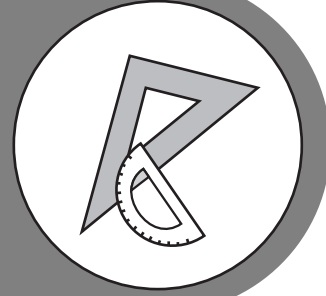


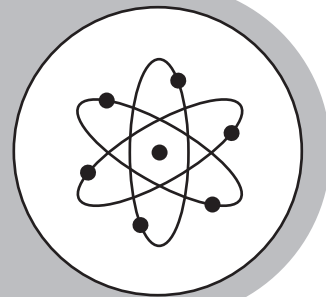
GEOMETRY



Study



Guide



Georgia End-Of-Course Tests



TABLE OF CONTENTS

INTRODUCTION.....	1
HOW TO USE THE STUDY GUIDE.....	2
OVERVIEW OF THE EOCT	4
PREPARING FOR THE EOCT	5
Study Skills	5
Time Management	6
Organization.....	6
Active Participation	7
Test-taking Strategies.....	7
Suggested Strategies to Use to Prepare for the EOCT.....	8
Suggested Strategies to Use the Day Before the EOCT	9
Suggested Strategies to Use the Morning of the EOCT	9
Top 10 Suggested Strategies to Use During the EOCT	10
TEST CONTENT	11
Studying the Content Domains.....	12
Content Domain I: Logic and Reasoning.....	13
Content Domain II: Points, Lines, Planes, and Angles.....	20
Content Domain III: Congruence and Similarity	31
Content Domain IV: Polygons and Circles	45
Content Domain V: Perimeter, Area, and Volume	63
Content Domain VI: Coordinate, Transformational, and Three-dimensional Geometry.....	74
APPENDICES	
APPENDIX A: EOCT Sample Overall Study Plan Sheet.....	87
APPENDIX B: Blank Overall Study Plan Sheet.....	88
APPENDIX C: EOCT Sample Daily Study Plan Sheet	89
APPENDIX D: Blank Daily Study Plan Sheet	90

**This Page is Intentionally
Left Blank**

INTRODUCTION

This study guide is designed to help students prepare to take the Georgia End-of-Course Test (EOCT) for ***Geometry***. This study guide provides information about the EOCT, tips on how to prepare for it, and strategies students can use to perform their best.

What is the EOCT? The EOCT program was created to improve student achievement through effective instruction and assessment of the standards in the Quality Core Curriculum specific to the eight EOCT core high school courses. The EOCT program also helps to ensure that all Georgia students have access to a rigorous curriculum that meets high performance standards. The purpose of the EOCT is to provide diagnostic data that can be used to enhance the effectiveness of schools' instructional programs.

The Georgia End-of-Course Testing program is a result of the A+ Educational Reform Act of 2000, O.C.G.A. §20-2-281. This act requires that the Georgia Department of Education create end-of-course assessments for students in grades nine through twelve for the following core high school subjects:

Mathematics

- Algebra I
- Geometry

Social Studies

- United States History
- Economics/Business/Free Enterprise

Science

- Biology
- Physical Science

English Language Arts

- Ninth Grade Literature and Composition
- American Literature and Composition

Getting started: The HOW TO USE THE STUDY GUIDE section, on page 2, outlines the contents in each section, lists the materials you should have available as you study for the EOCT, and suggests some steps for preparing for the ***Geometry*** EOCT.

**SUGGESTED STEPS FOR USING THIS STUDY GUIDE**

- 1** Familiarize yourself with the structure and purpose of this study guide.
(You should have already read the INTRODUCTION and HOW TO USE THE STUDY GUIDE. Take a few minutes to look through the rest of the study guide to become familiar with how it is arranged.)
- 2** Learn about the test and expectations of performance.
(Read OVERVIEW OF THE EOCT.)
- 3** Improve your study skills and test-taking strategies.
(Read PREPARING FOR THE EOCT.)
- 4** Learn what the test will assess by studying each domain and the strategies for answering questions that assess the standards in the domain.
(Read TEST CONTENT.)
- 5** Answer the sample questions at the end of each domain section. Check your answers against the annotated answers to see how well you did.
(See TEST CONTENT.)

HOW TO USE THE STUDY GUIDE

This study guide is designed to help you prepare to take the **Geometry EOCT**. It will give you valuable information about the EOCT, explain how to prepare to take the EOCT, and provide some opportunities to practice for the EOCT. The study guide is organized into three sections. Each section focuses on a different aspect of the EOCT.

The **OVERVIEW OF THE EOCT** section on page 4 gives information about the test: dates, time, question format, and number of questions that will be on the **Geometry EOCT**. This information can help you better understand the testing situation and what you will be asked to do.

The **PREPARING FOR THE EOCT** section that begins on page 5 provides helpful information on study skills and general test-taking skills and strategies. It explains how to prepare before taking the test and what to do during the test to ensure the best test-taking situation possible.

The **TEST CONTENT** section that begins on page 11 explains what the **Geometry EOCT** specifically measures. When you know the test content and how you will be asked to demonstrate your knowledge, it will help you better prepared for the EOCT. This section also contains specific test-taking strategies for successfully answering questions on the EOCT.

With time, determination, and guided preparation, you will be better prepared to take the **Geometry EOCT**.



GET IT TOGETHER

In order to make the most of this study guide, you should have the following:

Materials:

- ✓ This study guide
- ✓ Pen or Pencil
- ✓ Highlighter
- ✓ Paper
- ✓ Calculator (**optional**)

Resources:

- ✓ Dictionary
- ✓ Geometry textbook
- ✓ A teacher or other adult

Study Space:

- ✓ Comfortable (but not too comfortable)
- ✓ Good lighting
- ✓ Minimal distractions
- ✓ Enough work space

Time Commitment:

- ✓ When are you going to study?
- ✓ How long are you going to study?

Determination:

- ✓ Willingness to improve
- ✓ Plan for meeting goals

OVERVIEW OF THE EOCT

Good test takers understand the importance of knowing as much about a test as possible. This information can help you determine how to study and prepare for the EOCT and how to pace yourself during the test. The box below gives you a “snapshot” of the *Geometry* EOCT.



THE EOCT AT A GLANCE

Administration Dates:

The EOCT will be given three times a year; once in the spring, once in the summer, and once in the winter.

Administration Time:

Each EOCT is comprised of two sections; each section will take 45 to 60 minutes to complete. You will have from 100 to 135 minutes to complete each EOCT. You will be given a 5-minute stretch break between the two sections of the test.

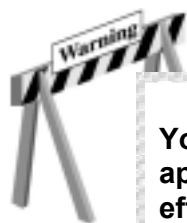
Question Format:

All the questions on the EOCT are multiple choice.

Number of Questions:

Each section of the EOCT contains 45 questions; there are a total of 90 questions on the EOCT.

If you have additional administrative questions regarding the EOCT, please visit the Georgia Department of Education website at www.doe.k12.ga.us, see your teacher, or see your school test coordinator.



PREPARING FOR THE EOCT



WARNING!

You cannot prepare for this kind of test in one night. Questions will ask you to apply your knowledge, not list specific facts. Preparing for the EOCT will take time, effort, and practice.

In order to do your best on the *Geometry* EOCT, it is important that you take the time necessary to prepare for this test and develop those skills that will help you take the EOCT.

First, you need to make the most of your classroom experiences and test preparation time by using good **study skills**. Second, it is helpful to know general **test-taking strategies** to ensure that you will achieve your best score.

Study Skills

A LOOK AT YOUR STUDY SKILLS



Before you begin preparing for this test, you might want to consider your answers to the following questions. You may write your answers here or on a separate piece of paper.

1. How would you describe yourself as a student?
Response: _____
2. What are your study skills strengths and/or weaknesses as a student?
Response: _____
3. How do you typically prepare for a geometry test?
Response: _____
4. Are there study methods you find particularly helpful? If so, what are they?
Response: _____
5. Describe an ideal study situation (environment).
Response: _____
6. Describe your actual study environment.
Response: _____
7. What can you change about the way you study to make your study time more productive?
Response: _____

Effective study skills for preparing for the EOCT can be divided into three categories.

- ◆ **Time Management**
- ◆ **Organization**
- ◆ **Active Participation**



Time Management

Do you have a plan for preparing for the EOCT? Often, students have good intentions for studying and preparing for a test, but without a plan, many students fall short of their goals. Here are some strategies to consider when developing your study plan. (See Appendices A–D for SAMPLE STUDY PLAN SHEETS that you can use to help you create your study plan.)

- ◆ Set realistic goals for what you want to accomplish during each study session and chart your progress.
- ◆ Study during your most productive time of the day.
- ◆ Study for reasonable amounts of time. Marathon studying is not productive.
- ◆ Take frequent breaks. Breaks can help you stay focused. Doing some quick exercises (e.g., sit-ups or jumping jacks) can help you stay alert.
- ◆ Be consistent. Establish your routine and stick to it.
- ◆ Study the most challenging test content first.
- ◆ For each study session, build in time to review what you learned in your last study session.
- ◆ Evaluate your accomplishments at the end of each study session.
- ◆ Reward yourself for a job well done.

Organization

You don't want to waste your study time. Searching for materials, trying to find a place to study, and debating what and how to study can all keep you from having a productive study session. Get organized and be prepared. Here are a few organizational strategies to consider.



- ◆ Establish a study area that has minimal distractions.
- ◆ Gather your materials in advance.
- ◆ Develop and implement your study plan (See Appendices A–D for SAMPLE STUDY PLAN SHEETS).

Active Participation



Students who actively study will learn and retain information longer. Active studying also helps you stay more alert and be more productive while learning new information. What is active studying? It can be anything that gets you to interact with the material you are studying. Here are a few suggestions:

- ◆ Carefully read the information and then **DO** something with it. Mark the important points with a highlighter, circle them with a pen, write notes on them, or summarize the information in your own words.
- ◆ Ask questions. As you study, questions often come into your mind. Write them down and actively seek the answers.
- ◆ Create sample test questions and answer them.
- ◆ Find a friend who is also planning to take the test and quiz each other.

Test-taking Strategies

There are many test-taking strategies that you can use before and during a test to help you have the most successful testing situation possible. Below are a few questions to help you take a look at your test-taking skills.

A LOOK AT YOUR TEST-TAKING SKILLS



As you prepare to take the EOCT, you might want to consider your answers to the following questions. You may write your answers here or on your own paper.

1. How would you describe your test-taking skills?

Response: _____

2. How do you feel when you are taking a test?

Response: _____

3. List the strategies that you already know and use when you are taking a test.

Response: _____


4. List test-taking behaviors you use when preparing for and taking a test that do not contribute to your success.

Response: _____

5. What would you like to learn about taking tests?

Response: _____

Suggested Strategies to Use to Prepare for the EOCT

 **Learn from the Past.** Think about your daily/weekly grades in your mathematics classes (past and present) to answer the following questions.

- In which specific areas of mathematics were you or are you successful?

Response: _____


- Is there anything that has kept you from achieving higher scores?


Response: _____

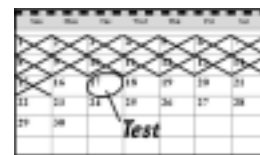
- What changes should you implement to achieve higher scores?

Response: _____

Before taking the EOCT, work toward removing or minimizing any obstacles that might stand in the way of you performing your best. The test preparation ideas and test-taking strategies in this section are designed to help guide you to accomplish this.

 **Be Prepared.** The best way to perform well on the EOCT is to be prepared. In order to do this, it is important that you know what standards/skills will be measured on the **Geometry EOCT** and then practice understanding and using those standards/skills. The standards that will be measured in this EOCT are located in the **Geometry Quality Core Curriculum (QCC)**. The OVERVIEW OF THE EOCT and TEST CONTENT sections of this study guide are designed to help you understand which specific standards are on the **Geometry EOCT** and give you suggestions for how to study the skills for those standards that will be assessed. Take the time to read through this material and follow the study suggestions. You can also ask your mathematics teacher for any suggestions he or she might offer on preparing for the EOCT.

 **Start Now.** Don't wait until the last minute to start preparing. Begin early and pace yourself. By preparing a little bit each day, you will retain the information longer and increase your confidence level. Find out when the EOCT will be administered so you can allocate your time appropriately.



Suggested Strategies to Use the Day Before the EOCT

- ✓ **Review what you learned from this study guide**
 1. Review the general test-taking strategies discussed in the TOP 10 SUGGESTED STRATEGIES TO USE DURING THE EOCT on page 10.
 2. Review the content domain-specific information discussed in the section, TEST CONTENT, beginning on page 11.
 3. Focus your attention on the domain, or domains, that you are most in need of improving.
- ✓ **Take care of yourself**
 1. Try to get a good night's sleep. Most people need an average of 8 hours, but everyone's sleep needs are different.
 2. Don't drastically alter your routine. If you go to bed too early, you might lie in bed thinking about the test. You want to get enough sleep so you can do your best.

Suggested Strategies to Use the Morning of the EOCT



Eat a good breakfast. Eat some food that has protein in it for breakfast (and for lunch if the test is given in the afternoon). Some examples of foods high in protein are peanut butter, meat, and eggs. Protein gives you long-lasting, consistent energy that will stay with you through the test to help you concentrate better. Some people believe it is wise to eat some sugar before a test, claiming it gives them an energy boost. In reality, the energy boost is very short lived, and you actually end up with less energy than before you ate the sugar. Also, don't eat too much. A heavy meal can make you feel tired. So think about what you eat before the test.



Dress appropriately. If you are too hot or too cold during the test, it can affect your performance. It is a good idea to dress in layers, so you can stay comfortable, regardless of the room temperature, and keep your mind on the EOCT.








Arrive for the test on time. Racing late into the testing room can cause you to start the test feeling anxious. You want to be on time and prepared.

TOP 10

Suggested Strategies to Use During the EOCT

These general test-taking strategies can help you do your best during the EOCT.

- 1 Focus on the test.**  Try to block out whatever is going on around you. Take your time and think about what you are asked to do. Listen carefully to all the directions.
- 2 Budget your time.**  Be sure that you allocate an appropriate amount of time to work on each question on the test.
- 3 Take a quick break if you begin to feel tired.** To do this, put your pencil down, relax in your chair, and take a few deep breaths. Then, sit up straight, pick up your pencil, and begin to concentrate on the test again. Remember that each test section is only 45 to 60 minutes.
- 4 Use positive self-talk.** If you find yourself saying negative things to yourself like, “I can’t pass this test,” it is important to recognize that you are doing this. Stop and think positive thoughts like, “I prepared for this test, and I am going to do my best.” Letting the negative thoughts take over can affect how you take the test and your test score.
- 5 Mark in your test booklet.**  Mark key ideas or things you want to come back to in your test booklet. Remember that only the answers marked on your answer sheet will be scored.
- 6 Read the entire question and the possible answer choices.** It is important to read the entire question so you know what it is asking. Read each possible answer choice. Do not mark the first one that “looks good.”
- 7 Use what you know.**  Draw on what you have learned in class, from this study guide, and during your study sessions to help you answer the questions.
- 8 Use content domain-specific strategies to answer the questions.** In the TEST CONTENT section, there are a number of specific strategies that you can use to help improve your test performance. Spend time learning these helpful strategies, so you can use them while taking the test.
- 9 Think logically.** If you have tried your best to answer a question but you just aren’t sure, use the process of elimination. Look at each possible answer choice. If it doesn’t seem like a logical response, eliminate it. Do this until you’ve narrowed down your choices. If this doesn’t work, take your best educated guess. It is better to mark something down than to leave it blank.
- 10 Check your answers.**  When you have finished the test, go back and check your work.

A WORD ON TEST ANXIETY

It is normal to have some stress when preparing for and taking a test. It is what helps motivate us to study and try our best. Some students, however, experience anxiety that goes beyond normal test “jitters.” If you feel you are suffering from test anxiety that is keeping you from performing at your best, please speak to your school counselor who can direct you to resources to help you address this problem.

TEST CONTENT



Up to this point in this study guide, you have been learning various strategies on how to prepare for and take the EOCT. This section focuses on what will be tested. It also includes a section of sample questions that will let you apply what you have learned in your classes and from this study guide.

This section of the study guide will help you learn and review the various mathematical concepts that will appear on the **Geometry EOCT**. Since *geometry* is a broad term that covers many different topics, the state of Georgia has divided it into six major areas of knowledge, called **content domains**. The content domains are broad categories. Each of the content domains is broken down into smaller ideas. These smaller ideas are called **content standards**, or just standards. Each content domain contains standards that cover different ideas related to its content domain. Each question on the EOCT measures an individual standard within a content domain.

UNDERSTANDING THE STANDARDS

One way to think about **content domains** and **standards** is to think about a supermarket. Supermarkets often group similar foods in the same aisles or areas of the store. For example, the section of the store marked “Fresh Fruits” will be a section filled with apples, oranges, and bananas, to name just a few. So the part of the store called “Fresh Fruits” is like the domain name, and all the various items—apples, oranges, bananas—are the standards that fall under that domain.

The six content domains for the **Geometry EOCT** are important for several reasons. Together, they cover the major skills and concepts needed to understand and solve mathematical problems involving geometry. These geometric skills have many practical applications in the real world. Another more immediate reason that the content domains are important has to do with test preparation. The best way to prepare for any test is to study and know the material measured on the test. Since the **Geometry EOCT** covers the six content domains and nothing else, isn't it a good idea to learn as much about these domains as you can? The more you understand about these domains, the greater your opportunity to be successful on the EOCT.

The chart below lists the six content domains for the **Geometry EOCT**.

CONTENT DOMAINS

- I. Logic and Reasoning
- II. Points, Lines, Planes, and Angles
- III. Congruence and Similarity
- IV. Polygons and Circles
- V. Perimeter, Area, and Volume
- VI. Coordinate, Transformational, and Three-dimensional Geometry

Studying the Content Domains

You should plan to study/review the standards for ALL the content domains. To learn what the EOCT will cover, work through this TEST CONTENT section. It is organized by the content domains into the following areas:

- **A Look at the Content Domain:** an overview of what will be assessed in the content domain
- **Spotlight on the Standards:** information about the specific standards that will be assessed (Note: The names of the standards may not be the exact names used by the Georgia Department of Education. Some of the names in this guide may have been modified to reflect the fact that this book is designed for students and not for professional educators.)
- **Sample Questions:** sample questions *similar* to those that appear on the EOCT
- **Answers to the Sample Questions:** in-depth explanations of the answers to the sample questions

Content Domain I: Logic and Reasoning



A LOOK AT CONTENT DOMAIN I

Test questions in this content domain will measure your understanding of logic and reasoning. Your answers to the questions will help show how well you can perform on the following standards:

- * Use inductive and deductive reasoning in a mathematical setting
- * Recognize and use if-then, converse, inverse, and contrapositive statements
- * Use formal or informal logical reasoning
- * Use reasoning to prove conjectures

The *Geometry* EOCT Formula Sheet

Many geometry problems involve the use of formulas, such as $A = s^2$ (the formula for the area of a square). Some—but not all—of these formulas will be listed at the beginning of your ***Geometry* EOCT** test booklet. Although these formulas can be helpful, you should not rely on them exclusively because not every formula you may need is listed on the formula sheet. You will be better prepared if you know a formula and do not need to depend on the formula sheet. Also, formulas at the beginning of the ***Geometry* EOCT** are not defined for the plane figure or solid to which they apply. They are written out like “ $V = \frac{4}{3}\pi r^3$.” This is useful only if you recognize this as the formula for the volume of a sphere.

The other content domains are filled with figures and formulas, but Content Domain I is different. It focuses on the mental reasoning skills needed to understand the figures and formulas encountered in geometry. There won’t be many questions showing triangles and other figures. Instead, you might see a series of mathematical statements and be asked, “What is the MOST logical order for these statements?” This question focuses on the mental reasoning behind the mathematics, not the mathematics itself.

There are also two standards that appear throughout all content domains. These two standards are “Solve problems using appropriate methods” and “Use algebraic skills to solve geometry problems.” These standards are incorporated into other standards. Although they might appear in some questions, there are no questions devoted exclusively to them. For example, using a geometry formula like $A = \pi r^2$ involves variables (an “algebraic skill”), but the question is still primarily a geometry question about the area of a circle.



Spotlight on the Standards

*** Use Inductive and Deductive Reasoning in a Mathematical Setting ***

Suppose you meet a man wearing a large red bowtie. He tells you that he works as an accountant. Two days later, you run into a woman wearing a large red bowtie; you learn that she is also an accountant. The next day your mathematics teacher tells you that Mr. Choi, an accountant, will be a guest speaker.

Picture Mr. Choi in your mind. If you have him wearing a large red bowtie, you are using inductive reasoning. **Inductive reasoning** assumes that something is generally true because individual, specific cases have been true. Since you have seen two specific cases of accountants wearing large red bowties, you used inductive reasoning to make the general conclusion that “all accountants wear large red bow ties.”

Inductive reasoning can be helpful, but conclusions based on inductive reasoning are not always true. Just because two accountants wear large red bowties does not mean all of them do. Since inductive reasoning leads to different types of conclusions, it is not often used in the precise world of mathematics.

Deductive reasoning, on the other hand, uses given information and leads to conclusions based on accepted statements, theorems, definitions, and other facts. Nothing is assumed. By using deductive reasoning about the accountants described above, you could only conclude, “At least two people who are accountants wear large red bowties.” This conforms to the facts given and goes no further than that.

Deductive reasoning is very important in geometry, because it allows mathematicians to prove that certain theorems are true by using a given set of already established facts.

Use deductive reasoning to answer the following question:

In the room, there are people from two countries. Russians are all wearing felt hats. Greeks are all wearing blue shoes. All the Russians use the Cyrillic alphabet. Russians with an *a* or *e* in their first name are from Volgograd. Greeks with a *z* or *y* in their last name are from Corinth.

Using deductive reasoning, what can you conclude about a Greek man in the room who is named Jerzy Koskis?

- A** He does not use the Cyrillic alphabet.
- B** He is from Corinth.
- C** He is wearing blue shoes.
- D** He is from Volgograd.

From the statements you were given, you know that Greeks in the room are all wearing blue shoes. Since Jerzy Koskis is Greek, he must be wearing blue shoes—choice C. Although his first name has a z and a y , his last name doesn't, so you cannot conclude he is from Corinth.

*** Recognize and Use If-Then, Converse, Inverse, and Contrapositive Statements ***

Everyone uses if-then statements at some point. You might say something like, “If the tickets are free, then I will go to the concert.” This if-then statement shows how an action will occur (“going to the concert”) only if certain conditions are true (“the tickets are free”).

Many geometric theories and postulates are constructed using the “if-then” model. The statement “If two points lie in a plane, then the line created by joining these points is also in that plane” uses the if-then format to show how the second “then” part is true as long as the first “if” part is true. This approach is important when constructing geometric proofs. You will not be asked to create geometric proofs on the test, but you will be expected to know the basic if-then format.

You can form the inverse, converse, and contrapositive of an if-then statement. The statement “If the tickets are free, then I will go to the concert” can be abbreviated with the use of variables to “If t , then g .” The converse of this statement reverses the parts that follow “if” and “then.”

Converse: If I go to the concert, then the tickets are free.
If g , then t .

The inverse of the statement keeps the original if-then form but negates each part that follows “if” and “then.” In written form, this is usually accomplished by adding the word *not* to the statement or removing *not* if it appears in the original statement.

Inverse: If the tickets are not free, then I will not go to the concert.”
If $\sim t$, then $\sim g$.

The contrapositive of the statement is a combination of both the converse and inverse. The parts that follow “if” and “then” are reversed and negated.

Contrapositive: If I will not go to the concert, then the tickets are not free.
If $\sim g$, then $\sim t$.

Learn all these terms. You will then be able to answer a question that gives an if-then statement and asks you to modify it into its converse, inverse, or contrapositive form.

*** Use Formal or Informal Logical Reasoning ***

Putting a series of related events in the correct order gives you practice in using logical reasoning skills. Read the following statements and put them in the correct order according to the events as they occurred.

- A I bought a small set of twelve acrylic paints.
- B Our teacher gave us an assignment to make a poster.
- C I went to the nearby art store.
- D I decided to make a poster of a rose garden.

Which statement describes the first event that occurred? It makes the most sense logically to have statement B come first. If you put D in front of B, you would have a student deciding on a project before the project is assigned. After B and D, you have statements A and C. It makes more sense to have the student *go* to the art store (C) and *then* buy the acrylic paints (A). If you place these the other way around, you have an illogical sequence of events. Therefore, the logical sequence of events is B, D, C, A.

Since the **Geometry EOCT** is a mathematics test, an appropriate question about logical reasoning is appropriate, like this one:

The measures of three angles in a triangle are 54, 65, and x degrees. The goal is to find the value of x . The following statements are parts of a proof. Which statement should be first?

- 1. $119 + x = 180$
- 2. $x = 61$
- 3. $54 + 65 + x = 180$
- 4. $119 - 119 + x = 180 - 119$

- A 1
- B 2
- C 3
- D 4

The answer would not be Statement 2, which is the value of x , the last statement of the proof. Statement 3 sets up the equation, and 1 and then 4 would follow it. Therefore, C is the correct answer.

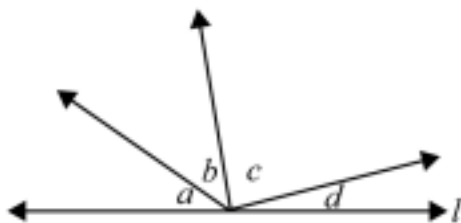
Understanding how to set up and solve an algebraic equation is a large part of this problem. This shows one way in which the standard “Use algebraic skills to solve geometry problems” appears in other standards.

*** Use Reasoning to Prove Conjectures ***

Although similar to the previous standard—in the sense that both test your reasoning skills—these questions often resemble a “Find the missing link” problem. You will be given a statement as a conjecture. You will need certain facts in order to prove that this conjecture is correct. The question will provide you with some of the facts, but there will always be one missing fact that you must find.

To find the correct missing link, you need to use your reasoning skills to determine the facts you have and what fact you need in order to prove that the conjecture is correct.

Suppose you are given four adjacent angles with a common vertex on line l , like the figure below. Given $m\angle d = 20^\circ$, prove that $m\angle c = 80^\circ$.



You have one fact at your disposal, $m\angle d = 20^\circ$. In order to prove that $m\angle c = 80^\circ$ (the conjecture), which of the following four facts is needed?

1. $m\angle a + m\angle b = m\angle c$
2. $m\angle a + m\angle b + m\angle c = 160^\circ$
3. $m\angle a = 35^\circ$
4. $m\angle b = 45^\circ$

Using parts 3 and 4 *together* would help you find the answer, but on a multiple-choice test like the **Geometry EOCT**, you need a single answer. The first choice provides the missing link, since

$$\begin{aligned}
 m\angle a + m\angle b + m\angle c + m\angle d &= 180^\circ && \text{(since there are 180 degrees in a straight angle)} \\
 m\angle a + m\angle b + m\angle c + 20 &= 180 \\
 m\angle a + m\angle b + m\angle c + 20 - 20 &= 180 - 20 \\
 m\angle a + m\angle b + m\angle c &= 160 \\
 \text{and since } m\angle a + m\angle b &= m\angle c, \text{ you have} \\
 m\angle c + m\angle c &= 160 \\
 2m\angle c &= 160 \\
 \frac{2m\angle c}{2} &= \frac{160}{2} \\
 m\angle c &= 80^\circ
 \end{aligned}$$

STRATEGY BOX – Complete the Puzzle

Think of this type of question as a puzzle. Identify the pieces of the puzzle. You have to find the piece that is missing.

The missing link of $m\angle a + m\angle b = m\angle c$ allowed you to prove that the conjecture is correct.

Sample Questions for Content Domain I

This section has some sample questions for you to try. After you have answered all of the questions, check your answers in the “Answers to the Content Domain I Sample Questions” section that follows. This section will give you the correct answer to each question and show you the appropriate steps for solving the problem.

- 1** Using the information given below, what is the sum of the first nine consecutive positive odd numbers?

- The sum of 1 and 3 is 4.
- The sum of 1, 3, and 5 is 9.
- The sum of 1, 3, 5, and 7 is 16.
- The sum of 1, 3, 5, 7, and 9 is 25.

- A 49
- B 64
- C 81
- D 100

- 2** Given the conjecture below, which statement is a step in the proof of the conjecture?

If $a + b = 180$, $c + d = 180$, and $a = c$, then $b = d$.

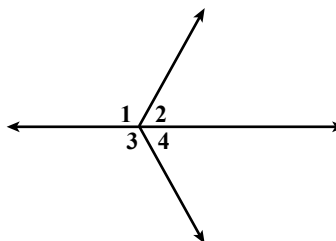
- A $a + b + c + d = 180$
- B $a + b = c + d$
- C $a = d$
- D $a = b$

- 3** What is the inverse of the following statement?

If the shape is a triangle, then the interior angles sum to 180° .

- A If the interior angles sum to 180° , then the shape is a triangle.
- B If the shape is a triangle, then the interior angles do not sum to 180° .
- C If the interior angles do not sum to 180° , then the shape is not a triangle.
- D If the shape is not a triangle, then the interior angles do not sum to 180° .

- 4** Marcy drew the diagram below with $\angle 2 \cong \angle 4$.



Which of the following is a logical conclusion about her figure?

- A $\angle 1$ and $\angle 2$ are complementary
- B $\angle 2$ and $\angle 4$ are complementary
- C $\angle 1 \cong \angle 3$
- D $\angle 1 \cong \angle 4$

Answers to the Content Domain I Sample Questions

1. Answer: **C** Standard: *Use inductive and deductive reasoning in a mathematical setting*

The text inside the box shows the sum of consecutive positive odd numbers from $1 + 3$ to $1 + 3 + 5 + 7 + 9$. These sums show a pattern—the sum of consecutive positive odd numbers is always a perfect square. (The sum of the first 2 consecutive positive odd numbers $= 2^2$, first 3 $= 3^2$, first 4 $= 4^2$, and first 5 $= 5^2$.) Therefore, the sum of the first 9 positive odd numbers is the same as 9^2 , choice **C**.

Another way to answer the question could be to add the first nine consecutive positive odd numbers: $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = 81$.

2. Answer: **B** Standard: *Use reasoning to prove conjectures*

This question asks you to “find the missing link.” Without that missing link, the conjecture wouldn’t be able to be proved true. First identify the “if” parts of the statement. These include $a + b = 180$, $c + d = 180$, and $a = c$. Next identify the “then” part of the statement: $b = d$. Now identify the missing link. Since we are told that $a + b = 180$ and $c + d = 180$ in the “if” statement, then we know that $a + b = c + d = 180$. Option **B** is $a + b = c + d$, which is necessarily true and would be able to be used when proving the conjecture.

3. Answer: **D** Standard: *Recognize and use if-then, converse, inverse, and contrapositive statements*

If you review the definitions of converse, inverse, and contrapositive statements on page 15, you will see that option **A** is the converse, option **C** is the contrapositive, and option **D** is the inverse. You might have thought that option **B** was the inverse because of the word “not” in the *then* portion of the statement. Remember that the inverse negates both the *if* and the *then* portions of the statement.

4. Answer: **C** Standard: *Use formal or informal logical reasoning*

Complementary angles sum to 90° , and there is no information given to show that the sum of $\angle 1$ and $\angle 2$ would equal 90° . This eliminates choice **A**. Choice **B** can also be eliminated, since there is no information given that the sum of $\angle 2$ and $\angle 4$ would equal 90° . This leaves **C** and **D**. **D** is impossible to prove with the information that is given. However, since the unnamed ray bisects angles 2 and 4, it could be proven that an opposite ray bisects angles 1 and 3. An angle bisector divides an angle into two congruent angles. Therefore $\angle 1 \cong \angle 3$, and the correct answer is choice **C**.

Content Domain II: Points, Lines, Planes, and Angles



A LOOK AT CONTENT DOMAIN II

Test questions in this content domain will measure your understanding of geometric terms. Your answers to the questions will help show how well you can perform on the following standards:

- ★ Identify fundamental geometric terms
- ★ Understand and identify various angles
- ★ Define properties associated with several fundamental geometric terms
- ★ Recognize angles created when a transversal crosses two lines
- ★ Apply basic facts about common geometric terms

Some geometric shapes—triangles, squares, rectangles, and trapezoids, for example—are groups of *lines* and *angles* lying in the same *plane*. Therefore, before you learn about these shapes, let's first look at the basic building blocks of geometry: points, lines, planes, and angles. Once you learn about these terms, you will use them to identify and define the many geometric shapes that will be discussed in Content Domains III, IV, V, and VI.



Spotlight on the Standards

★ Identify Fundamental Geometric Terms ★

The basic building blocks of geometry are *point*, *line*, and *plane*. The three ideas are accepted without definitions that rely only on geometric terms. Other geometric shapes can be explained, defined, and proven by referring to these terms.

Point, line, and plane are sometimes called *undefined terms* because the meaning of these terms is accepted without definition.

Fundamental Geometric Terms

1. Points. A point marks only a position in space. A point has no width, length, or height. It is merely a marker showing location. Graphically, points are represented by a dot and labeled with a capital letter. In the diagram below, A , B , C , D , E , and F are all points.

2. Lines. A line contains infinitely many points, is straight, and extends indefinitely in two directions. A line has length, but no width or height. Lines are named in two ways:

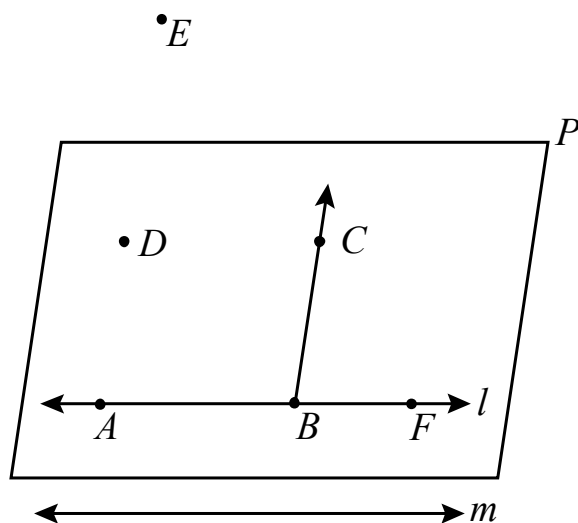
- 1) By using lowercase variables, like lines l and m
- 2) By identifying two points on a line and placing a \leftrightarrow over them.

Using this notation, line l may also be described as the line passing through Points A and F and written as \overleftrightarrow{AF} .

3. Planes. A plane represents a flat surface that extends indefinitely in all directions. This means a plane has infinite length and width, but no height. Imagine a sheet of paper—with no thickness—that extends outward in every direction. That paper represents a plane. The graphic representation of a plane is usually a four-sided figure with a capital letter, such as plane P .

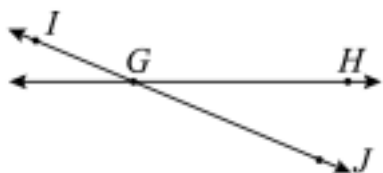
4. Line segments. Lines extend indefinitely, but line segments do not. A segment consists of its endpoints and all points on the line between these end points. Segments are denoted by their endpoints and by the symbol $\overline{\quad}$ above them. For example, \overline{AB} , \overline{BF} , and \overline{AF} are all line segments. The order of the endpoints is unimportant, so line segment \overline{AB} is the same as \overline{BA} .

5. Rays. A ray is part of a line with a single endpoint. A ray starts at its endpoint and continues indefinitely in one direction. Rays are denoted using the symbol \rightarrow , indicating the endpoint and one other point on the ray. \overrightarrow{BC} is an example of this. The endpoint of \overrightarrow{BC} is always placed under the non-arrow part of the ray symbol, since this shows the direction the ray is traveling.



After learning these terms, you will be better able to answer this question:

Name a ray with endpoint at I .



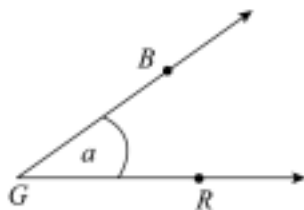
- A \overrightarrow{GI}
- B \overrightarrow{IJ}
- C \overrightarrow{IJ}
- D \overrightarrow{IG}

Only A and C use ray symbols, so choices B and D can be eliminated. Option A can be eliminated because point G is the endpoint of this ray. Choice C correctly names endpoint I first, and then names one other point on the ray. Choice C is the correct answer.

★ Understand and Identify Various Angles ★

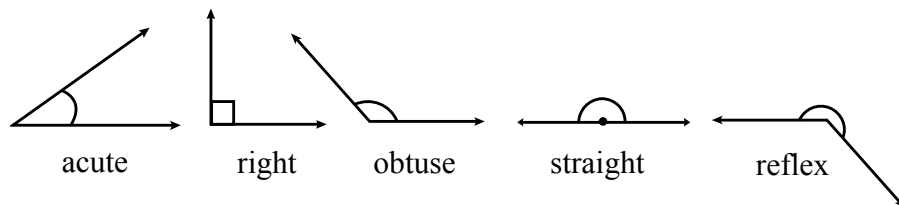
Two rays with the same endpoint form an **angle**. In an angle, the shared endpoint is called the **vertex**. The two rays are called the **sides** of the angle.

Angles are denoted using the symbol \angle . Variables and numbers can be used to name angles, but you can also use three points, such as $\angle BGR$. When referring to points, the middle point is always the vertex of the angle. The other two points are each on opposite sides of the angle.



Angles are measured in degrees. Using the measure of an angle, you can classify it into one of five categories: acute, right, obtuse, straight and reflex.

Acute angles have measures greater than 0° and less than 90° . **Right** angles have measures equal to 90° . They are usually designated with a \square at the vertex. **Obtuse** angles have measures between 90° and 180° . A **straight** angle has a measure equal to 180° . A **reflex** angle has a measure greater than 180° and less than 360° .



Angle measurements are not the only way to classify angles. The following list shows other terms that are used when discussing angles.

Terms Commonly Used With Angles

1. Complementary. Two angles are complementary if the sum of their measures is 90° . In the diagram below, $\angle a$ and $\angle b$ are complementary because $m\angle a + m\angle b = 90^\circ$.

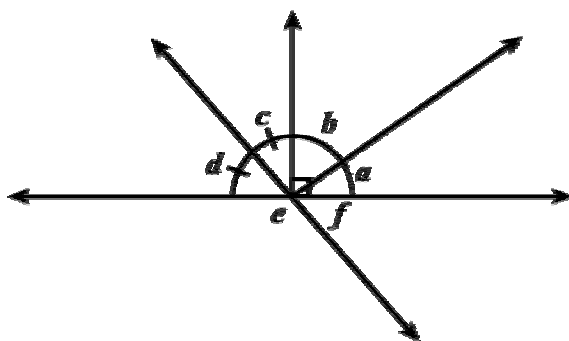
2. Supplementary. Two angles are supplementary if the sum of their measures is 180° . In the diagram, $\angle e$ and $\angle f$ are supplementary because $m\angle e + m\angle f = 180^\circ$.

Although the complementary and supplementary angles shown in the diagram are next to one another, this does not have to be the case. Two angles do not have to be touching to be complementary or supplementary. They just need to sum to the correct amount.

3. Adjacent. Adjacent angles have a common vertex and side but have no interior points in common. Pairs of adjacent angles in the diagram are $\angle a$ and $\angle b$, $\angle b$ and $\angle c$, . . .

4. Vertical. Vertical angles are formed when two lines intersect to form 4 angles. The angles that are not adjacent are vertical angles. In the diagram, $\angle d$ and $\angle f$ are vertical angles.

5. Congruent. Congruent means “the same as.” Congruent angles are angles that have the same measure. They are denoted by the symbol \cong . In the diagram, $\angle c$ and $\angle d$ are congruent ($\angle c \cong \angle d$). Congruent angles are shown with a short slash (also known as a tic mark).



STRATEGY BOX – Follow the Rules

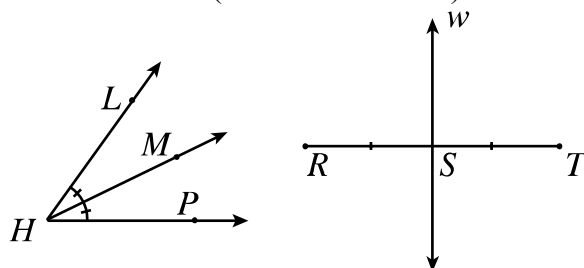
Some geometry questions will present you with a diagram and then give you the measure of one or more angles. Your job will be to find the measure of another angle in the diagram. This can be done if you apply the definitions of complementary, supplementary, adjacent, congruent, and vertical angles.

★ Define Properties Associated with Several Fundamental Geometric Terms ★

The first two standards define basic geometric terms like lines and angles. This standard focuses on some of their properties. For example, consider the prefixes *co-* and *non-*. The first prefix, *co-*, means “together” or “jointly,” while the prefix *non-* means “not.” Using these definitions, the term **collinear** means “existing jointly on the same line,” while **noncollinear** describes points *not* on the same line. Similarly, the term **coplanar** describes points within the same plane, while **noncoplanar** describes points *not* in the same plane.

Look at the diagram on page 21 to see some examples of these properties. Points *A*, *B*, and *F* are all collinear (Line *l*). Points *A*, *B*, and *C* are noncollinear. Points *D*, *C*, and *F* are all coplanar, but Points *D*, *C*, *F*, and *E* are noncoplanar.

The term **bisector** can refer to both angles and line segments. An angle bisector is a ray that divides an angle into two congruent angles. (See page 23 if you have forgotten the meaning of congruent.) In the following illustration, \overline{HM} bisects $\angle LHP$, forming two congruent angles, $\angle LHM$ and $\angle MHP$ ($\angle LHM \cong \angle MHP$).



Now that you know what the bisector of an angle is, it is easier to determine what a bisector of a line segment does—it cuts a segment into two equal-length segments. Line *w* above is a bisector of segment \overline{RT} , dividing it into congruent segments \overline{RS} and \overline{ST} . The point at which a bisector divides a segment is called the **midpoint**, in this case Point *S*. Point *S* is an equal distance from endpoints *R* and *T*.

The term bisector can refer to an angle or line segment. Similarly, the terms parallel and perpendicular can refer to lines or planes. **Parallel** lines are lines in the same plane that never intersect. Parallel lines are written as $a \parallel b$.

If two lines intersect to form a right angle, these lines are **perpendicular**. In the diagram on page 25, lines *c* and *a* are perpendicular. This is written as $c \perp a$.

The terms parallel, perpendicular, and intersecting have the same meaning for planes as they do for lines. Parallel planes never meet, and intersecting planes create a line.

If you learn these geometric properties, you will be better able to answer a term-heavy question such as the following:

If \overline{RB} bisects $\angle QRS$ and if $m\angle BRS = 45^\circ$, then which of the following must be true?

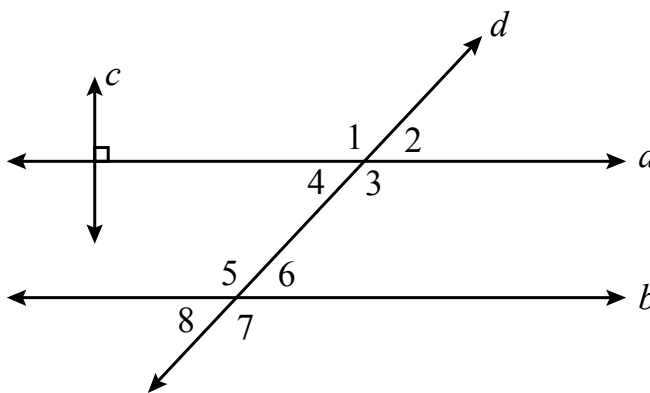
- A $\angle QRS$ is a straight angle.
- B $\angle QRS$ is a right angle.
- C $m\angle BRS = (\frac{1}{2}) m\angle QRB$
- D $m\angle QRB = 135^\circ$

Drawing a diagram will probably help you find the right answer. Try to use the facts given to make your diagram as accurate as possible. In this case, since $m\angle BRS = 45^\circ$, you should try to draw $\angle BRS$ as close to 45° as you can.

Since \overline{RB} bisects $\angle QRS$, then $m\angle QRB = m\angle BRS$. Since $m\angle BRS = 45^\circ$, then $m\angle QRB = 45^\circ$. Because $m\angle QRS = m\angle QRB + m\angle BRS$, then $m\angle QRS = 45^\circ + 45^\circ = 90^\circ$. Therefore, $\angle QRS$ is a right angle.

★ Recognize Angles Created When a Transversal Crosses Two Lines ★

Line d also crosses lines a and b , but not at a right angle. Line d is called a **transversal**, which is a line that crosses two lines at different points on those lines.



When a transversal crosses two lines, eight different angles are formed. The following chart shows all these angles and their respective names.

Angle Descriptions When a Transversal Crosses Two Lines

Name	Meaning	Example Angles
Interior angles	inside the two lines	3, 4, 5, 6
Exterior angles	outside the two lines	1, 2, 7, 8
Same-side angles	on the same side of the transversal	1, 4, 5, 8 or 2, 3, 6, 7
Alternate angles	nonadjacent angles on opposite sides of the transversal	1 and 7, 2 and 8, 3 and 5, 4 and 6
Corresponding angles	angles that appear in the same location relative to the four angles at a vertex	1 and 5, 2 and 6, 3 and 7, 4 and 8

The terms in the chart can also be combined. This gives you the following pairs of angles:

Alternate exterior angles	Alternate interior angles
$\angle 1$ and $\angle 7$	$\angle 4$ and $\angle 6$
$\angle 2$ and $\angle 8$	$\angle 3$ and $\angle 5$
Same-side interior angles	Same-side exterior angles
$\angle 4$ and $\angle 5$	$\angle 1$ and $\angle 8$
$\angle 3$ and $\angle 6$	$\angle 2$ and $\angle 7$

These terms are a mouthful, but they are not too difficult to understand since the words *exterior*, *interior*, *alternate*, and *same* should be familiar to you.

Things get even more interesting if the two lines cut by a transversal are parallel. If it is given that lines *a* and *b* in the diagram on page 25 are parallel, sets of congruent and supplementary angles are created. In this figure, $\angle 1$, $\angle 3$, $\angle 5$, and $\angle 7$ are congruent, and $\angle 2$, $\angle 4$, $\angle 6$, and $\angle 8$ are congruent. Furthermore, in this diagram, the sum of any one odd-numbered angle and one even-numbered angle is 180° . This second part might be more obvious for angles like $\angle 5$ and $\angle 6$, but it is also true for $\angle 1$ and $\angle 8$.

A transversal cutting across two parallel lines creates eight different angles. Learn the names for the different pairs of angles formed. When two parallel lines are cut by a transversal, all the acute angles formed are congruent, all the obtuse angles formed are congruent, and any acute angle is supplementary to any obtuse angle.

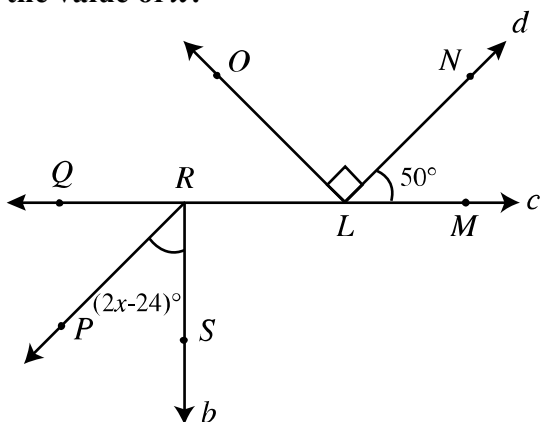
★ Apply Basic Facts About Common Geometric Terms ★

Now that you've learned all these facts about basic geometric terms, how well can you apply them? This standard is a transition between understanding a concept and using that concept to solve a problem.

The following problem takes you on a mini-journey of geometric terms. Stay on course, work one step at a time, and you will find the answer.

Try what you have learned by answering this question:

In the figure below, lines b and c are perpendicular. If $\angle OLQ \cong \angle QRP$, then what is the value of x ?



- A 37
- B 40
- C 50
- D 74

Line c —also called a straight angle ($\angle QLM$)—equals 180° . Since $m\angle NLM = 50^\circ$ and $\angle OLN$ is a right angle, you can find the measure of $\angle OLQ$ by the following steps:

$$\begin{aligned} m\angle OLQ + m\angle OLN + m\angle NLM &= 180^\circ \\ m\angle OLQ + 90 + 50 &= 180 \\ m\angle OLQ + 140 &= 180 \\ m\angle OLQ + 140 - 140 &= 180 - 140 \\ m\angle OLQ &= 40^\circ \end{aligned}$$

It is given that $\angle OLQ$ and $\angle QRP$ are congruent, so $m\angle QRP = 40^\circ$ as well. Because lines b and c are perpendicular, then $m\angle QRP + m\angle PRS = 90^\circ$.

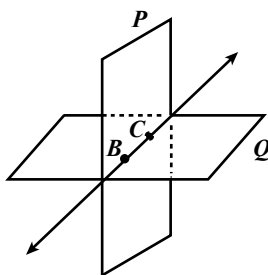
$$\begin{aligned} m\angle QRP + m\angle PRS &= 90^\circ \\ m\angle QRP + (2x - 24) &= 90 \\ 40 + (2x - 24) &= 90 \\ 40 - 40 + (2x - 24) &= 90 - 40 \\ (2x - 24) &= 50 \\ 2x - 24 &= 50 \\ 2x - 24 + 24 &= 50 + 24 \\ 2x &= 74 \\ \frac{2x}{2} &= \frac{74}{2} \\ x &= 37, \text{ choice A} \end{aligned}$$

Your knowledge of congruence, perpendicular lines, and straight angles, helps you to find the correct value for x .

Sample Questions for Content Domain II

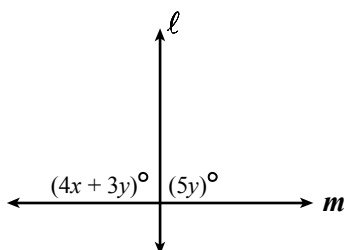
This section has some sample questions for you to try. After you have answered all of the questions, check your answers in the “Answers to the Content Domain II Sample Questions” section that follows. This section will give you the correct answer to each question and show you the appropriate steps for solving the problem.

- 1** What is the intersection of the two planes, P and Q , shown in the figure below?



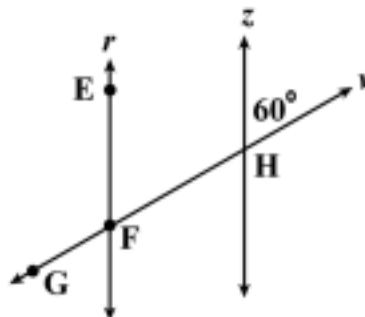
- A \overline{BC}
- B BC
- C \overleftrightarrow{BC}
- D \overline{BC}

- 2** If $\ell \perp m$, what is the value of x ?



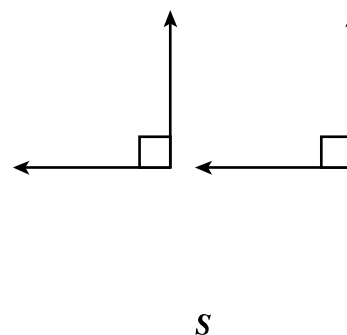
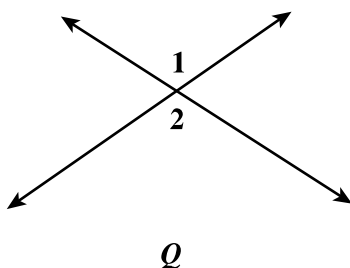
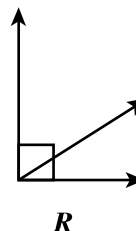
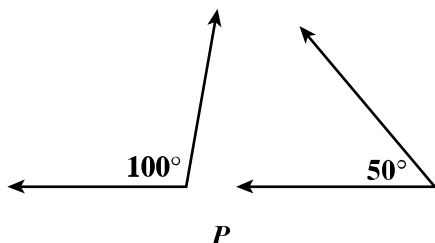
- A 9
- B 12
- C 15
- D 18

- 3** In the figure below, lines r and z are parallel. What is the measure of $\angle EFG$?



- A 30°
- B 60°
- C 120°
- D 180°

4 Which figure below shows a pair of marked supplementary angles?



- A** *P*
- B** *Q*
- C** *R*
- D** *S*

Answers to the Content Domain II Sample Questions

1. Answer: **C** Standard: *Identify fundamental geometric terms*
 Although planes are often depicted on paper as four-sided figures, planes have infinite length and width. This means the intersection extends along the line containing Points *B* and *C*. There are no endpoints for the intersection. This means choices **A**, **B**, and **D** are all incorrect because they indicate end points. Choice **C**, which describes a line, is correct. It is also what is shown in the diagram, because there are arrow signs on both ends of line *BC*.

2. Answer: **A** Standard: *Apply basic facts about common geometric terms*
 Since $l \perp m$, four right angles are formed at the intersection of these lines. Since all right angles are equal to 90° , $5y = 90$.

$$\begin{array}{r} 5y = 90 \\ \underline{5y \quad 90} \\ 5 \quad 5 \\ y = 18 \end{array}$$

With this value for y , you can now solve the equation $4x + 3y = 90$, for x .

$$\begin{aligned}4x + 3y &= 90 \\4x + 3(18) &= 90 \\4x + 54 &= 90 \\4x + 54 - 54 &= 90 - 54 \\4x &= 36 \\\frac{4x}{4} &= \frac{36}{4} \\x &= 9\end{aligned}$$

The answer is **A**.

3. Answer: **C** Standard: *Recognize angles created when a transversal crosses two lines*

When two parallel lines are cut by a transversal, many pairs of congruent and supplementary angles are created. The 60° angle is congruent to its corresponding angle, $\angle EFH$. Since $\angle EFH$ and $\angle EFG$ are a supplementary angle pair, the following is true:

$$\begin{aligned}m\angle EFH + m\angle EFG &= 180^\circ \\60 + m\angle EFG &= 180 \\60 - 60 + m\angle EFG &= 180 - 60 \\m\angle EFG &= 120^\circ\end{aligned}$$

The answer is **C**.

4. Answer: **D** Standard: *Understand and identify various angles*

Students who know that the term supplementary refers to two angles whose sum is 180° should be able to find the right answer on this problem. The two right angles in S are the correct choice, since $90^\circ + 90^\circ = 180^\circ$. Q is incorrect because these two angles are complementary, not supplementary.