problems involves understanding the following terms and how they appear on a graph: intercepts, slope, (x, y) coordinates, and linear equations in the form $y = mx + b$. If you know these terms, finding the right answer will become more straightforward.

◆ *Identify Specific Lines on a Graph* ◆

In addition to being able to recognize a linear equation in the form $y = mx + b$, there are also some special types of lines that may appear on the questions in this standard. These are horizontal and vertical lines, and they can be found on the graph below as lines *h* and *v*, respectively.



The horizontal line—that is, a line parallel to the x-axis—called h can be written as $y = 5$. No matter what the value of x is, $y = 5$. To find the slope of this horizontal line, you can take Points $F(2, 5)$ and $G(6, 5)$ and place them into the slope formula from page 59:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(5-5)}{(6-2)}$$

$$m = \frac{0}{4} = 0$$

Since all y values for any horizontal line will be the same, you will always end up with a zero in the numerator (top number of a fraction) of the slope formula. **Therefore, the slope of any horizontal line is always zero.**

In contrast, the slope of a vertical line like v creates a different situation. Line v can be described as $x = 2$ for the same reason the horizontal line h is $y = 5$. Using Points $F(2, 5)$ and $I(2, 10)$ to find the slope, you end up with the following:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(10-5)}{(2-2)}$$

$$m = \frac{5}{0} = \text{undefined slope}$$

You cannot have a zero in the denominator (bottom part) of a fraction. Since this will always be the case with vertical lines, **the slope of any vertical line is always undefined.**

> If you are wondering why a zero cannot be in the denominator of a fraction, consider the fact that a fraction means "something (the top part) divided by something else (the bottom part)." How can you divide anything by 0? For this reason we say that division by 0 is undefined.

Parallel and perpendicular lines can also make an appearance on some problems, and understanding slope is the key to finding the right answer. Look at line w, on page 65, which passes through Points A and B. Line w can be described using the equation $y = \frac{4}{3}x + 7$. Any line parallel to line w will have the same slope; if it didn't, the lines would eventually meet and that would mean they were not parallel. Line s, which passes through Points D and E, is defined by the equation $y = \frac{4}{3}x - 7$. Since the slope $\left(\frac{4}{3}\right)$ is the same for both lines w and s, they are parallel.

A perpendicular line is a line that crosses another line at a $90°$ angle. The slope of a perpendicular line is the **negative inverse** of the slope of the line it crosses. That might sound complicated, but negative inverse just means you flip the denominator and numerator, as well as change the positive/negative sign.

Using line w, $y = \frac{4}{3}x + 7$, as an example, the negative inverse could be found in the following way:

$$m = \frac{4}{3}$$

Inverse $m = \frac{3}{4}$

Negative inverse $m = -\frac{3}{4}$

Any line that has a slope of $-\frac{3}{4}$ will be perpendicular to line w. Line r, defined as $y = -\frac{3}{4}x + \frac{3}{4}$, fits this description.

Slope is the key concept when discussing horizontal, vertical, parallel, and perpendicular lines.

◆ *Identify the Equation of a Line from Its Various Properties* ◆

This standard focuses on linear equations and your ability to manipulate variables. For the most part, you will be given some pieces of information about a line, and your task will be to assemble that information into a linear equation that accurately describes the line.

Consider the fact that the linear equation $y = mx + b$ has three major parts:

1) a slope, m
2) a y-intercept, also called the numerical constant, b
3) two variables, x and y, that represent points using the form (x, y)

These are the three main pieces of the puzzle. For most problems, you will be given two of these parts, and your job will be to create the third part in order to complete the linear equation properly. You might be given a slope (part 1) and a point (part 3). Since you are missing part 2, the y-intercept value, you know you must find it to complete the equation.

You might be asked to create a linear equation using only two points. (A double serving of part 3!) This is multi-step process. Do one part at a time.

Which is an equation of the line that passes through points (– 4, 4) and (– 2, 1)?

A $y = \frac{2}{3}x - \frac{2}{3}$

B $y = -\frac{3}{2}x - 2$

C $y = -\frac{2}{3}x + 1$

D $y = -\frac{3}{2}x + 4$

It's often easier to find the slope (part 1) first, and then use that information to find a value for *b*. Taking the points listed in the question, you can use the slope formula from page 59:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(1-4)}{(-2-(-4))}$$

$$m = -\frac{3}{2}$$

Once you have found the slope $-\frac{3}{2}$, you can eliminate choices A and C since they do not have this slope. To find the correct value for *b*, just pick one of the points given, such as (–2, 1), and place the information you know into the formula below:

$$y = mx + b$$

$$y = -\frac{3}{2}x + b \qquad \text{substitute the slope}$$

$$1 = -\frac{3}{2}(-2) + b \qquad \text{substitute the } x \text{ and } y \text{ values}$$

$$1 = 3 + b$$

$$1 - 3 = 3 - 3 + b$$

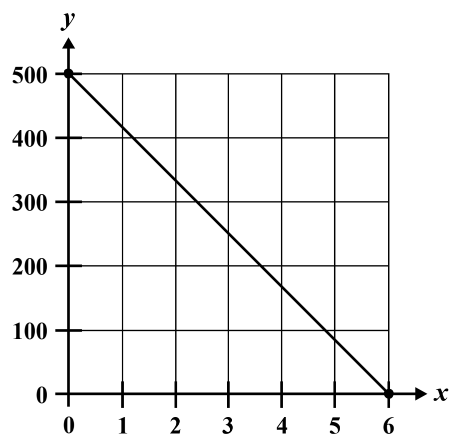
$$-2 = b$$

Having solved for both *m* and *b*, you have the equation $y = -\frac{3}{2}x - 2$, choice B.

Sample Questions for Content Domain IV

This section has some sample questions for you to try. After you have answered all of the questions, check your answers in the "Answers to the Content Domain IV Sample Questions" section that follows. This section will give you the correct answer to each question and show you the appropriate steps for solving the problem.

1 **What is the range for the function represented by the graph below?**



A $\{0 \le x \le 6\}$
B $\{0 \le x \le 500\}$
C $\{0, 1, 2, 3, 4, 5, 6\}$
D $\{0, 100, 200, 300, 400, 500\}$

2 **What is the equation of the line that has a *y*-intercept of 6 and contains (–2, 7)?**

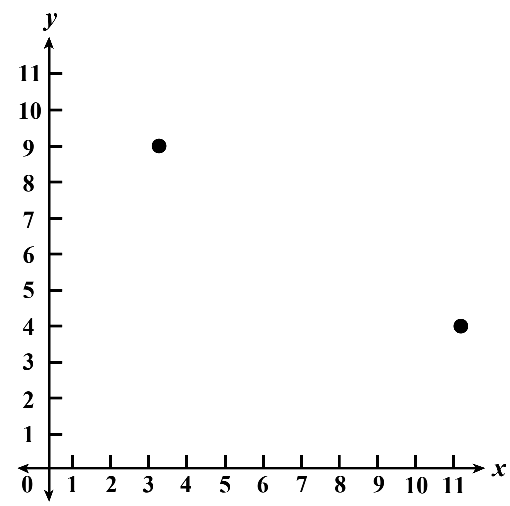
A $y = -\frac{1}{2}x + 6$

B $y = \frac{1}{2}x + 6$

C $y = -\frac{7}{8}x + 6$

D $y = \frac{7}{8}x + 6$

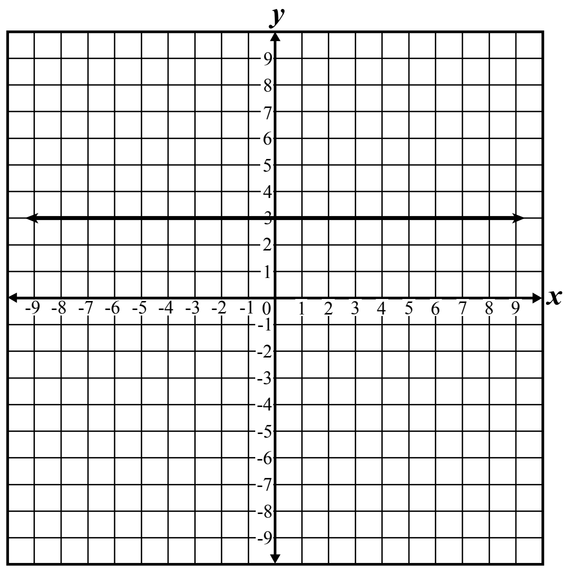
3 **Points *A* and *B* are graphed on the coordinate grid below. Point *A* is located at (3, 9).**

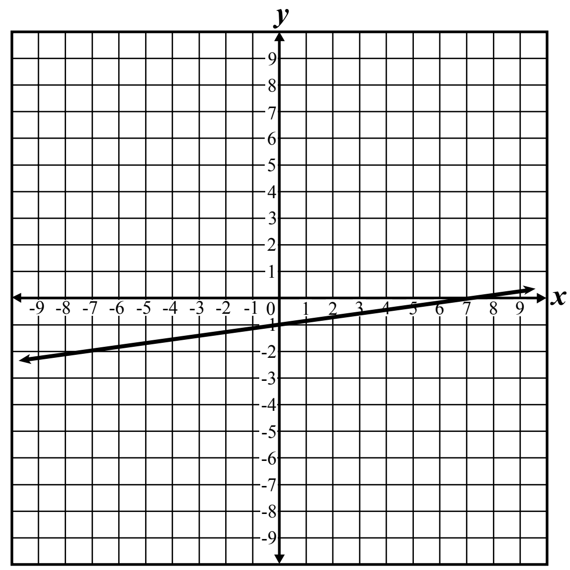


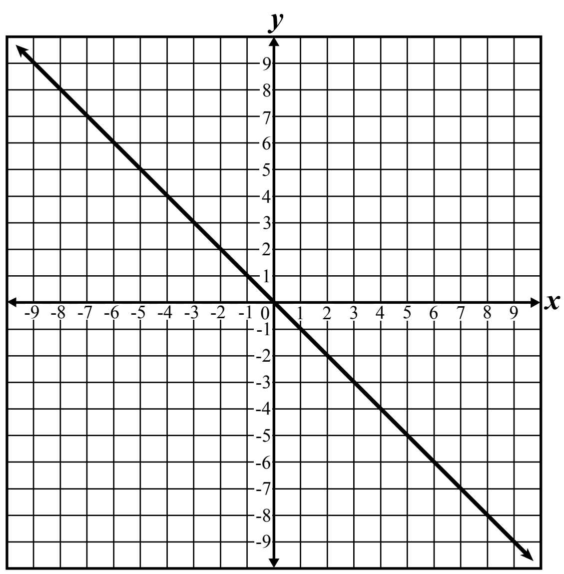
Which ordered pair names the coordinates of Point *B*?

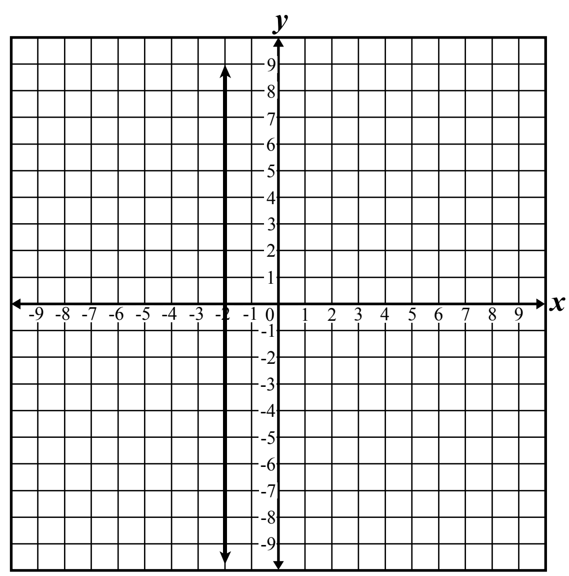
A (3, 4)
B (4, 11)
C (11, 4)
D (4, 3)

4 Which of the following is the graph of a line with an undefined slope?

A


C


B


D


Answers to the Content Domain IV Sample Questions

1. Answer: **B** Standard: *Recognize functions and related terms*
The range is the "output" part of the function, the y value acquired by inserting an "input" x value into the function. The values along the vertical y-axis start at 0 and increase all the way to 500. The graph covers all these numbers, so the range of the graph is $0 \le x \le 500$, choice **B**.

2. Answer: **A** Standard: *Identify the slope and intercepts of a linear equation*
The goal is to find the correct linear equation in the form $y = mx + b$. The question stem gives you the y-intercept value (b) as well as values for x and y with the point (–2, 7). All that remains is finding a value for the slope, m.

$$y = mx + b$$
$$7 = m(-2) + 6 \qquad \text{insert the } y\text{-intercept and } x \text{ and } y \text{ values}$$
$$7 - 6 = -2m + 6 - 6$$
$$1 = -2m$$
$$\frac{1}{-2} = \frac{-2m}{-2}$$
$$\frac{-1}{2} = m \text{ or } m = -\frac{1}{2}$$

Since $m = -\frac{1}{2}$ and $b = 6$, the correct equation is $y = -\frac{1}{2}x + 6$, choice **A**.

3. Answer: **C** Standard: *Graph points on a coordinate plane*
The first value in an ordered pair gives the x value, or the value along the horizontal x-axis. Point B is 11 spaces to the right of the origin. The second value of an ordered pair gives its altitude, or vertical y value. Point B is four spaces above the x-axis. Therefore, the order pair that names Point B is (11, 4). This is answer choice **C**.

4. Answer: **D** Standard: *Understand and use information on a linear equation graph*
It is easy to confuse choices **A** and **D**, since one of them has a slope of zero and the other has an undefined slope. From page 66, you learned why vertical lines have undefined slopes. Choice **D** is a vertical line, so it is the correct response.

Content Domain V: Connections and Applications

A LOOK AT CONTENT DOMAIN V

Test questions in this content domain will measure your understanding of applying algebraic properties including proportions, the Pythagorean Theorem, elementary statistics, and probability. Your answers to the questions will help show how well you can perform on the following standards:

- Solve practical problems using algebraic concepts
- Solve problems involving direct and inverse variation
- Solve problems using ratios, proportions, and percents
- Apply the Pythagorean Theorem to problems
- Summarize data using mean, median, mode, and range
- Understand simple probabilities
- Understand compound probabilities

The seven standards in this content domain cover an array of subjects that are not as closely related to each other as the standards from the other content domains. However, all of the algebraic concepts discussed in Domain V have practical elements that can be applied in your everyday life. The goal of this domain is to show the many different ways that algebra can help you analyze real-world problems. Since this is the goal, most of the questions in this domain will present you with a real-world situation of some sort. In broad terms, your strategy for these questions will be to take the real-world situation, apply algebraic concepts—like equations and variables—to it, and then come up with a suitable answer.

Spotlight on the Standards

▲ *Solve Practical Problems Using Algebraic Concepts* ▲

Suppose you have been given the chore of vacuuming five rooms in a house. Three rooms are bedrooms of the same size, another room is a bathroom half the size of a bedroom, and the last room is a living room three times the size of a bedroom. It takes you ten minutes to vacuum the first bedroom. Your friends are coming over to pick you up in one hour. If you vacuum the other rooms at the same speed, will you be finished vacuuming by the time they arrive?

You can use algebra to determine whether or not you will finish in time. First, you need to assign a variable or two:

Bedroom = b = 10 minutes to clean
Bathroom = w = half the space of b = $(0.5)b$
Living room = l = three times the size of a bedroom = $3b$

You already finished one bedroom, so there are only two bedrooms, the bathroom, and the living room to go. Algebraically, this can be shown in the following way:

Rooms left = $b + b + w + l$
Rooms left = $b + b + (0.5)b + 3b$
Rooms left = $5.5b$

b = 10 minutes to clean, so

$5.5b = 5.5 \times (10 \text{ minutes}) = 55$ minutes

Your friends are coming over in an hour (60 minutes), so you should finish vacuuming with five minutes to spare.

The problem above illustrates how to use algebra in a real-world situation. Questions on the ***Algebra I*** **EOCT** will require the same approach. Therefore, when you encounter a question like this, think about it in terms of these questions:

Questions To Help Convert a Real-World Situation into Algebra

1. What are the facts?

2. Can I assign variables to these facts? Or, what variables need to be created in order to make an equation?

3. How would these variables and facts fit together in an equation?

4. Does this equation provide me with the information I need to answer the question? If not, does it provide useful information that will help me find the final answer?

Asking yourself these questions should get you started in the right direction on a real-word problem. Remember, you do not need to jump directly from the written words to a purely mathematical equation. Writing out intermediate steps can help you understand the problem better.

▲ *Solve Problems Involving Direct and Inverse Variation* ▲

Light bulbs come in various wattages. For low light you might use a 40-watt bulb. If you need more light, you could use a 100-watt bulb. The higher the wattage, the brighter the light. This is an example of **direct variation**—as one increases, the other increases; as one decreases, the other decreases.

With **inverse variation**, the opposite is true. As one factor increases, the other decreases. Think about squeezing a small, spongy ball in your hand. As you increase the pressure with your hand, the size of the ball decreases. If you decrease the pressure by letting go, the size of the ball increases as it pops back to its usual shape. The size of the ball is inversely related to the amount of pressure placed on it.

> Another example of direct variation that you have probably heard relates to studying and school grades. It turns out that studying and doing well in school are directly related: the more you study, the better your grades are.
>
> Most students desperately wish that studying and grades were inversely related: the less they study, the better they do in school. If only!

Most questions on the ***Algebra I* EOCT** will concern direct variation, but it's always a good idea to know what inverse variation means, just in case. For the direct variation problems, the best approach is often to set up two equal ratios, called a proportion.

The amount of money a charity receives varies directly with the number of guests that come to its annual dinner party. Last year 216 guests came to the dinner, and the charity received $972 in donations. This year 252 guests came to the dinner. How much money should the charity receive in donations this year?

A $833.14
B $1,134.00
C $1,188.00
D $1,224.00

The **ratio** shows the relationship between two quantities. It is often expressed as one quantity divided by the other quantity. Since you know the ratio of guests to money for last year, you can set up the ratio as follows:

$$\frac{\text{Number of guests last year}}{\text{Amount of money received last year}}$$

$$\frac{216 \text{ guests}}{\$972}$$

$$\frac{216}{972}$$

That's the ratio for last year. Since the two quantities vary directly, you can create a ratio for this year's event and make it equal to last year's ratio. Of course, you don't know what the exact amount of money is for this year. This is why you must use a variable like *a*.

$$\frac{\text{Number of guests this year}}{\text{Amount of money received this year}}$$

$$\frac{252 \text{ guests}}{a}$$

Set the two ratios equal to each other.

$$\frac{252}{a} = \frac{216}{972}$$

You now have a proportion, with a variable that needs to be determined. The quickest way to do this is to cross-multiply, multiplying the denominator of one ratio with the numerator of the other ratio, and vice versa. Then solve the equation.

$$\frac{252}{a} = \frac{216}{972}$$

$$(216)a = (252)(972)$$

$$216a = 244{,}944$$

$$\frac{216a}{216} = \frac{244{,}944}{216}$$

$$a = 1{,}134, \text{ or } \$1{,}134$$

The correct answer is choice B.

STRATEGY BOX – Set Up Ratios

For direct variation problems, the key is to set up matching ratios. (These two ratios can equal each other because they vary directly.) The set of these ratios should contain a single variable that is the amount you are looking for. After solving for this variable, you will have your answer.

▲ *Solve Problems Using Ratios, Proportions, and Percents* ▲

The concept of ratio was just discussed, but talking about it again in terms of proportion can help you understand both concepts better. To do this, imagine that two brothers—Darnell and Richard—split a pizza. The pizza was cut into eight equal-sized slices. Darnell ate 5 pieces, and Richard ate 3.

The ratio of "Pieces Darnell Ate" to "Pieces Richard Ate" is 5 to 3, which can also be written $\frac{5}{3}$ or 5:3. The ratio compares the two quantities *to each other*. In contrast, a proportion compares a quantity *to the whole*. In this case, the whole would be considered the eight slices of the entire pizza. Therefore, the proportion of pizza Darnell ate is $\frac{5}{8}$, since this number shows the amount he ate (5) compared to entire, whole amount (8). The proportion that Richard ate is $\frac{3}{8}$, for the same reason.

Consider the difference between proportion and ratio on the following problem.

In a recent poll of 210 students, 140 said they were opposed to voluntary summer school, 25 students said they were in favor of it, and 45 were undecided. What proportion of the students polled were in favor of voluntary summer school?

A $\frac{5}{42}$

B $\frac{5}{28}$

C $\frac{5}{9}$

D $\frac{2}{3}$

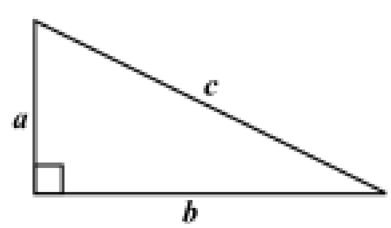
To create the correct proportion, you must find the number of students in favor of voluntary summer school and place it over (divide it by) the total number of students polled.

This gives you the proportion $\frac{25}{210}$, which can be reduced to $\frac{5}{42}$, choice A.

The third part of the standard, **percent**, means "divide by 100." This means 8% is really $\frac{8}{100}$ and "seventy-five percent" is $\frac{75}{100}$ (or $\frac{3}{4}$ when simplified). If a percent is given in a problem, be sure to convert it right away. Then do the computations.

▲ *Apply the Pythagorean Theorem to Problems* ▲

The Pythagorean Theorem applies to all right triangles (triangles that contain a right angle). The side opposite the right angle is called the **hypotenuse**, and it is the longest side of the right triangle. The Pythagorean Theorem states that the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides of the triangle. An algebraic way to describe the Pythagorean Theorem is $a^2 + b^2 = c^2$, as in the illustration:



Answering questions in this standard will require two major steps:

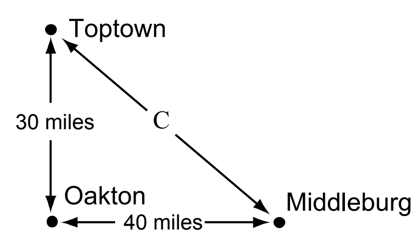
1) First, you have to recognize that the Pythagorean Theorem relates to the problem you are looking at. In other words, can you find the right triangle? These questions will often describe real-world situations, so don't expect to be given an illustration of a right triangle as a clue. Instead, you might have the height of a tree and the length of its shadow. These two values could act as the smaller two sides of a right triangle. You could then use this information to find the hypotenuse. In this example, the distance from the top of the tree to where its shadow ends on the ground. (You can look at the illustration on the previous page if you do not understand this. Make *a* the height of a tree, and *b* its shadow on the ground.)

2) Once you recognize that you have a right triangle, you will be given two of the three sides. Place these values into the Pythagorean Theorem ($a^2 + b^2 = c^2$) in order to find the length of the third side. Use this information to answer the following question.

Oakton is exactly 30 miles due south of Toptown. Middleburg is precisely 40 miles east of Oakton. If there were a direct road between Toptown and Middleburg, how many miles long would this road be?

A 10
B 26.46
C 50
D 70

If you map out where these three towns are, you will see that you have a right triangle. You have been given the two shorter sides, which are 30 and 40 miles each. The direct road between Toptown and Middleburg is the hypotenuse.



$$a^2 + b^2 = c^2$$
$$30^2 + 40^2 = c^2$$
$$900 + 1{,}600 = c^2$$
$$2{,}500 = c^2$$
$$\sqrt{2500} = \sqrt{c^2}$$
$$50 = c$$

The answer is 50 miles, option C.

▲ *Summarize Data Using Mean, Median, Mode, and Range* ▲

Mean, median, mode, and range are four different ways to look at a set of data and try to gain some meaning about the data. You will see these concepts in one form or another any time you read a newspaper. One article might talk about the median household

income while another might talk about the mean cost of a new house. To understand these articles, you need to understand the terms. The best way to do this is to start with a set of numbers.
Twelve students recently completed a pop quiz in Ms. Diaz's class. There were ten questions. Here are the students' scores:

Students' Scores on Recent Pop Quiz

2 3 4 4 4 5 6 7 8 9 10 10

The lowest value is 2 and the highest is 10. The **range** of the scores is the difference between the highest and lowest numbers. Therefore, the range for this set of data is 8 since 10 – 2 = 8. The next term, mean, describes the centrality of the numbers. The **mean** is found by adding all the values and then dividing that total by the number of values.

$$\text{Mean} = \frac{\text{total combined score of all students}}{\text{number of students taking test}}$$

$$\text{Mean} = \frac{(2+3+4+4+4+5+6+7+8+9+10+10)}{12}$$

$$\text{Mean} = \frac{72}{12}$$

$$\text{Mean} = 6$$

The mean score is 6.

The line down the middle of a highway is called a median, and the **median** value of a set of numbers is the middle value. The median is another measure of centrality. The median is found by listing all numbers from least to greatest, and then crossing off the lowest number and the highest number, then the next lowest and next highest, and so on and so on until only a middle value remains. In this pop quiz case, this would look like:

$$\cancel{2}\ \cancel{3}\ \cancel{4}\ \cancel{4}\ \cancel{4}\ 5\ \ 6\ \cancel{7}\ \cancel{8}\ \cancel{9}\ \cancel{10}\ \cancel{10}$$

Since there were an even number of scores, two numbers (5 and 6) remain. The median is then found by taking the mean of these two middle numbers. The median in this case is 5.5. In other words, the median is the middle value.

Suppose that one of the students in the class was absent on the day of the quiz. She made up the quiz and scored a 10. Now there are 13 values. Would that change the median? Try it yourself—include another 10-point score on the list above and find the new median. You should be left with a single number (6) as the median.

The **mode** of any set of numbers is the number that appears most often within the set. The score of 4 is the mode of the pop quiz scores since it appears three times.

> **HINT!** Mean, median, and mode all summarize data, but each one does so in a slightly different way. As you learn to use these measures, you will be able to select and use them to represent data. If you understand the various methods used to find each of these terms, you should be able to answer any ***Algebra I*** **EOCT** questions about them.

▲ *Understand Simple Probabilities* ▲

In broad terms, you can use the idea of proportions to think about simple mathematical probabilities. For the most part, simple probabilities follow this proportion:

$$\frac{\text{Number of specific outcomes}}{\text{Total number of possible outcomes}}$$

Take a regular six-sided number cube. What is the probability that you will roll a four? There are six possible sides, but only one side with a four. The probability can be found in the following way:

$$\frac{\text{Number of specific outcomes}}{\text{Total number of possible outcomes}} = \frac{1}{6}$$

Every single time you roll that number cube, you have a one in six chance of rolling a four. The odds are better for rolling an even number, since there are three even numbers (2, 4, and 6) on a six-sided number cube. Therefore, the probability of rolling an even number is $\frac{3}{6}$, or $\frac{1}{2}$ after you reduce the fraction.

There are 11 Siamese kittens in a box, 7 Manx kittens, and 6 Persian kittens. If a person reaches into the box without looking and pulls out one kitten, what is the probability that the kitten will be Persian?

A $\frac{1}{3}$

B $\frac{1}{4}$

C $\frac{1}{6}$

D $\frac{1}{24}$

To find the right proportion, you have to figure out the total number of possible outcomes. In this case, it is the total number of kittens that could be pulled out. This number is 24, since 11 Siamese + 7 Manx + 6 Persian = 24. This is the denominator of the fraction. The numerator is the number of Persian kittens in the box, since any one of these kittens could be picked. This makes the probability:

To find a simple probability, you take the number of times an event might occur and place this number over (divide it by) the total number of events that could occur.

$\frac{6}{24} = \frac{1}{4}$, choice B.

▲ Understand Compound Probabilities ▲

You only need to set up one proper proportion for a simple probability. As the name suggests, compound probabilities require more than one proportion.

Bringing back the six-sided number cube will help explain compound probabilities. You already know that the probability of rolling a 3 *once* is $\frac{1}{6}$. But what is the probability of rolling a 3 twice in a row? To find this, you must find the probability of rolling a three each time, and then multiply the two to obtain the compound probability.

$\frac{1}{6}$ (probability of rolling a 3 the first time) $\times$ $\frac{1}{6}$ (probability of rolling a 3 the second time) = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$, or 1 in 36.

As you can see, the probability of rolling three two times in a row $\left(\frac{1}{36}\right)$ is smaller than it is to roll a three just once.

STRATEGY BOX – Is It AND or OR?

To find a compound probability, the word "or" requires addition and the word "and" requires multiplication. In the example above, rolling a 3 twice in a row is the same as rolling a 3 AND a 3. If the question asked for the probability of rolling a 3 at least once in two rolls, the probability would be $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$.
(Check for whether the answer is reasonable—the probability is twice that of rolling a 3 in one roll.)

If you wanted to find the probability of rolling a three the first time, and then rolling even numbers each of the next two times, your compound probability would look like this:

$$\frac{1}{6}\times\frac{1}{2}\times\frac{1}{2}$$

This shows you have a one in six chance of getting a three the first time and a one in two chance of rolling an even number the next two times. After multiplying the fractions, you have a $\frac{1}{6}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{24}$ chance.

Compound probabilities can be tricky because sometimes the first step changes the following steps. For example, look back at the question about the kittens in the box. There were 24 kittens to start with, so the probability of picking out a Persian is $\frac{6}{24}=\frac{1}{4}$ or one in four. Now find the probability of picking *two* Persians kittens in a row.

For the first kitten, you still have a $\frac{6}{24}$ chance. After that, however, the total number of kittens has changed. There are now only 23 kittens left in the box, and only 5 of them are Persian. This second proportion takes into account that one Persian kitten has been taken out in the first step. The second time, there are only 5 Persians remaining out of 23 total kittens. This means the probability of picking out two Persians is:

$$\frac{6}{24}\times\frac{5}{23}=\frac{1}{4}\times\frac{5}{23}=\frac{1\times 5}{4\times 23}=\frac{5}{92}$$

The probability of pulling out two Persian kittens in a row is 5 out of 92.

Sample Questions for Content Domain V

This section has some sample questions for you to try. After you have answered all of the questions, check your answers in the "Answers to the Content Domain V Sample Questions" section that follows. This section will give you the correct answer to each question and show you the appropriate steps for solving the problem.

1 **Last month Sam spent $200 dining out and $250 on groceries. If he plans to cut his total monthly food budget by 20%, what is the total he should spend for groceries and dining out?**

A $90
B $200
C $360
D $441

2 **Ali compared the number of calories per cookie for 7 different kinds of cookies.**

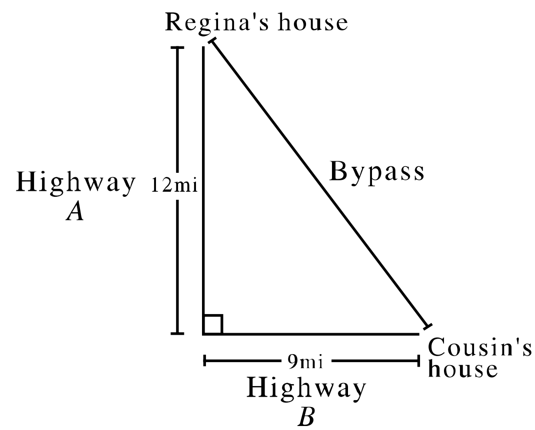
Calorie Comparison

Cookie	Number of Calories
A	50
B	40
C	70
D	65
E	50
F	60
G	55

What is the median for the number of calories per cookie?

A 50
B 55
C 56
D 65

3 **To drive from her house to her cousin's house, Regina can take Highway *A* to Highway *B* or she can take the bypass.**



If she takes the bypass, how many miles will Regina travel?

A 15
B 16
C 17
D 18

4 **A cooler contains 12 cans of apple juice, 8 cans of grape juice, and 6 cans of orange juice. A person removes 2 cans, one at a time, out of the cooler without looking or replacing them. What is the probability that they will both be orange juice?**

A $\frac{3}{13}$

B $\frac{6}{13}$

C $\frac{11}{26}$

D $\frac{3}{65}$

Answers to the Content Domain V Sample Questions

1. Answer: **C** Standard: *Solve problems using ratios, proportions, and percents*
The first sentence states how much money Sam spends on both groceries and dining out. Together this equals his total monthly food budget of $450 (or 200 + 250). Since Sam plans to reduce this number by 20%, you convert 20% into $\frac{20}{100}$ and multiply this by $450.

$$\frac{20}{100} \times 450 =$$

$$\frac{1}{5} \times 450 =$$

90, or $90

This is choice A. However, the question asks how much is "the total he should spend for groceries and dining out" *after* reducing his spending by 20%, or $90. This total can be found by taking what he used to spend ($450) and subtracting 20%, or $90.
$450 – $90 = $360. Choice **C** is the answer.

2. Answer: **B** Standard: *Summarize data using mean, median, mode, and range*
There are seven values in the data set. To find the median, you must write out the values in consecutive order and then find the middle value.
It would look like:

$\cancel{40}$
$\cancel{50}$
$\cancel{50}$
55
$\cancel{60}$
$\cancel{65}$
$\cancel{70}$

The median value is 55, choice **B**. (Incidentally, the mode value is 50, choice **A**.)

3. Answer: **A** Standard: *Apply the Pythagorean Theorem to problems*
By taking the bypass, Regina is driving along the hypotenuse of a right triangle whose smaller sides are 12 miles and 9 miles. To find the length of the bypass, substitute the values of *a* and *b*.

$$a^2 + b^2 = c^2$$
$$12^2 + 9^2 = c^2$$
$$144 + 81 = c^2$$
$$225 = c^2$$
$$\sqrt{225} = \sqrt{c^2}$$
$$15 = c$$

The bypass is 15 miles long, choice **A**.

4. Answer: **D** Standard: *Understand compound probabilities*
In this compound probability, actions taken in the first stage will affect subsequent actions. At the start, there are 26 cans total (12 apple + 8 grape + 6 orange). The probability of pulling out an orange can the first time is $\frac{6}{26}$, or $\frac{3}{13}$.

Now there are only 5 cans of orange juice remaining among the 25 remaining cans. So the next time a can is pulled out, the probability of it being orange juice is $\frac{5}{25}$, or $\frac{1}{5}$.

Multiplying these two probabilities will give you the probability of pulling out two cans, both of which are orange juice:

$\frac{3}{13} \times \frac{1}{5} = \frac{3}{65}$, choice **D**.

Appendix A

EOCT Sample Overall Study Plan Sheet

Here is a sample of what an OVERALL study plan might look like. You can use the Blank Overall Study Plan Sheet in Appendix B or create your own.

Materials/Resources I May Need When I Study:
(You can look back at page 2 for ideas.)

1. *This study guide*
2. *Pens*
3. *Highlighter*
4. *Notebook*
5. *Dictionary*
6. *Calculator*
7. *Algebra I textbook*

Possible Study Locations:

- First Choice: *The library*
- Second Choice: *My room*
- Third Choice: *My mom's office*

Overall Study Goals:

1. *Read and work through the entire study guide*
2. *Answer the sample questions and study the answers*
3. *Do additional practice with an algebra book*

Number of Weeks I Will Study: *6 weeks*

Number of Days a Week I Will Study: *5 days a week*

Best Study Times for Me:

- Weekdays: *7:00 p.m. – 9:00 p.m.*
- Saturday: *9:00 a.m. – 11:00 a.m.*
- Sunday: *2:00 p.m. – 4:00 p.m.*

Appendix B

Blank Overall Study Plan Sheet

Materials/Resources I May Need When I Study:
(You can look back at page 2 for ideas.)

1. ______________________________
2. ______________________________
3. ______________________________
4. ______________________________
5. ______________________________
6. ______________________________
7. ______________________________

Possible Study Locations:

- First Choice: ______________________________
- Second Choice: ______________________________
- Third Choice: ______________________________

Overall Study Goals:

1. ______________________________
2. ______________________________
3. ______________________________

Number of Weeks I Will Study: ______________________________

Number of Days a Week I Will Study: ______________________________

Best Study Times for Me:

- Weekdays: ______________________________
- Saturday: ______________________________
- Sunday: ______________________________

Appendix C

EOCT Sample Daily Study Plan Sheet

Here is a sample of what a DAILY study plan might look like. You can use the Blank Daily Study Plan Sheet in Appendix D or create your own.

Materials I May Need Today:

1. *Study Guide*
2. *Pen*
3. *Notebook*
4. *Calculator*

Today's Study Location: *the desk in my room*

Study Time Today: *From 7:00 p.m. to 8:00 p.m. with a short break at 7:30 p.m.*
(Be sure to consider how long you can actively study in one sitting. Can you sit for 20 minutes? 30 minutes? An hour? If you say you will study for three hours, but get restless after 40 minutes, anything beyond 40 minutes may not be productive—you will most likely fidget and daydream your time away. "Doing time" at your desk doesn't count for real studying.)

If I start to get tired or lose focus today, I will: *do some sit-ups.*

Today's Study Goals and Accomplishments: (Be specific. Include things like number of pages, sections, or standards. The more specific you are, the better able you will be to tell if you reached your goals. Keep it REALISTIC. You will retain more if you study small "chunks" or blocks of material at a time.)

Study Task	*Completed*	*Needs more work*	*Needs more information*
1. *Review what I learned last time*	X		
2. *Study the first standard in Content Domain I*	X		
3. *Study the second standard in Content Domain I*		X	

What I learned today:

1. *Reviewed meaning of variable*
2. *The importance of checking that the answer "makes sense" by estimating first*
3. *How to use math symbols*

Today's reward for meeting my study goals: *Eating some popcorn*

Appendix D

Blank Daily Study Plan Sheet

Materials I May Need Today:

1. ______________________________________
2. ______________________________________
3. ______________________________________
4. ______________________________________

Today's Study Location: ______________________________________

Study Time Today: ______________________________________

(Be sure to consider how long you can actively study in one sitting. Can you sit for 20 minutes? 30 minutes? An hour? If you say you will study for three hours, but get restless after 40 minutes, anything beyond 40 minutes may not be productive—you will most likely fidget and daydream your time away. "Doing time" at your desk doesn't count for real studying.)

If I start to get tired or lose focus today, I will: ________________________________

Today's Study Goals and Accomplishments: (Be specific. Include things like number of pages, sections, or standards. The more specific you are, the better able you will be to tell if you reached your goals. Keep it REALISTIC. You will retain more if you study small "chunks" or blocks of material at a time.)

Study Task	*Completed*	*Needs more work*	*Needs more information*
1.			
2.			
3.			

What I learned today:

1. ______________________________________
2. ______________________________________
3. ______________________________________

Today's reward for meeting my study goals: ________________________________