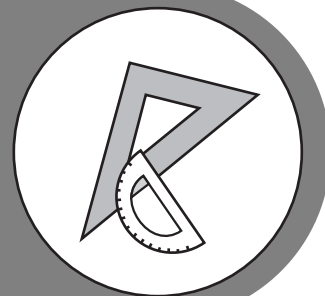


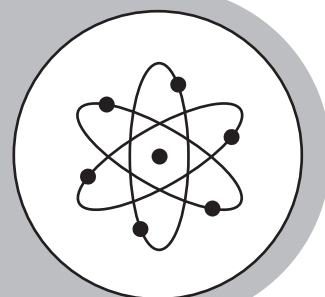
ALGEBRA 1



Study



Guide



Georgia End-Of-Course Tests



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INTRODUCTION

This study guide is designed to help students prepare to take the Georgia End-of-Course Test (EOCT) for *Algebra I*. This study guide provides information about the EOCT, tips on how to prepare for it, and some suggested strategies students can use to perform their best.

What is the EOCT? The EOCT program was created to improve student achievement through effective instruction and assessment of the standards in the Quality Core Curriculum specific to the eight EOCT core high school courses. The EOCT program also helps to ensure that all Georgia students have access to a rigorous curriculum that meets high performance standards. The purpose of the EOCT is to provide diagnostic data that can be used to enhance the effectiveness of schools' instructional programs.

The Georgia End-of-Course Testing program is a result of the A+ Educational Reform Act of 2000, O.C.G.A. §20-2-281. This Act requires that the Georgia Department of Education create end-of-course assessments for students in grades nine through twelve for the following core high school subjects:

Mathematics

- Algebra I
- Geometry

Social Studies

- United States History
- Economics/Business/Free Enterprise

Science

- Biology
- Physical Science

English Language Arts

- Ninth Grade Literature and Composition
- American Literature and Composition

Getting started: The HOW TO USE THE STUDY GUIDE section on page 2 outlines the contents in each section, lists the materials you should have available as you study for the EOCT, and suggests some steps for preparing for the *Algebra I* EOCT.

HOW TO USE THE STUDY GUIDE

This study guide is designed to help you prepare to take the *Algebra I* EOCT. It will give you valuable information about the EOCT, explain how to prepare to take the EOCT, and provide some opportunities to practice for the EOCT. The study guide is organized into three sections. Each section focuses on a different aspect of the EOCT.

The **OVERVIEW OF THE EOCT** section on page 4 gives information about the test: dates, time, question format, and number of questions that will be on the *Algebra I* EOCT. This information can help you better understand the testing situation and what you will be asked to do.

The **PREPARING FOR THE EOCT** section that begins on page 5 provides helpful information on study skills and general test-taking skills and strategies. It explains how to prepare before taking the test and what to do during the test to ensure the best test-taking situation possible.

The **TEST CONTENT** section that begins on page 11 explains what the *Algebra I* EOCT specifically measures. When you know the test content and how you will be asked to demonstrate your knowledge, it will help you be better prepared for the EOCT. This section also contains some test-taking strategies for successfully answering questions on the EOCT.

With some time, determination, and guided preparation, you will be better prepared to take the *Algebra I* EOCT.



GET IT TOGETHER

In order to make the most of this study guide, you should have the following:

Materials:

- ✓ This study guide
- ✓ Pen or Pencil
- ✓ Highlighter
- ✓ Paper
- ✓ Calculator (**optional**)

Resources:

- ✓ Dictionary
- ✓ Algebra I textbook
- ✓ A teacher or other adult

Study Space:

- ✓ Comfortable (but not too comfortable)
- ✓ Good lighting
- ✓ Minimal distractions
- ✓ Enough work space

Time Commitment:

- ✓ When are you going to study?
- ✓ How long are you going to study?

Determination:

- ✓ Willingness to improve
- ✓ Plan for meeting goals

**SUGGESTED STEPS FOR USING THIS STUDY GUIDE**

- 1** Familiarize yourself with the structure and purpose of the study guide.
(You should have already read the INTRODUCTION and HOW TO USE THE STUDY GUIDE. Take a few minutes to look through the rest of the study guide to become familiar with how it is arranged.)
- 2** Learn about the test and expectations of performance.
(Read OVERVIEW OF THE EOCT.)
- 3** Improve your study skills and test-taking strategies.
(Read PREPARING FOR THE EOCT.)
- 4** Learn what the test will assess by studying each domain and the strategies for answering questions that assess the standards in the domain.
(Read TEST CONTENT.)
- 5** Answer the sample questions at the end of each domain section. Check your answers against the annotated answers to see how well you did.
(See TEST CONTENT.)

OVERVIEW OF THE EOCT

Good test takers understand the importance of knowing as much about a test as possible. This information can help you determine how to study and prepare for the EOCT and how to pace yourself during the test. The box below gives you a “snapshot” of the *Algebra EOCT*.



THE EOCT AT A GLANCE

Administration Dates:

The EOCT will be given three times a year; once in the spring, once in the summer, and once in the winter.

Administration Time:

Each EOCT is comprised of two sections; each section will take 45 to 60 minutes to complete. You will have from 100 to 135 minutes to complete each EOCT. You will be given a 5-minute stretch break between the two sections of the test.

Question Format:

All the questions on the EOCT are multiple choice.

Number of Questions:

Each section of the EOCT contains 45 questions; there are a total of 90 questions on the EOCT.

If you have additional administrative questions regarding the EOCT, please visit the Georgia Department of Education website at www.doe.k12.ga.us, see your teacher, or see your school test coordinator.



PREPARING FOR THE EOCT



WARNING!

You cannot prepare for this kind of test in one night. Questions will ask you to apply your knowledge, not list specific facts. Preparing for the EOCT will take time, effort, and practice.

In order to do your best on the *Algebra I* EOCT, it is important that you take the time necessary to prepare for this test and develop those skills that will help you take the EOCT.

First, you need to make the most of your classroom experiences and test preparation time by using good **study skills**. Second, it is helpful to know general **test-taking strategies** to ensure that you will achieve your best score.

Study Skills

A LOOK AT YOUR STUDY SKILLS



Before you begin preparing for this test, you might want to consider your answers to the following questions. You may write your answers here or on a separate piece of paper.

1. How would you describe yourself as a student?
Response: _____
2. What are your study skills strengths and/or weaknesses as a student?
Response: _____
3. How do you typically prepare for an algebra test?
Response: _____
4. Are there study methods you find particularly helpful? If so, what are they?
Response: _____
5. Describe an ideal study situation (environment).
Response: _____
6. Describe your actual study environment.
Response: _____
7. What can you change about the way you study to make your study time more productive?
Response: _____

Effective study skills for preparing for the EOCT can be divided into three categories.

- ◆ **Time Management**
- ◆ **Organization**
- ◆ **Active Participation**



Time Management

Do you have a plan for preparing for the EOCT? Often students have good intentions for studying and preparing for a test, but without a plan, many students fall short of their goals. Here are some strategies to consider when developing your study plan. (See Appendices A–D for SAMPLE STUDY PLAN SHEETS that you can use to help you create your study plan.)

- ◆ Set realistic goals for what you want to accomplish during each study session and chart your progress.
- ◆ Study during your most productive time of the day.
- ◆ Study for reasonable amounts of time. Marathon studying is not productive.
- ◆ Take frequent breaks. Breaks can help you stay focused. Doing some quick exercises (e.g., sit-ups or jumping jacks) can help you stay alert.
- ◆ Be consistent. Establish your routine and stick to it.
- ◆ Study the most challenging test content first.
- ◆ For each study session, build in time to review what you learned in your last study session.
- ◆ Evaluate your accomplishments at the end of each study session.
- ◆ Reward yourself for a job well done.

Organization

You don't want to waste your study time. Searching for materials, trying to find a place to study, and debating what and how to study can all keep you from having a productive study session. Get organized and be prepared. Here are a few organizational strategies to consider.



- ◆ Establish a study area that has minimal distractions.
- ◆ Gather your materials in advance.
- ◆ Develop and implement your study plan (See Appendices A–D for SAMPLE STUDY PLAN SHEETS).

Active Participation



Students who actively study will learn and retain information longer. Active studying also helps you stay more alert and be more productive while learning new information. What is active studying? It can be anything that gets you to interact with the material you are studying. Here are a few suggestions:

- ◆ Carefully read the information and then **DO** something with it. Mark the important points with a highlighter, circle them with a pen, write notes on them, or summarize the information in your own words.
- ◆ Ask questions. As you study, questions often come into your mind. Write them down and actively seek the answers.
- ◆ Create sample test questions and answer them.
- ◆ Find a friend who is also planning to take the test and quiz each other.

Test-taking Strategies

There are many test-taking strategies that you can use before and during a test to help you have the most successful testing situation possible. Below are a few questions to help you take a look at your test-taking skills.



A LOOK AT YOUR TEST-TAKING SKILLS

As you prepare to take the EOCT, you might want to consider your answers to the following questions. You may write your answers here or on your own paper.

1. How would you describe your test-taking skills?

Response: _____

2. How do you feel when you are taking a test?

Response: _____

3. List the strategies that you already know and use when you are taking a test.

Response: _____


4. List test-taking behaviors you use when preparing for and taking a test that do not contribute to your success.

Response: _____

5. What would you like to learn about taking tests?

Response: _____

Suggested Strategies to Use to Prepare for the EOCT

 **Learn from the Past.** Think about your daily/weekly grades in your mathematics classes (past and present) to answer the following questions.

- In which specific areas of mathematics were you or are you successful?

Response: _____


- Is there anything that has kept you from achieving higher scores?


Response: _____

- What changes should you implement to achieve higher scores?

Response: _____

Before taking the EOCT, work toward removing or minimizing any obstacles that might stand in the way of you performing your best. The test preparation ideas and test-taking strategies in this section are designed to help guide you to accomplish this.

 **Be Prepared.** The best way to perform well on the EOCT is to be prepared. In order to do this, it is important that you know what standards/skills will be measured on the **Algebra I EOCT** and then practice understanding and using those standards/skills. The standards that will be measured in this EOCT are located in the **Algebra I Quality Core Curriculum (QCC)**. The **OVERVIEW OF THE EOCT** and **TEST CONTENT** sections of this study guide are designed to help you understand which specific standards are on the **Algebra I EOCT** and give you suggestions for how to study the standards that will be assessed. Take the time to read through this material and follow the study suggestions. You can also ask your mathematics teacher for any suggestions he or she might offer on preparing for the EOCT.

 **Start Now.** Don't wait until the last minute to start preparing. Begin early and pace yourself. By preparing a little bit each day, you will retain the information longer and increase your confidence level. Find out when the EOCT will be administered, so you can allocate your time appropriately.



Suggested Strategies to Use the Day Before the EOCT

✓ Review what you learned from this study guide

1. Review the general test-taking strategies discussed in the TOP 10 SUGGESTED STRATEGIES TO USE DURING THE EOCT on page 10.
2. Review the content domain-specific information discussed in the section, TEST CONTENT, beginning on page 11.
3. Focus your attention on the domain, or domains, that you are most in need of improving.

✓ Take care of yourself

1. Try to get a good night's sleep. Most people need an average of 8 hours, but everyone's sleep needs are different.
2. Don't drastically alter your routine. If you go to bed too early, you might lie in bed thinking about the test. You want to get enough sleep so you can do your best.

Suggested Strategies to Use the Morning of the EOCT



Eat a good breakfast. Eat some food that has protein in it for breakfast (and for lunch if the test is given in the afternoon). Some examples of foods high in protein are peanut butter, meat, and eggs. Protein gives you long-lasting, consistent energy that will stay with you through the test to help you concentrate better. Some people believe it is wise to eat some sugar before a test, claiming it gives them an energy boost. In reality, the energy boost is very short lived, and you actually end up with less energy than before you ate the sugar. Also, don't eat too much. A heavy meal can make you feel tired. So think about what you eat before the test.



Dress appropriately. If you are too hot or too cold during the test, it can affect your performance. It is a good idea to dress in layers, so you can stay comfortable, regardless of the room temperature, and keep your mind on the EOCT.








Arrive for the test on time. Racing late into the testing room can cause you to start the test feeling anxious. You want to be on time and prepared.

TOP 10

Suggested Strategies to Use During the EOCT

These general test-taking strategies can help you do your best during the EOCT.

- 1 Focus on the test.**  Try to block out whatever is going on around you. Take your time and think about what you are asked to do. Listen carefully to all the directions.
- 2 Budget your time.**  Be sure that you allocate an appropriate amount of time to work on each question on the test.
- 3 Take a quick break if you begin to feel tired.** To do this, put your pencil down, relax in your chair, and take a few deep breaths. Then, sit up straight, pick up your pencil, and begin to concentrate on the test again. Remember that each test section is only 45 to 60 minutes.
- 4 Use positive self-talk.** If you find yourself saying negative things to yourself like, “I can’t pass this test,” it is important to recognize that you are doing this. Stop and think positive thoughts like, “I prepared for this test, and I am going to do my best.” Letting the negative thoughts take over can affect how you take the test and your test score.
- 5 Mark in your test booklet.**  Mark key ideas or things you want to come back to in your test booklet. Remember that only the answers marked on your answer sheet will be scored.
- 6 Read the entire question and the possible answer choices.** It is important to read the entire question so you know what it is asking. Read each possible answer choice. Do not mark the first one that “looks good.”
- 7 Use what you know.**  Draw on what you have learned in class, from this study guide, and during your study sessions to help you answer the questions.
- 8 Use content domain-specific strategies to answer the questions.** In the TEST CONTENT section, there are a number of specific strategies that you can use to help improve your test performance. Spend time learning these helpful strategies, so you can use them while taking the test.
- 9 Think logically.** If you have tried your best to answer a question but you just aren’t sure, use the process of elimination. Look at each possible answer choice. If it doesn’t seem like a logical response, eliminate it. Do this until you’ve narrowed down your choices. If this doesn’t work, take your best educated guess. It is better to mark something down than to leave it blank.
- 10 Check your answers.**  When you have finished the test, go back and check your work.

A WORD ON TEST ANXIETY

It is normal to have some stress when preparing for and taking a test. It is what helps motivate us to study and try our best. Some students, however, experience anxiety that goes beyond normal test “jitters.” If you feel you are suffering from test anxiety that is keeping you from performing at your best, please speak to your school counselor who can direct you to resources to help you address this problem.

TEST CONTENT

Up to this point in this study guide, you have been learning various strategies on how to prepare for and take the EOCT. This section focuses on what will be tested. It also includes a section of sample questions that will let you apply what you have learned from your classes and this study guide.

This section of the study guide will help you learn and review the various mathematical concepts that will appear on the *Algebra I EOCT*. Since *algebra* is a broad term that covers many different topics, the state of Georgia has divided it into five major areas of knowledge, called **content domains**. The content domains are broad categories. Each of the content domains is broken down into smaller ideas. These smaller ideas are called **content standards**, or just standards. Each content domain contains standards that cover different ideas related to its content domain. Each question on the EOCT measures an individual standard within a content domain.

The five content domains for the *Algebra I EOCT* are important for several reasons. Together, they cover the major skills and concepts needed to understand and solve mathematical problems involving algebra. These algebraic skills have many practical applications in the real world. Another more immediate reason that the content domains are important has to do with test preparation. The best way to prepare for any test is to study and know the material measured on the test. Since the *Algebra I EOCT* covers the five content domains and nothing else, isn't it a good idea to learn as much about these domains as you can? The more you understand about these domains, the greater your opportunity to be successful on the EOCT.

The chart below lists the five content domains for the *Algebra I EOCT*.

CONTENT DOMAINS

- I. Algebraic Fundamentals
- II. Operations on Real Numbers and Algebraic Expressions
- III. Solving Equations and Inequalities
- IV. Functions and Their Graphs
- V. Connections and Applications



UNDERSTANDING THE STANDARDS

One way to think about **content domains** and **standards** is to think about a supermarket. Supermarkets often group similar foods in the same aisles or areas of the store. For example, the section of the store marked "Fresh Fruits" will be a section filled with apples, oranges, and bananas, to name just a few. So the part of the store called "Fresh Fruits" is like the domain name, and all the various items—apples, oranges, bananas—are the standards that fall under that domain.

Studying the Content Domains

You should plan to study/review the standards for ALL the content domains. To learn what the EOCT will cover, work through this TEST CONTENT section. It is organized by the content domains into the following areas:

- **A Look at the Content Domain:** an overview of what will be assessed in the content domain
- **Spotlight on the Standards:** information about the specific standards that will be assessed (Note: The names of the standards may not be the exact names used by the Georgia Department of Education. Some of the names in this guide may have been modified to reflect the fact that this book is designed for students and not for professional educators.)
- **Sample Questions:** sample questions *similar* to those that appear on the EOCT
- **Answers to the Sample Questions:** in-depth explanations of the answers to the sample questions

Content Domain I: Algebraic Fundamentals



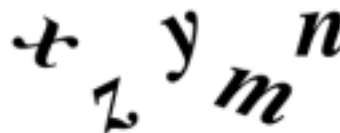
A LOOK AT CONTENT DOMAIN I

Test questions in this content domain will measure your understanding of basic algebraic concepts and skills. Your answers to the questions will help show how well you can perform on the following standards:

- ★ Solve mathematical problems and estimate
- ★ Communicate ideas by using mathematical language and symbols
- ★ Use algebraic variables and numbers

Before you run, you must learn to walk. Before handling algebraic concepts like the slope of a line, you must first understand the basics of algebra, which is what Content Domain I emphasizes.

Most people think about variables when they hear the word “algebra.” You will certainly see your share of variables on the *Algebra I* EOCT, but not every question will contain a string of letters and numbers. Many of the questions in Content Domain I focus on your ability to set up mathematical equations properly, with or without variables. If you don’t understand algebraic terms, you will have trouble figuring out what some questions are asking.



A **variable** is a letter or symbol used in place of an actual number, such as the variable n in the equation $n + 3 = 5$. In this example, you can see that the variable n is equal to 2 because $2 + 3 = 5$. Keep in mind that most questions on the *Algebra I* EOCT that use variables will not be this simple.



Spotlight on the Standards

- ★ **Solve Mathematical Problems and Estimate** ★

Estimation is a useful skill to have when working algebra problems because it allows you to spot major computational errors. Consider the equation:

$$p + 6,794 = 16,985$$

By estimating, you can tell that p has to be a large number around 10,000. Therefore, if you do the arithmetic and come up with $p = 2.5$, you know you have made a mistake. Estimating helped you know there was a computational error.

The computational error is fairly simple. Instead of **subtracting** 6,794 from both sides to find the proper value of p , 6,794 was somehow **divided** into both sides. This led to the incorrect answer of 2.5, when p is actually equal to 10,191.

Estimation questions may seem unusual because you are often asked to find a precise answer on most mathematics problems. Estimation questions ask for an approximate answer, which some students find a bit strange. However, if you realize beforehand that there might be an estimation question or two on the EOCT, you will not be bothered when you come across one like this:

Bradford has four different piggy banks. Two of the banks have \$5.87 in them, one has \$1.12, and the last one has \$3.23. Which of the following is the BEST estimate of how much money Bradford has in all?

- A \$11
- B \$13
- C \$16
- D \$18



HINT! On questions involving dollars and cents, the best approach usually involves rounding the amounts to the nearest dollar.

Therefore, \$5.87 becomes \$6, \$1.12 becomes \$1, and \$3.23 is closer to \$3 than to \$4. Since two of the banks have roughly \$6 in them, estimating the answer will get you $6 + 6 + 1 + 3 = 16$, answer choice C.

STRATEGY BOX – Make a Good Guess

Estimation questions do not require an exact answer, just a good guess. For each problem, cross out the exact number and replace it with an estimate. Rounding an exact number to the nearest whole number or unit of 10, 100, or 1,000 will often get you a good estimated value.

★ Communicate Ideas By Using Mathematical Language and Symbols ★

Mathematics uses symbols in place of language, allowing us to write “ $7 + 4$ ” instead of “seven plus four.” Questions in this standard assess whether or not you can use the mathematical “language” of symbols correctly. This standard also assesses how well you can set up an equation with these symbols.

Adriana is 17 years old. In seven more years she will be exactly twice the age of her younger sister Claudine. Which expression can be used to find Claudine’s age?

- A** $(17 + 7) \div 2$
- B** $(17 - 7) \times 2$
- C** $(17 \times 2) + 7$
- D** $(17 + 7) \times 2$

Like the previous standard, you are not asked to find a numerical answer. Instead, you are simply asked to determine the proper expression that would produce the correct answer. This means you will have to take the words in the question and “translate” them into the correct mathematical language.

The initial number, 17, is a good place to start. The question uses the phrase, *in seven more years*. Ask yourself, “Should I add, subtract, multiply, or divide the number 7 to 17?” Since it says *seven more years*, you should realize that you must add the two numbers together. At this age, $(17 + 7)$, Adriana will be exactly twice the age of Claudine. Therefore, should you multiply or divide by 2 in order to find Claudine’s age? Keep in mind that Claudine is the *younger sister*. If you multiply $(17 + 7)$ by 2, you will get a higher number, making Claudine much older than Adriana. Since Claudine is half her older sister’s age, you need to divide $(17 + 7)$ by 2. The correct answer is A.



HINT! As you work a question, you can always take advantage of the process of elimination to help you cross out incorrect answers.

For instance, once you determine that you must add 7 and 17, you can cross out any choices that do not have $(17 + 7)$. This gets rid of B and C. Even if you got stuck at this point, you could at least take an educated guess between choices A and D.

STRATEGY BOX – Become a Translator

Think of yourself as a translator to answer questions for this standard. All you need to do is take the words from the question stem and translate them into the proper mathematical language.

★ Use Algebraic Variables and Numbers ★

The “Algebra as language” lesson continues under this standard. There are many terms to review, and you will also see some variables scattered throughout questions in this standard. Determining the value of any unknown variables will be the key to these problems. Here is an example of one such equation.

If $r + 2u - 5 = g$, and if $r = 4$ and $u = 8$, then which of the following is equivalent to $g - r$?

- A 7
- B 11
- C 12
- D 15

An **equation** means that there is an equal sign in the mathematical “sentence.” The values on both sides of an equation must be equal, so if you have the equation $g + 11 = 16$, you know that $g + 11$ must be equal to 16. This means that $g = 5$.

A mathematical sentence like $g + 2x + 7$ contains no equal sign. It is referred to as an **expression**.

This problem contains three variables: r , u , and g . Values for the first two are given, so the first thing you must do is find the value of the third variable, g . Inserting the given values into the equation, you get:

$$\begin{aligned} r + 2u - 5 &= g \\ 4 + 2(8) - 5 &= g \\ 4 + 16 - 5 &= g \\ 15 &= g \end{aligned}$$

The answer choice D gives the value for the variable g . However, the question asks for the equivalent of $g - r$, which would be $15 - 4 = 11$. The correct answer is B.

Pick a Number (System)

Normally, when you ask someone to “pick a number,” they do not respond with, “negative square root of 3.” This is because when most people think of numbers, they think of simple numbers like 1, 2, 3, and so on. These simple numbers are called **natural numbers**, and they are just one of six different number systems commonly used in algebra. As you might expect, there are differences between these number systems, and pitfalls await the student who does not know what each number system represents.

Number Systems

1. **Natural Numbers.** This is the simplest category. *Natural numbers are the numbers 1, 2, 3, 4, 5, and so on.* They are sometimes called the *counting numbers* since they are the numbers you count with. Zero and negative numbers are not natural numbers.

When you first learned to count, you probably used your fingers to help you. To help your memory, you can associate natural numbers with your fingers, which are also natural. Any number you can count on your natural fingers is a natural number.



2. **Whole Numbers.** There is only one difference between whole and natural numbers, and that is the number zero. *Whole numbers are all the natural numbers and zero.*

Again, bring your fingers into play to help you remember this fact. Curve your “natural” fingers around to touch your thumb so that your hand forms a circle. The circle represents zero, since it has the same shape. This should help remind you that whole numbers are all natural numbers and zero.



3. Integers. *Integers include all the whole numbers and negative whole numbers.*

Another way to state this is to say that integers include:

- Zero
- The natural numbers
- Negative numbers (the opposites of the natural numbers, like -6 and -45)

The term “integer” is fairly simple, but you don’t really hear it very often in general conversation. (Has anyone ever asked you for your “phone integer”?) Fractions are *not* integers. The number system that includes fractions is called...

...-2,-1,0,1,2...

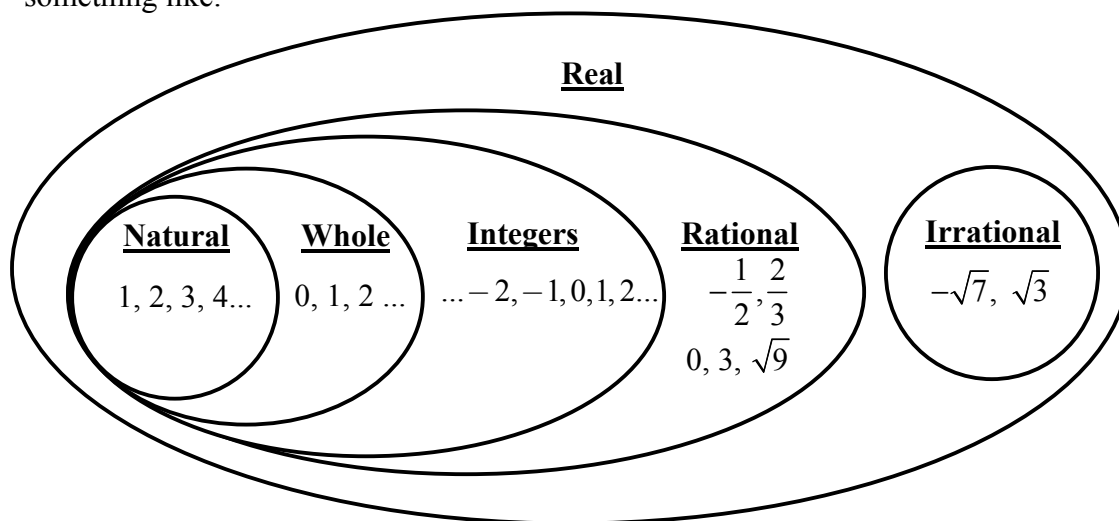
4. Rational Numbers. *Rational numbers can be expressed as the ratio of two integers. They are all terminating or repeating decimals. This includes positive and negative fractions.*

For instance, the fraction $\frac{3}{5}$ is the ratio of two whole numbers, 3 and 5.

Rational numbers also include all integers, whole numbers, and natural numbers. For instance, -6 can be written as $-\frac{42}{7}$. Rational numbers cannot have a denominator of zero.

5. Irrational Numbers. *Numbers that cannot be expressed as the ratio of two integers are called irrational numbers.* Two examples of irrational numbers are $\sqrt{2}$ and $-\sqrt{5}$. Pi (π) is also an irrational number. Every number can be classified as either a rational number or an irrational number. No number can be in both groups.

6. Real Numbers. Everybody’s invited! *All of the numbers in the number systems listed above are called the set of real numbers.* If you are a visual learner, you could draw figures to represent the different number systems and their relationships. It would look something like:



Sample Questions for Content Domain I

This section has some sample questions for you to try. After you have answered all of the questions, check your answers in the “Answers to the Content Domain I Sample Questions” section that follows below. This section will give you the correct answer to each question and show you the appropriate steps for solving the problem.

- 1** If $x = 6$, what is the value of $x^2 + 3x + 7$?

A 28
B 37
C 52
D 61

- 2** Tom ran 12.25 miles every day for 15 days. Which is the MOST reasonable total for the number of miles Tom ran?

A 18.375
B 183.75
C 1837.5
D 18,375

- 3** It rained 1.92 centimeters in the town of Hillside in April, and 5.14 centimeters in May. About how many more centimeters did it rain in May than in April?

A 2.5
B 3.0
C 3.5
D 4.0

- 4** Which value is equivalent to $5 - 3(6 - 7) - 9$?

A 7
B 1
C -1
D -7

Answers to the Content Domain I Sample Questions

1. Answer: **D** Standard: *Use algebraic variables and numbers*

An expression is given, $x^2 + 3x + 7$, along with a value for x equal to 6. The correct approach is to substitute the numerical value into the expression and then make sure to do the math correctly.

$$\begin{aligned}(6)^2 + 3(6) + 7 &= \\ 36 + 18 + 7 &= \end{aligned}$$

61 This is the value for choice **D**.

2. Answer: **B** Standard: *Use algebraic variables and numbers*

Placement of the decimal point is the key to this problem. First, estimate the total amount of miles Tom ran. Rounding 12.25 miles down to 10, you have

$$(10 \text{ miles}) \times (15 \text{ days}) = 10 \times 15 = 150 \text{ miles.}$$

The answer choice closest to this estimate is **B**. It is the most reasonable guess. The actual number, 183.75 miles, is higher than 150 miles, but this makes sense when you recall that

you rounded *down* from 12.25 miles to 10. This means the actual number would be larger than your estimate.

3. Answer: **B** Standard: *Solve mathematical problems and estimate*

You can round 1.92 to 2 and 5.14 to 5. Since the question asks “how many *more* centimeters” of rain fell, you need to subtract $5 - 2 = 3$. This is answer choice **B**.

4. Answer: **C** Standard: *Communicate ideas by using mathematical language and symbols*

Your ability to use arithmetic to simplify is being tested here, regardless of the fact that there are no variables. You should have the following sequence:

$$5 - 3(6 - 7) - 9 =$$

$$5 - 3(-1) - 9 =$$

$$5 + 3 - 9 =$$

$$8 - 9 =$$

$$-1 \quad \text{This is the value for choice C.}$$

Content Domain II: Operations on Real Numbers and Algebraic Expressions



A LOOK AT CONTENT DOMAIN II

Test questions in this content domain will measure your understanding of mathematical and algebraic operations. Your answers to the questions will help show how well you can perform on the following standards:

- * Identify basic properties of the real number system
- * Identify polynomial expressions and associated terms
- * Manipulate polynomials arithmetically
- * Simplify expressions with exponents
- * Factor and divide polynomials
- * Factor quadratic expressions
- * Simplify simple algebraic fractions
- * Simplify square roots and radicals
- * Manipulate radicals arithmetically

To give you an example of what this content domain is like, think about baking bread. To make a loaf of bread, a good baker has to take a hunk of dough and knead, twist, pull, flatten, and mold the dough until it's ready to be placed in the oven. Depending on what type of bread is being made, the baker will have to do different things to the dough in order to get the desired results.

For Content Domain II, the hunk of dough is an algebraic expression, and you are the baker. Your job is to manipulate an expression—by *factoring*, *simplifying*, or just plain *solving*—in order to get the desired results.

So roll up your sleeves and get ready to work on some algebraic expressions and equations. Some of the stickier concepts may be difficult at first, but if you work hard at it, you should be able to taste success on the ***Algebra I EOCT***.



Spotlight on the Standards

* **Identify Basic Properties of the Real Number System** *

It's well known that $2 + 2 = 4$. It's also true that $1 + 3 = 3 + 1$. While you won't see the first equation on the ***Algebra I EOCT***, you may see some form of the second equation since it illustrates what is known as the **Commutative Property of Addition**. This

property states that *two numbers added together always result in the same sum, no matter what order you add the numbers in*. Using variables, the Commutative Property of Addition is written:

$$a + b = b + a$$

In addition to the previous property, there are four other basic properties that you should know for the *Algebra I EOCT*:

Five Basic Properties of the Real Number System

1. Commutative Property of Addition. $a + b = b + a$

The property is discussed above.

2. Commutative Property of Multiplication. $ab = ba$

This property states that *no matter what order two numbers are multiplied together in, the product is the same*. As you can see, this is the same idea as the Commutative Property of Addition, only with multiplication instead of addition. Note that subtraction and division are not commutative operations.

There are more properties than the five listed here. However, these are the five basic properties that appear most often in questions on the *Algebra I EOCT*.

3. Associative Property of Addition. $(a + b) + c = a + (b + c)$

The name has changed, but the central idea stays the same: *no matter what order three (not two) numbers are added together in, the sum is always the same*.

4. Associative Property of Multiplication. $(a \times b) \times c = a \times (b \times c)$

You can probably figure out what this property means. *No matter what order three numbers are multiplied in, the product remains the same*.

5. Distributive Property of Multiplication Over Addition. $a(b + c) = ab + ac$

On the left side of the equation, you *add two numbers together, $b + c$, and then multiply them by a third number, a* . The Distributive Property shows you that you can also *multiply each number by a , and then add the results together to get the same sum*. Using real numbers, you can see that:

$$\begin{aligned} 3(4 + 5) &= 3(4) + 3(5) \\ 3(9) &= 12 + 15 \\ 27 &= 27 \end{aligned}$$

Be sure to remember this property. As you go through this content domain, you will see it become your best friend.

Knowing these properties can help you manipulate certain equations, since they allow you the ability to move some terms around on problems with addition and multiplication. Learning these properties can also make answering questions easier.

The equation $(65t + 43h) + 91w = 65t + (43h + 91w)$ is an illustration of which property?

- A Associative Property of Addition
- B Commutative Property of Addition
- C Associative Property of Multiplication
- D Commutative Property of Multiplication

By the Associative Property of Addition, $(65t + 43h) + 91w$ is equivalent to $65t + (43h + 91w)$. The answer is A.

*** Identify Polynomial Expressions and Associated Terms ***

Knowing the correct algebraic language is very important in answering these questions correctly. For example, once you understand what is being said, you will be able to find the *leading coefficient of a polynomial*.

Before jumping into something like $19x^3 - 4x^2 + 7x - 5$, let's start with a single x . The letter x is a **variable**, a letter used to represent an unknown number.

Placing a 7 in front of the x gives you $7x$, which is another way of saying “seven multiplied by the variable x .” (The multiplication sign is rarely used in algebra, because the sign \times is easily confused with the variable x .) The number in front of a variable is called the **coefficient**. In this case, the coefficient is 7.

Both x and $7x$ are **monomials**, expressions with only one term. The fact that there are no addition or subtraction signs is one indication that an expression is a monomial. Adding or subtracting another term—whether or not the term is a plain number, a variable, or a variable with a coefficient—will give you a **binomial**.

The expression $3x + 6x$ is not a binomial, since this can be simplified into the monomial $9x$. Since both $3x$ and $6x$ share the same variable, x , they are called **like terms**. Only two unlike terms make up a binomial.

Adding a third term to a binomial gives you a **trinomial**, like $-4x^2 + 7x - 5$. At first, some might think that $-4x^2$ and $7x$ are like terms because they both have the variable x . However, x^2 is not the same as x , since it has the exponent 2. (Exponents will be covered in greater detail starting on page 25.)

Here are some binomials:

$$\begin{array}{ll} 7x - 5 & \text{(two terms, } 7x \text{ and } -5) \\ -4x^2 + x & \text{(two terms, } -4x^2 \text{ and } x) \end{array}$$



HINT! Looking at the word origins can help you remember some of these different terms.

Since the root word *-nomial* means *number* (or term), the prefixes hold the key to each word. *Mono-* means *one*, *bi-* means *two*, and *tri-* means *three*. Therefore, trinomial means “three numbers,” and a trinomial can be recognized as having three distinct sets of terms.

While the words binomial and trinomial refer to a specific number of terms, the word **polynomial** simply means “many terms.” Therefore, a polynomial expression could have six, three, five, or eleven million terms. As long as an expression has more than a single (monomial) term, it can be called a polynomial.

Polynomials have different vocabulary words associated with them. Three major concepts are *degree*, *leading coefficient*, and *constant term*. We can use the expression $19x^3 - 4x^2 + 7x - 5$ to explain each of these concepts.

- **Degree** refers to the largest power of x in the expression. In this case, $19x^3$ has the largest power of x , so the degree of this expression is 3.
- The **leading coefficient** is the number in front of the term with the highest power of x . Since $19x^3$ has the highest power, the leading coefficient is 19.
- Finally, the **constant term** is the term that remains unchanged. The constant is -5 in this expression.

Once you master the language of algebra, you will have an easier time understanding a question like:

In the polynomial $25r^4 + 9r^2 - r - 8$,
what is the constant term?

- A 25
- B $-r$
- C -8
- D 4

Negative 8 is not multiplied or divided by any variable. The answer is C.

STRATEGY BOX – Learn the Language of Algebra

You need to speak the language of algebra well if you want to answer the **Algebra I EOCT** questions correctly. Since many questions are designed to assess your understanding of algebraic terms, learning these terms should make these questions easier to answer.

* Manipulate Polynomials Arithmetically *

Now that you understand the terms used to describe polynomials, it's time to start mixing and matching them. This standard focuses on your ability to add, subtract, and multiply polynomials.

Adding and subtracting polynomials requires two things: a knowledge of (1) like terms and (2) the Distributive Property of Multiplication over Addition. Both ideas have already been covered. Like terms are

Terms like *sum*, *difference*, and *product* may appear in the questions for this standard. If you are unfamiliar with these words, remember that *sum* calls for addition, *difference* asks for subtraction, and *product* requires multiplication. A question that asks “What is the sum of $5t$ and $2t$?” would have $7t$ as its answer, while the question “What is the difference of $5t$ and $2t$?” would have $3t$ as its answer.

important because **only like terms may be added or subtracted**. You can add $8w$ to $4w$, but you can't add $8w$ to $6p$. Like water and oil, $8w$ and $6p$ are unlike terms that do not mix.

Furthermore, even though $8w^3$ and $7w$ both have the variable w , the fact that $8w^3$ includes w raised to the third power makes these unlike terms. Like terms must have the same variable raised to the same power, so $6w^3$ and $5w^3$ are like terms. For example, $6w^3 + 5w^3$ is equivalent to $11w^3$ because $6w^3$ and $5w^3$ are like terms. Because $8w^3$ and $7w$ are not like terms, they may not be combined by addition or subtraction.

To add like terms, recall the Distributive Property of Multiplication over Addition, which states $a(b + c) = ab + ac$. Since both sides are equal, the equation can also be written as $ab + ac = a(b + c)$. Using $8w$ and $4w$ as our examples, you can see:

$$\begin{aligned} 8w + 4w &= w(8 + 4) \quad \text{The Distributive Property at work!} \\ &= 12w \end{aligned}$$

Subtraction follows the same process as addition:

$$9y - 4y = y(9 - 4) = 5y$$

Multiplication of polynomials can be a little more challenging because exponents are often involved. As you may know, mathematicians like shortcuts, and exponents are a quick way to show a number or variable multiplied by itself more than once. Instead of having to write $7 \times 7 \times 7 \times 7$ or $d \times d \times d$, mathematicians use exponents—those raised numbers—and just say 7^4 or d^3 . The term 7^4 uses the exponent 4 to say “seven multiplied by itself four times” in a neat and tidy manner.

Exponents become important in multiplication because you do not need like terms in order to multiply. So, you can multiply $8p$ and $6w$. The result is found by multiplying the coefficients together and then multiplying the variables.

$$(8p)(6w) = 48pw \text{ because } 8 \times 6 \text{ is } 48 \text{ and } p \times w \text{ is } pw.$$

Multiplying $8w^3$ and $7w$ equals $56w^4$, and this should make sense if you remember what exponents do. Think about how $8w^3$ is really just $8 \times w \times w \times w$. If you write it out this way, you will see:

$$\begin{aligned} (8w^3)(7w) &= \\ (8 \times w \times w \times w)(7 \times w) &= \text{First multiply the coefficients 8 and 7.} \\ 56 \times w \times w \times w \times w &= \text{Change to exponent notation.} \\ 56w^4 & \end{aligned}$$

Try your hand at the problem on the next page that requires addition, subtraction, and multiplication. If you write everything out clearly and do the math correctly, you will get the correct answer.

The expression $8k(2k - 6) + 5(-3k^2 - 5)$ is equivalent to which of the following?

- A $-32k^2 - 15k^2 - 30$
- B $k^2 - 11$
- C $12k^2 - 14k - 15$
- D $k^2 - 48k - 25$

Writing out each step of the process will help eliminate careless errors when working the math.

$$\begin{aligned} 8k(2k - 6) + 5(-3k^2 - 5) &= && \text{by the Distributive Property} \\ 16k^2 - 48k + (-15k^2) - 25 &= && \text{There are two like terms, } 16k^2 \text{ and } -15k^2, \text{ that can be} \\ &&& \text{added together to get } 1k^2, \text{ or simply } k^2. \end{aligned}$$

$$k^2 - 48k - 25, \text{ which is answer choice D}$$

STRATEGY BOX – Look for Like Terms

In order to add or subtract polynomials, you must have like terms. This is not the case when multiplying polynomials.

* Simplify Expressions with Exponents *

As you have seen, exponents are a shorthand way of writing repeated multiplication. The raised number is called the **exponent**, while the number or variable being multiplied is called the **base**. There may also be a coefficient. In the expression $6j^3$, 6 is the coefficient, j is the base, and 3 is the exponent. The expression $6j^3$ means the same thing as $6 \times j \times j \times j$ or “six times j times j times j .”

If you have two terms with the same base, there is an easy way to multiply the terms together: simply **add the exponents** together. Here is an example:

$$(g^5)(g^2) = g^{(5+2)} = g^7$$

If this seems strange to you, just write out g^5 and g^2 , and you will see how it works.

$$(g^5)(g^2) = (g \times g \times g \times g \times g)(g \times g) = g \times g \times g \times g \times g \times g \times g = g^7$$

You can also raise a variable with an exponent to an even higher power, like $(d^3)^3$.

This is just the shorthand way of writing out $(d^3)(d^3)(d^3)$. When you raise a variable with an exponent to a higher power using another exponent, there is an easy way to find the final term: simply **multiply the exponents** together.

Here is an example of raising a variable with an exponent to a higher power.

$$(d^3)^3 = d^{(3 \times 3)} = d^9$$

Again, you can see this answer is correct if you write out all the variables.

$$(d^3)^3 = (d^3)(d^3)(d^3) = (d \times d \times d)(d \times d \times d)(d \times d \times d) = d^9$$

Understanding when to add exponents and when to multiply them is the main decision you must make to answer the following problem:

Which expression is equivalent to $(2b^3c^4)^3$?

- A $8b^9c^{12}$
- B $6b^6c^7$
- C $6b^6c^{12}$
- D $8b^9c^7$

Let's first write this out the long way:

$$(2b^3c^4)^3 = (2b^3c^4)(2b^3c^4)(2b^3c^4)$$

| | | |
|---------------------------|---------|--------------------------------------|
| multiply the coefficients | 2: | $2 \times 2 \times 2 = 8$ |
| multiply the variables | b^3 : | $b^3 \times b^3 \times b^3 = b^9$ |
| multiply the variables | c^4 : | $c^4 \times c^4 \times c^4 = c^{12}$ |

Putting all of this together, you get the answer $8b^9c^{12}$ —answer choice A.

Let's work the same problem by raising just the coefficient to a higher power:
 $2^3 = 8$, which eliminates B and C as possible answers.

Multiplying the exponents correctly, $b^{(3 \times 3)}$ and $c^{(4 \times 3)}$, also gives you answer choice A.

* Factor and Divide Polynomials *

Factoring polynomials calls for the use of our old friend, the Distributive Property of Multiplication over Addition, also known as $ab + ac = a(b + c)$. Speaking in abstract terms, you will be given polynomials that resemble the $ab + ac$ part, and your job will be to factor out the a part in the form of a monomial.

If you understood that previous statement perfectly, congratulations! You are mastering the language of algebra. If not, don't worry because now we're going to look at that abstract statement using real numbers. The Distributive Property is a good place to start.

$$ab + ac = a(b + c)$$

Now suppose you are to find a monomial factor of $45m^5vr^3 - 30m^3v$.



HINT! A common mistake people make when simplifying an expression like $(d^3)^3$ is to add the exponents together instead of multiplying them. This gives them the result $d^{3+3} = d^6$, which is incorrect.

Many people make this mistake because they add the exponents when multiplying two variables together, as seen with the example $(g^5)(g^2) = g^{(5+2)} = g^7$. However, if this problem were rewritten to be $(g^5)^2$, then the answer would be $g^{(5 \times 2)} = g^{10}$.

To do this, let's start with the two coefficients 45 and 30. What is the greatest common factor of both 45 and 30? The answer is 15, since $(15)(3) = 45$ and $(15)(2) = 30$. In terms of the Distributive Property,

$$ab + ac = a(b + c)$$

$$(15)(3) + (15)(2) = (15)(3 + 2)$$

We can factor out the 15 from the original polynomial $45m^5vr^3 - 30m^3v$, giving us $15(3m^5vr^3 - 2m^3v)$. But there's more work to do, since m^5 and m^3 also share a common factor. This greatest common factor is m^3 , since $m^5 = (m^3)(m^2)$ and $m^3 = m^3 \times (1)$.

In terms of the Distributive Property again,

$$ab + ac = a(b + c)$$

$$(m^3)(m^2) + (m^3)(1) = (m^3)(m^2 + 1)$$

This means we can write $15m^3(3m^2vr^3 - 2 \times 1 \times v)$, or just $15m^3(3m^2vr^3 - 2v)$ since the 1 does not need to be written.

Finally, the variable v is in both terms $3m^2vr^3$ and $2v$, so it can be factored out as well. This leaves us with

$$45m^5vr^3 - 30m^3v = 15m^3v(3m^2r^3 - 2)$$

The monomial term $15m^3v$ has been successfully factored out of the polynomial

$$45m^5vr^3 - 30m^3v$$

Only one term in the polynomial contains the variable r , so there is no way it could be factored out of both terms. Only variables that appear in every term of a polynomial can be factored out.

* Factor Quadratic Expressions *

Before trying to factor a quadratic expression, it's a good idea to know what a quadratic expression is in the first place. Algebraically, a quadratic expression is $ax^2 + bx + c$, where a is a real number other than zero. What this means is that there must be one variable raised to the second power (which is why a cannot be zero), but no variables have exponents larger than 2. The term c is a numerical constant.

From now on, many definitions will be given using as much algebraic terminology as possible. This should help you become more familiar with the unique language of algebra.

The following chart includes some examples of quadratic expressions. In the first binomial example, the middle term, bx , is not present. In the second binomial example, the constant is equal to zero and is therefore not written. Nevertheless, both of these expressions are quadratic expressions because they satisfy the definition of a quadratic expression given earlier.

| Some Examples of Quadratic Expressions | |
|--|-------------|
| Trinomial | Binomial |
| $10x^2 + 11x + 3$ | $x^2 - 49$ |
| $-14x^2 - 7x - 6$ | $2x^2 - 7x$ |

There are different ways you can factor a quadratic expression. Here are three methods that are often used.

1. Factoring quadratic expressions that lack a value for c .

These expressions take the form $ax^2 + bx$. Factor out the x first.

$$2x^2 - 7x$$

$$x(2x - 7)$$

Check to see whether there are any other common factors for the terms. In this case, there are not. If you want to review an example of several common factors, turn back to page 26 and review the “Factor and divide polynomials” standard.

2. Factoring quadratic expressions that lack a middle term ($b = 0$).

If $b = 0$, then there is no middle term. These quadratic expressions look like the example $x^2 - 49$ in the earlier chart. To factor them, find the factors of both terms. In many cases these expressions are not factorable.

The factors of the first term are $(x)(x)$. The factors of the second term are $(-7)(7)$. Now combine the first set of factors with the second set of factors to form the following two binomials:

$$(x - 7)(x + 7)$$

When a is 1, b is 0, c is a perfect square, and the expression has a subtraction sign, use the square root of c when you factor. The middle terms will drop out when you check your work! For more about square roots, look at pages 30 and 31.

3. Factoring quadratic expressions in which a , b , and c do not equal zero.

These expressions look like the example $10x^2 + 11x + 3$, and they tend to require more work than the previous two types. You can often figure out these binomials by doing a little factoring and some testing. That means trying and retrying several times. Factoring these kinds of polynomials is often like trying to solve as puzzle.

In order to factor $10x^2 + 11x + 3$, first list all the factors of the numerical term c . In this case, $c = 3$, and there is only one set of factors



HINT! Multiplying two binomials is often accomplished through the process known as FOIL.

Multiply the **F**irst two terms,
then the **O**utside terms,
then the **I**nside terms,
and finally the **L**ast two terms.

This process has been used on the two binomials on this page.

for 3, $(3)(1)$. Now consider all the factors for the first term, $10x^2$. It has two sets of factors, $(2x)(5x)$ and $(10x)(x)$. Now combine the first set of these factors with the single set of factors for c to form the following two binomials.

$$(2x + 1)(5x + 3)$$

You now must multiply these binomials together to check and see if you get the original quadratic expression $10x^2 + 11x + 3$. If you do, you have found the right combination. If not, you can always try the other set of factors for $10x^2$.

$$\begin{aligned}(2x + 1)(5x + 3) \\ 10x^2 + 6x + 5x + 3 \\ 10x^2 + 11x + 3\end{aligned}$$

Since the product of the two binomials, $(2x + 1)$ and $(5x + 3)$, equals the original expression, you know you have found the correct two binomials.

Try the following problem. The method to find the correct binomial factor is the same as discussed for the last type of quadratic expression.

Which of the following binomials is a factor of the expression $15b^2 - 11b - 14$?

- A $(15b - 14)$
- B $(3b - 2)$
- C $(5b - 7)$
- D $(b - 11)$

You need to first find the factors of -14 and then the factors of $15b^2$. A little experimenting—if you're lucky, not too much experimenting—will eventually lead you to find that $(3b + 2)(5b - 7)$ are the two binomial factors. The second factor is choice C.

*** Simplify Simple Algebraic Fractions ***

Is $\frac{11}{12}$ equal to $\frac{1}{2}$? You should be able to answer “no” to this question without having to use a calculator. However, what seems obvious with real numbers is often harder to see when variables are involved. This is why some students might look at the expression

$\frac{2w+1}{2w+2}$ and think, “Well, I can just cross out the ‘top and bottom’ $2w$ and have a

simplified $\frac{1}{2}$.” This is not correct. Just make $w = 5$, and you will see that it is the same

thing as saying $\frac{11}{12} = \frac{1}{2}$. You can reduce fractions, but only when terms are being

multiplied, not when they are being added or subtracted.

Time to use the **Distributive Property of Multiplication over Addition**. If you factor out some terms, you can then reduce an algebraic fraction like this one:

$$\frac{16y^4 + 4y^2}{8y^2 + 16y}$$

Apply the Distributive Property to both the numerator (top term) and the denominator (bottom term) in order to factor out a monomial. This factoring process is the same as the standard that begins on page 27.

$$\frac{4y^2(4y^2 + 1)}{8y(y + 2)}$$

You can reduce $4y^2$ and $8y$ by taking $4y$ out of both terms. The final result is your simplified algebraic fraction:

$$\frac{y(4y^2 + 1)}{2(y + 2)}$$

* Simplify Square Roots and Radicals *

The **square** of a number is simply that number multiplied by itself. For example, “ 3^2 ” is read as “three squared.” The **square root** of a number works in the opposite direction. When you take the square root of a number like 9, you are looking to find the number that, multiplied by itself, will give you 9. Using a square root sign ($\sqrt{\quad}$), this looks like:

$$\sqrt{9} = ?$$

The $\sqrt{\quad}$ is called a **radical**. The term under the radical is known as a **radicand**. In this example, the radicand is 9. Radicands may also include variables.

It turns out the number 9 is a **perfect square**, meaning the square root of the number is an integer. (Turn to page 17 if you don’t remember what an integer is.) The number 3 multiplied by itself, yields the number 9. This means the square root of 9 is 3. Another square root of 9 is -3 . For this section, the focus will be on the positive square roots.

$$\sqrt{9} = 3 \text{ because } 3^2 = 9$$

Learning the basic perfect squares is a good idea, as it will help you with various questions on the EOCT.

| Squares and Square Roots | |
|--------------------------|-------------------|
| $1^2 = 1$ | $\sqrt{1} = 1$ |
| $2^2 = 4$ | $\sqrt{4} = 2$ |
| $3^2 = 9$ | $\sqrt{9} = 3$ |
| $4^2 = 16$ | $\sqrt{16} = 4$ |
| $5^2 = 25$ | $\sqrt{25} = 5$ |
| $6^2 = 36$ | $\sqrt{36} = 6$ |
| $7^2 = 49$ | $\sqrt{49} = 7$ |
| $8^2 = 64$ | $\sqrt{64} = 8$ |
| $9^2 = 81$ | $\sqrt{81} = 9$ |
| $10^2 = 100$ | $\sqrt{100} = 10$ |

Can you use your knowledge of perfect squares to help you approximate the square root of a number that isn't a perfect square?

There is no simple, sure-fire method of finding the square root of a number that is not a perfect square . . . unless you use your calculator, of course. While you might use your calculator on the EOCT, learning about perfect squares is still very important for some factoring questions, so it will be emphasized over calculator use.

Which of the following answers is the BEST estimate for $\sqrt{17}$?

- A more than 3
- B a little over 4
- C almost 5
- D over 5

The number 17 is not a perfect square. If you find the nearest perfect squares, however, you can guess at the answer. The square root of 17 is between $\sqrt{16}$ and $\sqrt{25}$. These are the perfect squares for 4 and 5, respectively. This means $\sqrt{17}$ is going to be a real number between 4 and 5. Since $\sqrt{17}$ is only a little more than $\sqrt{16}$, choice B is the best answer.

*** Manipulate Radicals Arithmetically ***

Now that you understand what radicals are, it's time to start adding, subtracting, multiplying, and dividing them. Adding and subtracting radicals requires the assistance of the **Distributive Property of Multiplication over Addition**. You can see the Distributive Property at work in the problem below:

$$\begin{aligned} 23\sqrt{5} - 9\sqrt{5} + 18\sqrt{5} &= \\ (23 - 9 + 18)\sqrt{5} &= \\ 32\sqrt{5} \end{aligned}$$

As you might expect, you cannot add $4\sqrt{3}$ and $5\sqrt{2}$ together because the radicands are unlike. Only numbers with identical radicands may be added or subtracted. On the **Algebra I EOCT**, only terms with common radicands will be used on addition and subtraction problems, so you will not encounter any question asking you to subtract $a\sqrt{b}$ from $c\sqrt{d}$.

Multiplying radicals is a three-step process:

- 1) Multiply any numbers in front of the radicals together first.
- 2) Multiply the numbers under the radical sign together, keeping the product under a radical sign.
- 3) Simplify the number under the radical sign, if possible, by factoring out any perfect squares.

Apply this three-step process to the following question:

$$(4\sqrt{10})(8\sqrt{15}) =$$

- A $12\sqrt{150}$
- B $32\sqrt{25}$
- C $160\sqrt{6}$
- D 4800

First, multiply the numbers in front of the radicals together ($4 \times 8 = 32$). Now, multiply the radicands together so that $(\sqrt{10})(\sqrt{15}) = \sqrt{150}$. This gives you the answer $32\sqrt{150}$.

This is not an answer option. You should now try to factor out any perfect squares under the radical. To do this, take the factors of 150 and see if there are any perfect squares. You will find that

$25 \times 6 = 150$, and 25 is a perfect square. This means you can factor 25 out from under the radical in the following way.

$$\begin{aligned} 32\sqrt{150} &= \\ 32\sqrt{25} \times \sqrt{6} &= \\ (32 \times 5)\sqrt{6} &= \end{aligned}$$

Since the square root of 25 is 5, the 5 comes out from under the radical sign.

$$160\sqrt{6}, \text{ answer choice C}$$

When dividing radicals, you use the same three-step process, only you divide instead of multiply.

Sample Questions for Content Domain II

This section has some sample questions for you to try. After you have answered all of the questions, check your answers in the “Answers to the Content Domain II Sample Questions” section that follows. This section will give you the correct answer to each question and show you the appropriate steps for solving the problem.

- 1** The expression $5j(8 + 4j) - 6(3j^2 + 7)$ is equivalent to which of the following?

A $-18j^2 + 40j - 28$
B $2(j^2 + 20j - 21)$
C $4(j^2 - 20j - 42)$
D $4j^2 + 38j + 42$

- 2** Which expression equals $(x^4 y^{-6})(x^2 y^3)$?

A $x^2 y^{-2}$
B $x^2 y^{-9}$
C $x^6 y^{-3}$
D $x^8 y^{-18}$

- 3** What are the factors of the expression $x^2 - 8x$?

A $x(x - 8)$
B $x(1 - 8)$
C $8x(x^2 - 1)$
D $x^2(1 - 8x)$

- 4** What is the product of $\sqrt{7}$ and $\sqrt{2x}$?

A $\sqrt{14x}$
B $\sqrt{9x}$
C $\sqrt[4]{14x}$
D $7\sqrt{x}$

- 5** The expression $\frac{4b^2x + 12ab^2}{16bx - 72ab}$ can be simplified to

A $\frac{b}{5}$
B $\frac{b^2(4x + 12a)}{16x - a}$
C $\frac{b^2x + a}{bx - 72a}$
D $\frac{b(x + 3a)}{2(2x - 9a)}$

Answers to the Content Domain II Sample Questions

1. Answer: **B** Standard: *Manipulate polynomials arithmetically*

A key idea here is *like terms*. After multiplying the two binomials with their respective monomials, like terms must be added or subtracted. Therefore,

$$\begin{aligned} &5j(8 + 4j) - 6(3j^2 + 7) \\ &40j + 20j^2 - 18j^2 - 42 \quad \text{multiplying the binomials} \\ &\quad 2j^2 + 40j - 42 \quad \text{combining the like } j^2 \text{ terms} \\ &2(j^2 + 20j - 21) \quad \text{factoring out the 2 from each term} \end{aligned}$$

This gives you choice **B**, the answer.

2. Answer: **C** Standard: *Simplify expressions with exponents*

When two terms have the same base, you must add the exponents together when the terms are multiplied. The two bases here are x and y , so

$$\begin{aligned} &(x^4y^{-6})(x^2y^3) \\ &x^{(4+2)}y^{(-6+3)} \quad \text{adding the exponents together} \\ &x^6y^{-3} \quad \text{This is answer choice C.} \end{aligned}$$

3. Answer: **A** Standard: *Factor quadratic expressions*

In order to factor the expression $x^2 - 8x$, you must use the Distributive Property of Multiplication over Addition to factor out the similar terms within the binomial. What do $x^2 - 8x$ share? The answer is x , so this can be extracted, leaving you with $x(x - 8)$, choice **A**.

4. Answer: **A** Standard: *Manipulate radicals arithmetically*

Although the radicals can make a problem look confusing, this is little more than a straightforward multiplication problem (the word *product* in the question denotes multiplication). Keeping everything under the radical sign, $\sqrt{7} \times \sqrt{2x} = \sqrt{14x}$. If any perfect squares could be extracted from under the radical sign, this would be the time for it. However, $14x$ has no perfect square factors, so it is the correct answer, choice **A**.

5. Answer: **D** Standard: *Simplify simple algebraic fractions*

To simplify the fraction, you must first find the monomial factors of both the top and bottom polynomials. After that, you can simplify these monomials.

$$\begin{aligned} &\frac{4b^2x + 12ab^2}{16bx - 72ab} \\ &\frac{4b^2(x + 3a)}{8b(2x - 9a)} \quad \text{factoring out the monomials } 4b^2 \text{ and } 8b \\ &\frac{b(x + 3a)}{2(2x - 9a)} \quad \text{simplifying the monomials. This gives you answer D.} \end{aligned}$$

Content Domain III: Solving Equations and Inequalities



A LOOK AT CONTENT DOMAIN III

Test questions in this content domain will measure your understanding of solving a variety of equations and inequalities. Your answers to the questions will help show how well you can perform on the following standards:

- ✧ Solve linear equations
- ✧ Solve problems involving linear equations
- ✧ Solve quadratic equations with factoring
- ✧ Solve a system of linear equations
- ✧ Solve problems using systems of linear equations
- ✧ Solve one-variable inequalities and graph the solution on a number line
- ✧ Solve linear inequalities and graph the solution on a coordinate plane
- ✧ Solve simple rational equations
- ✧ Solve simple radical equations and problems involving radical equations

The first word of every standard above is “Solve.” This should give you a good idea about what you will encounter on Content Domain III problems. This domain will contain many algebraic equations. You will encounter three major types of questions.

Content Domain III Question Types

Type 1: You are given the algebraic equations. These questions will simply provide you with an equation of some kind. Your task will be to find the value of the unknown variable(s). These problems will not have any context, that is, any words describing why the equations are presented. They will simply give you an equation and ask you something like, “What is the value of b in this equation?”

Type 2: You make the algebraic equations. These problems will not have an algebraic equation. Instead, the problem will describe a real-world situation. To solve these problems, you must create (and then solve) the correct algebraic equation that describes the situation mentioned in the problem.

Type 3: You represent the equations visually. These questions start out like Type 1 questions, but you must then translate the equation into a graphic form. Type 3 questions will be easy to recognize, since the answer choices will have either number lines or coordinate planes.



Spotlight on the Standards

✧ Solve Linear Equations ✧

In a linear equation, the exponent of a variable is always 1. This means that $4x + 11 = 79$ is a linear equation, while $4x^2 + 11 = 79$ is not, because the variable x is raised to the second power.

Linear equations are also called *first degree* equations. Look back on page 23 for the definition of degree if you do not understand why the terms “linear” and “first degree” are interchangeable.

Many linear equations require two steps:

- 1) First, isolate the variable and its coefficient on one side of the equation. This step often requires the addition or subtraction of a constant. For example, with $4x + 11 = 79$, you would subtract 11 from both sides.

$$\begin{aligned} 4x + 11 &= 79 \\ 4x + 11 - 11 &= 79 - 11 \\ 4x &= 68 \end{aligned}$$

- 2) For the second step, isolate the variable. This step may involve division or multiplication. For this example, 4 must be divided into both sides in order to isolate x .

$$\begin{aligned} 4x &= 68 \\ \frac{4x}{4} &= \frac{68}{4} \\ x &= 17 \end{aligned}$$

To see if you did this correctly, substitute 17 (called the root) into the original equation.

$$\begin{aligned} 4(17) + 11 &= 79 \\ 68 + 11 &= 79 \end{aligned}$$

This type of checking can help you understand what you did.

You have successfully found the **root**, a value for x , that satisfies the linear equation $4x + 11 = 79$.

Some linear equations will be more difficult than the example listed above because of the arithmetic required. However, if you understand that the goal is to isolate the variable on one side, then you should be able to answer a problem like this one:

In the equation $\frac{8 + 3k}{4} = 23$, what is the value of k ?

- A 5
- B 23
- C 28
- D 31

You can proceed two ways on this problem. You can either multiply each side by 4 in order to eliminate the denominator 4 on the left side on the equation, or you can split up the term $\frac{8+3k}{4}$ into $\frac{8}{4} + \frac{3k}{4}$. As you can see below, both methods lead you to the same result.

Method I

$$\begin{aligned}\frac{8+3k}{4} &= 23 \\ \frac{8+3k}{4} \times 4 &= 23 \times 4 \\ 8+3k &= 92 \\ 8-8+3k &= 92-8 \\ 3k &= 84 \\ \frac{3k}{3} &= \frac{84}{3} \\ k &= 28\end{aligned}$$

Method II

$$\begin{aligned}\frac{8+3k}{4} &= 23 \\ \frac{8}{4} + \frac{3k}{4} &= 23 \\ 2 + \frac{3k}{4} &= 23 \\ 2-2 + \frac{3k}{4} &= 23-2 \\ \frac{3k}{4} &= 21 \\ \frac{3k}{4} \times \frac{4}{3} &= 21 \times \frac{4}{3} \\ k &= \frac{84}{3} \\ k &= 28\end{aligned}$$

The root of $\frac{8+3k}{4} = 23$ is 28. Using either method, the correct answer is C.

When solving linear equations, the goal is to isolate the variable on one side of the equation to find its numerical value, which should be on the other side of the equation, if you do all the arithmetic correctly.

✧ **Solve Problems Involving Linear Equations** ✧

Problems under this standard will fall under the Type 2 category described on page 35. You will be given a description of a problem, and your job will be to create the correct linear equation that describes the problem. You might then be asked to solve this linear equation.

Adele is a local orange grower. She has four regular orchards with the same number of trees in each orchard. Adele also has a smaller orchard with only 11 trees. If Adele has 79 orange trees in all, how many trees are in each regular orchard?

- A 11
- B 15.8
- C 17
- D 18

Creating a linear equation from a written description can be difficult. Many students make it even harder on themselves by trying to jump directly from the description to a completed algebraic equation. A better approach is to start slowly, adding as many intermediate steps as needed in order to fully understand the problem. Look at this first equation:

$$\text{Total number of orange trees Adele has} = 79$$

This is correct, and a nice simple start. Now consider information you know about Adele's orchards.

$$4 \text{ regular orchards} + 1 \text{ small orchard} = \text{Total number of orange trees Adele has}$$

Now combine these two equations to get:

$$4 \text{ regular orchards} + 1 \text{ small orchard} = 79$$

You know the small orchard has only 11 trees. Substitute that value into the equation:

$$4 \text{ regular orchards} + 11 = 79$$

Finally, you need to assign a variable that stands for "number of regular trees in a regular orchard." Let's make this variable x . If x is the number of trees in a regular orchard, and Adele has four regular orchards, then you can write:

$$\begin{aligned} &4 \text{ regular orchards} + 11 = 79 \\ &4 \times (\text{number of trees in each regular orchard}) \\ &+ 11 = 79 \\ &4x + 11 = 79 \end{aligned}$$

Look familiar? This is the same linear equation you solved on page 36, so you should know that $x = 17$. Therefore, Adele has 17 orange trees in each of her regular orchards, making choice C correct.

Most students are not able to read the description of Adele's orchards and then just blurt out, "Oh, that's obviously $4x + 11 = 79$." If you can, good for you. If not, don't sweat it. Start simple, laying out the problem using words. Gradually, you should be able to substitute numbers and variables for words, so that you can go from "Total number of orange trees Adele has = 79" to " $4x + 11 = 79$."



HINT! There's no need to rush when converting a word problem into an algebraic equation. Write out what you know using words, and then gradually replace the words with numbers (from the problem) and variables.

✧ **Solve Quadratic Equations with Factoring** ✧

This section is very similar to the “Factor quadratic expressions” standard found in Content Domain II starting on page 27. The main difference is that these are quadratic *equations*, not just *expressions*, so that they take the basic form:

A quadratic equation can have either 0, 1, or 2 roots. No quadratic equation can ever have more than 2 roots.

$$ax^2 + bx + c = 0 \quad \text{Again, } a \text{ is some number other than zero.}$$

Since the quadratic expression is now an equation, you can now solve the equation to find the roots. Here you will see one way in which quadratic equations differ from linear equations. While linear equations like $4x + 11 = 79$ have only one root, quadratic equations like $10x^2 + 11x + 3 = 0$ can have more than one root. Looking at the three different types of quadratic equations, you can see how to factor *and then solve* these types of equations.

1. Solving quadratic equations that lack a value for c .

These equations take the form $x^2 + bx = 0$. Factor out the x first.

$$2x^2 - 7x = 0$$

$$x(2x - 7) = 0$$

Keep in mind that any number multiplied by zero is zero. Since the right side of the above quadratic equation is equal to zero, either $x = 0$ or $2x - 7 = 0$ or both x and $2x - 7 = 0$. You have one solution for x ($x = 0$) already, and you can solve the linear equation $2x - 7 = 0$ in order to find the second value.

$$2x - 7 = 0$$

$$2x - 7 + 7 = 0 + 7$$

$$2x = 7$$

$$\frac{2x}{2} = \frac{7}{2}$$

$$x = 3\frac{1}{2}$$

The two values of x that satisfy the equation $2x^2 - 7x = 0$ are $x = 0$ and $x = 3\frac{1}{2}$.

These are the roots of the equation.

2. Solving quadratic equations that lack a middle term ($b = 0$).

If $b = 0$, then there is no middle term. These quadratic equations look like the example $x^2 - 49 = 0$. To solve them, first find the factors of both terms.

The factors of the first term are $(x)(x)$. The factors of the second term are $(-7)(7)$. Now combine the first set of factors with the second set of factors to form the following two binomials in the equation:

$$(x - 7)(x + 7) = 0$$

Again, since the product of the two factors equals zero, one or both of these two binomials must equal zero. Setting each binomial into a linear equation like $x - 7 = 0$ and $x + 7 = 0$ will give you the two values of x that satisfy the original quadratic equation. In this case, $x = 7$ and $x = -7$ are the two roots.

3. Solving quadratic equations in which a , b , and c do not equal zero.

These expressions look like $10x^2 + 11x + 3 = 0$. You can see how to factor the expression $10x^2 + 11x + 3$ on page 29. You are left with two binomials so that:

$$10x^2 + 11x + 3 = 0$$

$$(2x + 1)(5x + 3) = 0$$

This gives you two binomials whose product is zero, so one or both of them must equal zero. Solving requires you to place each binomial into a linear equation equal to zero, and then solve to find the correct values. If you do this, you should be able to correctly answer the problem below.

Which values of x satisfy the equation $10x^2 + 11x + 3 = 0$?

A $x = -\frac{1}{2}$ and $x = -\frac{3}{5}$

B $x = 0$ and $x = -\frac{3}{2}$

C $x = \frac{1}{2}$ and $x = 2\frac{1}{3}$

D $x = -10$ and $x = -3$

Since you have already factored the quadratic $10x^2 + 11x + 3 = 0$ into the two binomials $(2x + 1)(5x + 3) = 0$, all that remains is for you to apply your knowledge of solving linear equations.

$$\begin{aligned} 2x + 1 &= 0 \\ 2x + 1 - 1 &= 0 - 1 \\ 2x &= -1 \\ \frac{2x}{2} &= \frac{-1}{2} \\ x &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 5x + 3 &= 0 \\ 2x + 3 - 3 &= 0 - 3 \\ 5x &= -3 \\ \frac{5x}{5} &= \frac{-3}{5} \\ x &= -\frac{3}{5} \end{aligned}$$

The answer is choice A.

✧ Solve a System of Linear Equations ✧

So far, you have only seen linear equations involving one variable. However, a linear equation can have more than one variable, such as $4x - 2y = 1$.

This linear equation has two variables, x and y . The addition of a second variable makes a single answer for this equation uncertain, since both x and y can have an infinite number of solutions.

To see how the variables have an infinite number of values, try $x = -5$, $x = 7$, and $x = 85,345.2$. Once you assign a value for x , you can find a numerical value for y . No matter what real number you pick for x , you can find a value for y that satisfies the equation.

However, if you add another linear equation with the same two variables, then you can actually solve for both x and y . Pairs of linear equations that contain the same two variables are called **systems of linear equations**.

You can solve a linear equation system three different ways:

- 1) by adding or subtracting the equations
- 2) by substitution
- 3) by graphing

Using the equations $3x - 2y = 1$ and $x + 3y = 15$ as an example, each of these techniques will be shown.

1. By adding or subtracting the equations. When solving linear equation systems, you can't find the values for both variables at the same time. Instead, you must find a solution for one of the variables, and then use that value to find a solution for the remaining variable. One way to accomplish this is by adding or subtracting both equations in such a way that one variable is eliminated. To see how this is done, first line up both equations, then add or subtract each like term:


$$\begin{array}{r} 3x - 2y = 1 \\ x + 3y = 15 \\ \hline \end{array}$$

Adding these together would give you $4x + y = 16$, while subtracting them would lead to the equation $2x - 5y = -14$. Neither of these equations helps. Therefore, use multiplication to make the coefficients of one of the variables the same in both equations. For example, you can multiply every term in the lower equation by 3:

$$\begin{array}{r} x + 3y = 15 \\ (3)x + (3)3y = (3)15 \\ 3x + 9y = 45 \end{array}$$

Take this new equation, and place it under $3x - 2y = 1$. Then subtract the equation (subtract every term in the equation). You will see the x term drop out, and you can solve for y .

$$\begin{array}{r}
 3x - 2y = 1 \\
 -(3x + 9y = 45), \text{ which is the same as } -3x - 9y = -45 \\
 \hline
 -11y = -44
 \end{array}$$



$$\begin{array}{r}
 -11y = -44 \\
 \frac{-11y}{-11} = \frac{-44}{-11} \\
 y = 4
 \end{array}$$

Once you have the solution $y = 4$, you can substitute this into either of the original equations to find the solution for x . The equations below show that you will get the roots if you substitute 4 for y in either equation.

$$\begin{array}{r}
 3x - 2y = 1 \\
 3x - 2(4) = 1 \\
 3x - 8 = 1 \\
 3x - 8 + 8 = 1 + 8 \\
 3x = 9 \\
 \frac{3x}{3} = \frac{9}{3} \\
 x = 3
 \end{array}
 \qquad
 \begin{array}{r}
 x + 3y = 15 \\
 x + 3(4) = 15 \\
 x + 12 = 15 \\
 x + 12 - 12 = 15 - 12 \\
 x = 3
 \end{array}$$

2. By substitution. By solving one variable in terms of the other, you can then substitute that value into the second equation and find a solution for the other variable. As with many things algebraic, this sounds much more difficult than it really is.

Let's go back to our original two equations, $3x - 2y = 1$ and $x + 3y = 15$. Take the second equation, $x + 3y = 15$, and solve for x .

$$\begin{array}{r}
 x + 3y = 15 \\
 x + 3y - 3y = 15 - 3y \\
 x = 15 - 3y
 \end{array}$$

You have solved for x in terms of y , that is, you have an equation that gives a value for x in terms of the other variable.

After using an equation to solve for one variable in terms of the other, you must always substitute this value into the *other* equation. To see why, try substituting $x = 15 - 3y$ into the equation $x + 3y = 15$.

Now, if you substitute this solution for x into the second equation, you will find a solution for y .

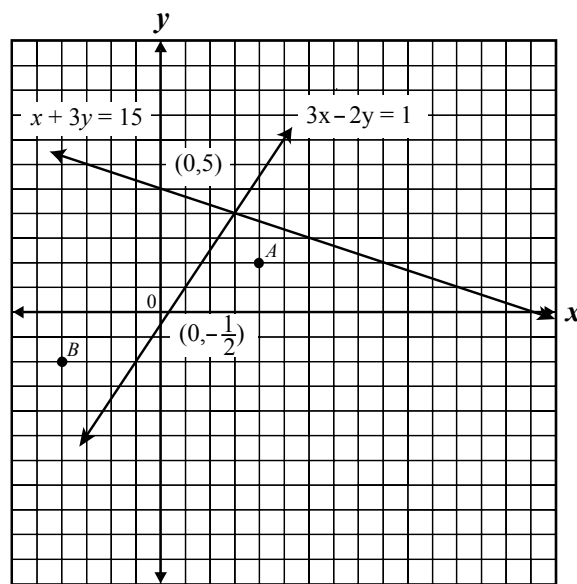
$$\begin{array}{rcl}
 3x - 2y & = & 1 \quad (\text{the other linear equation in the system}) \\
 3(15 - 3y) - 2y & = & 1 \quad (\text{substituting the value of } x \text{ in terms of } y) \\
 45 - 9y - 2y & = & 1 \\
 45 - 11y & = & 1 \\
 45 - 45 - 11y & = & 1 - 45 \\
 -11y & = & -44 \\
 \frac{-11y}{-11} & = & \frac{-44}{-11} \\
 y & = & 4
 \end{array}$$

Now that you have the solution for y , look back to $x = 15 - 3y$. Substituting the value of y into the equation gives you $x = 15 - 3y = 15 - 3(4) = 15 - 12 = 3$, so $x = 3$.

3. By graphing. Solving a linear equation system through graphing requires the use of a coordinate plane like the one shown below. As you probably know, a coordinate plane contains two perpendicular lines. The horizontal line is called the **x-axis**, while the vertical line is the **y-axis**. The point where the two lines meet is called the **origin**, and has the coordinates $(0, 0)$.

On the x -axis, everything to the right of the origin is positive, while everything to the left of it is negative. On the y -axis, everything above the origin is positive, while everything below it is negative.

Using a coordinate plane, you can graph points using the format (x, y) . Point A is at $(4, 2)$, meaning four spaces to the right of the origin on the x -axis and then two spaces up. Point B is at $(-4, -2)$, illustrating the difference between positive and negative values on a coordinate plane. In addition to graphing points, you can also graph equations on a coordinate plane. As the name suggests, *linear* equations form lines when graphed on a plane.



Start with the equation $3x - 2y = 1$. By substituting a numerical value for one variable, you can find a numerical value for the other, and then graph that point. You only need two points to make a line, so after finding two points, you can connect them to draw a line that represents the linear equation $3x - 2y = 1$. It is always a good idea to label the line with the equation you are graphing.

| | |
|--|---|
| $\begin{aligned} \text{If } x &= 0 \\ 3x - 2y &= 1 \\ 3(0) - 2y &= 1 \\ -2y &= 1 \\ y &= -\frac{1}{2} \\ \text{Point 1: } &\left(0, -\frac{1}{2}\right) \end{aligned}$ | $\begin{aligned} \text{If } y &= 0 \\ 3x - 2y &= 1 \\ 3x - 2(0) &= 1 \\ 3x &= 1 \\ x &= \frac{1}{3} \\ \text{Point 2: } &\left(\frac{1}{3}, 0\right) \end{aligned}$ |
|--|---|

Setting $x = 0$ and then $y = 0$ is a common way to find two points for a line. When $x = 0$, the point will be on the y -axis. When $y = 0$, the point will be on the x -axis. Knowing this usually makes the points easier to determine and to graph.

Now take the second equation, $x + 3y = 15$, and find two points by making $x = 0$ one time and $y = 0$ the other. You should come up with the points $(15, 0)$ and $(0, 5)$. Connecting these points gives you the second graph in your linear equation system.

The point where the two lines intersect gives the solution for the system. The two lines meet at point $(3, 4)$, so this means that $x = 3$, and $y = 4$. These are the same solutions you found using the previous two methods.

You now have three different ways to solve any system of linear equations that might appear on the *Algebra I* EOCT. One method is all you will need to solve any single problem, but it's always nice to have options.

✧ **Solve Problems Using Systems of Linear Equations** ✧

Questions under this standard will be in the Type 2 category discussed on page 35. Each question will describe a situation, and it is up to you to translate the words into the appropriate algebraic equations. The plural “equations” is used because this standard involves solving systems of linear equations, so there should be two equations with the same two variables.

A question under this standard might look something like this:

A piggy bank contains 90 coins, all of which are either dimes or nickels. If the contents of the piggy bank total \$7.00, how many dimes are in the piggy bank?

- A** 40
- B** 50
- C** 70
- D** 90

You know the goal is to create two equations with the same two variables. If you decide on what the two variables are, this should help you figure out the two equations. Or, you might write out one equation and then realize you have to use the same two variables to create

another equation. Either method works, since both of them move you towards the goal of two equations with the same two variables.

Think about the first sentence. This could be written as:

$$\text{Number of dimes in piggy bank} + \text{number of nickels in bank} = 90$$

Assigning the variables d (dimes) and n (nickels) to this equation, you have $d + n = 90$.

Now that you have your two variables, figure out a way to make them into another equation.

You have the total dollar amount of the piggy bank as \$7.00, or 700 cents.

Think about how much a dime is worth (10¢), and how much a nickel is worth (5¢), and then create an equation like:

$$(\text{value of dime}) \times (\text{number of dimes in bank, } d) + (\text{value of nickel}) \times (\text{number of nickels in bank, } n) = 700$$

$$10d + 5n = 700$$

You now have your two equations, $d + n = 90$ and $10d + 5n = 700$. Since the question asks “how many dimes,” you might choose to use the substitution method.

Take the first equation, $d + n = 90$, and change it to $n = 90 - d$.

Now substitute this value into the second equation:

$$10d + 5n = 700$$

$$10d + 5(90 - d) = 700$$

$$10d + 450 - 5d = 700$$

$$5d + 450 = 700$$

$$5d + 450 - 450 = 700 - 450$$

$$5d = 250$$

$$\frac{5d}{5} = \frac{250}{5}$$

$$d = 50$$

There are 50 dimes in the piggy bank, which is answer choice B.

STRATEGY BOX – Know the Goal

To create a system of linear equations, you need a pair of linear equations with the same two variables. Therefore, if you can solve one equation, then you have the two variables you need for the second equation. Your overall goal is always two equations with the same two variables.