Content Domain III: Congruence and Similarity

A LOOK AT CONTENT DOMAIN III

Test questions in this content domain will measure your understanding of congruence and similarity. Your answers to the questions will help show how well you can perform on the following standards:

- Identify congruent figures
- Identify congruent triangles using basic congruence theorems
- Use concepts of congruence to test triangles
- Identify similar figures
- Use ratio and proportion with similar polygons
- Use proportion with parallel lines
- Solve problems involving similar polygons
- Compare area and volume of similar figures

Congruent figures have the same size and shape. Determining whether two figures are congruent is accomplished in one of two ways. It can be done visually, as you look at one figure and then spin, flip, or rotate it in your mind to determine whether it is the same size and shape as another figure. The other way is to use the knowledge about angles and sides that you learned in Content Domain II to prove mathematically that two figures are congruent.

Similar figures are proportionally the same. This means that the angle measures are the same, but the sides of a similar figure are proportionally larger or smaller. For this reason, ratios and proportions are applied to these problems.

Domain III will address congruence and similarity in depth, with specific examples. You will also have an opportunity to apply your algebra skills in a geometric context.

Spotlight on the Standards

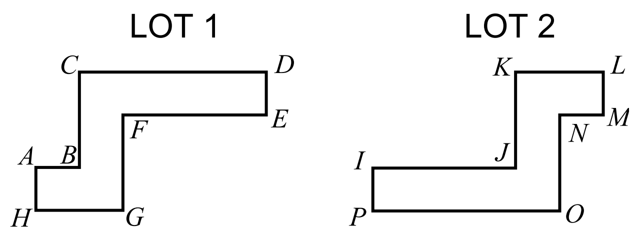
✧ Identify Congruent Figures ✧

Look at a capital letter R. When it is rotated, you probably still recognize it, especially if you tilt your head. Comparing the “flipped” R to the original, it appears as though the bottom of the letter changed places with the top of the letter. Place a “reversed” R in a mirror, and you can see the original R in the reflection.

Since the shape or size of the R never changes, all the above Rs are congruent. The only thing that changes is the position of the R relative to the page, since it is rotated, flipped, and reversed.

If you can rotate, flip, and reverse geometric shapes as easily as you can a capital letter R, you will be able to answer the questions in this standard. After you find the two congruent shapes, you might be asked to match a congruent side to another, or to find congruent angles in the figures. The following question is an example of this type of problem.

The figures below show two congruent parking lots.



Which segment of Lot 2 is congruent to segment *AH* in Lot 1?

A $\overline{DE}$
B $\overline{IP}$
C $\overline{NM}$
D $\overline{ML}$

You might think that the answer is choice B, since $\overline{IP}$ appears to be in the same position as $\overline{AH}$. However, Lot 2 is Lot 1 rotated 180°. This makes $\overline{LM}$, or $\overline{ML}$, the segment that is congruent to $\overline{AH}$. D is the correct answer.

> **STRATEGY BOX – Move the Figure**
> When comparing two figures for congruence, take the first figure and rotate, flip, and/or reverse it in order to determine whether it is the same shape as the second figure.

✧ *Identify Congruent Triangles Using Basic Congruence Theorems* ✧

There are several different theorems that can be used to show that two triangles are congruent. They are often mentioned using only their abbreviated names, like SAS. (S stands for *side* and A stands for *angle*.) Below is an overview of some congruence theorems. You may want to refer to your geometry textbook for a more detailed explanation.

Congruence Theorems for Triangles

1. SSS Theorem. If each side of one triangle is congruent to the corresponding side of a second triangle, then the two triangles are congruent.

2. SAS Theorem. If two sides of one triangle are congruent to the corresponding sides of another triangle, and if the angles between these sides are also congruent, then the two triangles are congruent.

3. ASA Theorem. If two angles of one triangle are congruent to the corresponding angles of another triangle, and if the sides between these angles are also congruent, then the two triangles are congruent.

4. AAS Theorem. If two angles of one triangle are congruent to the corresponding angles of another triangle, and if a side not between these angles is also congruent to its corresponding side in the other triangle, then the two triangles are congruent.

These theorems work for all triangles. There are also theorems for right triangles, which are triangles with one right angle. The side opposite the right angle is called the **hypotenuse**, and the other two sides are the **legs** of a right triangle.

Here is a chart showing the corresponding parts needed to show congruence in right triangles.

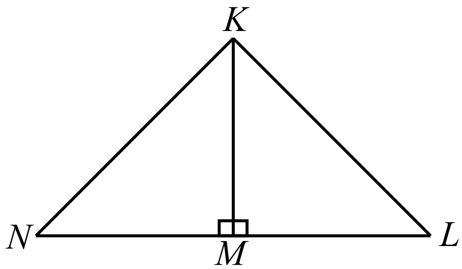
Congruence Theorems for Right Triangles

Theorem	Corresponding Congruent Parts Needed for Congruence
HL	hypotenuse and one leg
HA	hypotenuse and one acute angle
LL	both legs
LA	one leg and one acute angle

STRATEGY BOX – What's the Angle?

Since all right triangles share a congruent angle (the right angle), the theorems are special versions of those that apply to all triangles. For example, the LL Theorem of a right triangle is another way of stating the SAS Theorem, because the included angle of the LL Theorem is the right angle.

In order to prove $\Delta KMN \cong \Delta KML$ using the LA Theorem, what other piece of information would you need?



A $\overline{KN} \cong \overline{KL}$
B $\overline{NM} \cong \overline{ML}$
C $\angle KNM \cong \angle KLM$
D $\angle KMN \cong \angle KML$

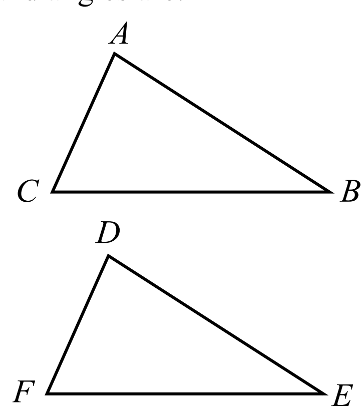
The LA Theorem requires that the two right triangles have a pair of corresponding congruent legs and a pair of corresponding congruent acute angles. Since both triangles share $\overline{KM}$, it is corresponding and congruent to itself. This eliminates A and B (although with choice A you could have proved congruency using the HL Theorem). Choice D is the right angle, but choice C is the acute angle. Therefore, C is the answer.

✧ Use Concepts of Congruence to Test Triangles ✧

For the previous standard, you had to gather enough facts to prove that two triangles were congruent. For this standard, the process is reversed: If you know that two triangles are congruent to start with, what facts do you know about their sides and angles?

For congruent triangles, you know that corresponding sides and angles are the same in each triangle. This means you must pay very careful attention to the order of the points of the triangle. The order determines which sides and angles are congruent to one another. For example, for ΔABC and ΔDEF, corresponding sides and angles are:

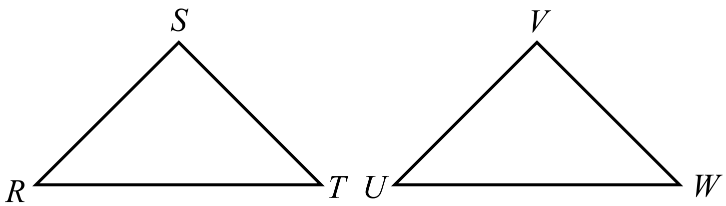
First point to second point:	$\overline{AB}$ and $\overline{DE}$
Second point to third point:	$\overline{BC}$ and $\overline{EF}$
Third point to first point:	$\overline{CA}$ and $\overline{FD}$
Angle with vertex at first point:	$\angle CAB$ and $\angle FDE$
Angle with vertex at second point:	$\angle ABC$ and $\angle DEF$
Angle with vertex at third point:	$\angle BCA$ and $\angle EFD$



Other than these pairs of corresponding sides and angles, there is no way to know if any other pair of angles or sides is congruent. You could not say, "angle *CAB* is congruent to angle *DEF*," since that would be comparing an angle from the first triangle with a *non-corresponding* angle from the second triangle.

The questions may also ask you to analyze information about congruent triangles, like this one:

In the figure below, $\Delta RST \cong \Delta UVW$. Which of the following statements may NOT be true?



A $\angle SRT \cong \angle TRS$
B $\angle SRT \cong \angle WUV$
C $\overline{RS} \cong \overline{VU}$
D $\overline{SR} \cong \overline{VW}$

STRATEGY BOX – Order Matters

With congruent triangles, *corresponding* sides and angles are congruent. Always pay close attention to the order in which points are written for two congruent triangles, since this order tells you which sides and angles are corresponding.

Choice A has the same point, *R*, at the vertex of both angles, so this is just the same angle written two different ways. Choice B names angles whose vertices are both listed first in the congruence statement, so these are corresponding angles and are therefore congruent. Choice B is true. Choice C names sides whose endpoints are the first two vertices named in the congruence statement, so these are corresponding sides and are therefore congruent. Choice C is true. However, Choice D compares non-corresponding segments on each triangle, so it may not be true. D is the answer.

✧ *Identify Similar Figures* ✧

Similar figures have corresponding sides that are in proportion. If you have a $45°\text{-}45°\text{-}90°$ right triangle with sides of length $1{:}1{:}\sqrt{2}$, it is similar to a $45°\text{-}45°\text{-}90°$ right triangle with sides of length $50{:}50{:}50\sqrt{2}$. These two triangles are not congruent because the lengths of the sides of the second triangle are 50 times larger than the lengths of the sides of the first one. However, the two triangles are similar because corresponding angles are congruent and corresponding sides are proportional.

Setting up ratios and finding corresponding side proportions is the key to understanding similarity.

Set up a ratio and answer this question:

An illustration in a textbook shows a triangle with sides of 8 cm, 12 cm, and 16 cm. Which of the following triangles is similar to the triangle in the textbook?

A A triangle with sides of 2 cm, 3 cm, and 4 cm
B A triangle with sides of 12 cm, 24 cm, and 32 cm
C A triangle with sides of 20 cm, 30 cm, and 32 cm
D A triangle with sides of 16 cm, 20 cm, and 28 cm

The ratio of the sides of the textbook triangle is 8:12:16. The correct answer will have three values in the same ratio. Looking at choice D, you can see that the first value, 16 cm, is twice the corresponding side, 8 cm. If the rest of the sides are twice the length of the textbook triangle, then this is the right answer.

8:12:16	original ratio
$(8\times2):(12\times2):(16\times2)$	multiplying by 2
16:24:32	

The ratio in choice D is 16:20:28, so even though the length of the first side is correct, the other two are not. This means the triangle in choice D is not similar.

For choice A, the length of the first side is one-fourth of 8 cm. If the other two sides are proportionally one-fourth, then A is the right answer.

8:12:28	original ratio
$\frac{8}{4}:\frac{12}{4}:\frac{16}{4}$	dividing by 4
2:3:4	

The sides of the triangle in choice A are one-fourth the length of the sides of the triangle in the textbook. The triangles are similar, and A is the right answer.

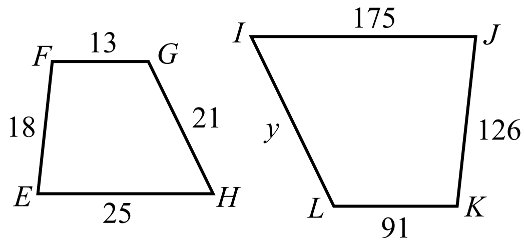
✧ *Use Ratio and Proportion with Similar Polygons* ✧

A **polygon** is a closed plane figure whose sides are line segments. Polygons have at least three sides, but they may have more. Since triangle similarity was addressed already, this standard will focus on similar figures with more than three sides.

Your basic approach to similar figures will be much like your work with triangles. The key is to find the correct proportion between the corresponding sides of the two figures, and then set up ratios to find the lengths of the unknown sides. It is important to line up the corresponding sides correctly. After that, the rest is arithmetic with a dash of algebra.

Set up a proportion to answer this question:

The two trapezoids below are similar. What is the length of *y*?



A 76.44 units
B 108 units
C 147 units
D 283 units

To find the correct proportion between the two trapezoids, you need to match corresponding sides. If you think 13 and 175—segments $\overline{FG}$ and $\overline{IJ}$, respectively—are corresponding sides, then you are ignoring the fact that the second trapezoid is upside down relative to the first. You would be comparing the "top" of the first trapezoid with the "bottom" of the second one. In fact, the corresponding sides are:

Ratios of Corresponding Sides

First Trapezoid	$FG = 13$	$EH = 25$	$EF = 18$	$GH = 21$
Second Trapezoid	$LK = 91$	$IJ = 175$	$JK = 126$	$IL = y$

The ratios are all the same, so it doesn't matter which one you use to solve for *y*. For example, take the first ratio, $\frac{FG}{LK}$, and use it to solve for *y*.

$\frac{FG}{LK} = \frac{GH}{IL}$ setting up the proportion

$\frac{13}{91} = \frac{21}{y}$ substituting the values

$91 \times 21 = 13y$ cross-multiplying the ratios

$1911 = 13y$

$\frac{1911}{13} = \frac{13y}{13}$

$147 = y$

This is choice C.

Since ratios of corresponding sides in similar figures are the same, you did not have to use the $\frac{FG}{LK}$ ratio. Starting with equation $\frac{EH}{IJ} = \frac{GH}{IL}$ gives you the same answer, $y = 147$.

STRATEGY BOX – Set Up Ratios

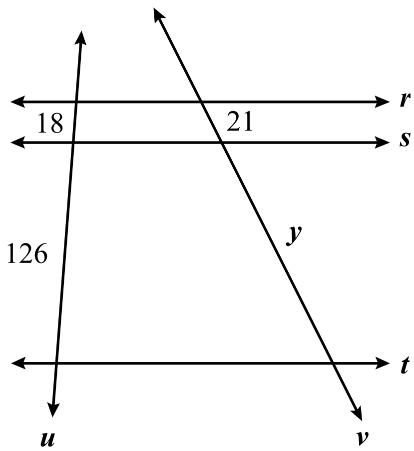
Like similar triangles, similar polygons all have corresponding sides in the same ratio. Setting up the right proportions will help you solve for any unknown values, regardless of how many sides the similar polygon has.

✧ *Use Proportion with Parallel Lines* ✧

These problems cover parallel lines and transversals, so you might see several lines in the diagrams of these problems. You will need to know the following theorem: If three parallel lines intersect two transversals, then they divide the transversals proportionally.

Using the theorem above, set up a proportion and find the answer to this question:

In the figure below, line *r* || line *s* || line *t*. Lines *u* and *v* are transversals.



What is the length of *y*?

A 76.44
B 108
C 147
D 283

Notice that there are two trapezoids, one above the other. The top trapezoid has non-parallel sides of 18 and 21, while the larger, lower trapezoid has corresponding sides of 126 and y. Place these into a proportion:

$$\frac{\text{shorter side of higher trapezoid}}{\text{shorter side of lower trapezoid}} = \frac{\text{longer side of higher trapezoid}}{\text{longer side of lower trapezoid}}$$

$\frac{18}{126} = \frac{21}{y}$ substituting the corresponding lengths from the figure

$(126)(21) = 18y$ cross-multiplying the ratios

$2646 = 18y$

$\frac{2646}{18} = \frac{18y}{18}$

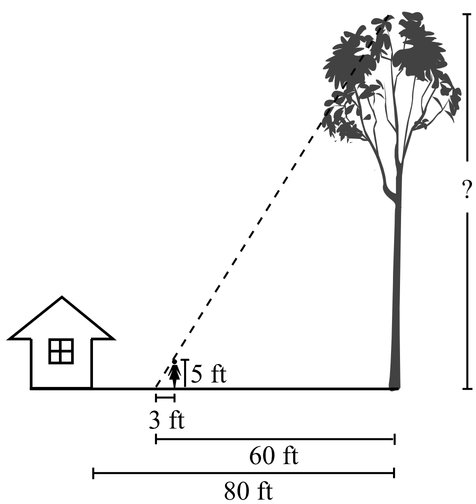
$147 = y$

Since the trapezoids in the last two problems have the same corresponding sides, it should be no surprise that you came up with same answer. The same trapezoids were used to show that even though the diagrams may be different, the same approach is used to solve both problems. Keep this in mind if you find yourself faced with a diagram that looks bewildering to you at first. If you are working with similar figures, all you need to do is find corresponding sides and set up a ratio. This ratio will enable you to create a proportion that you can use to solve for any unknown values.

✧ *Solve Problems Involving Similar Polygons* ✧

The phrase "solve problems" typically means that these questions will include a real-world element to them. Even so, you will still use the same approach that you used on the last several standards. The real-world element in the following question shows you how solving for the unknown length in similar polygons can have useful applications in everyday life.

There is a large tree 80 feet from a house. The owners are worried that the tree might hit their house if it fell. On a sunny day, the tree casts a 60-foot shadow at the same time as a 5-foot tall woman casts a 3-foot shadow.



What is the approximate height of the tree?

A 60 ft
B 80 ft
C 100 ft
D 120 ft

Using shadows and height, you can create the following proportion and then substitute the numbers you know:

$$\frac{\text{height of woman}}{\text{height of tree}} = \frac{\text{length of woman's shadow}}{\text{length of tree's shadow}}$$

Using the proportion from page 39, substitute the values given in the problem.

$$\frac{5}{t} = \frac{3}{60}$$

$$5 \times 60 = 3t$$

$$300 = 3t$$

$$\frac{300}{3} = \frac{3t}{3}$$

$$100 = t$$

The tree and the woman create two similar triangles. The woman and her shadow create two legs of a triangle with the dotted line as the hypotenuse. The tree and its shadow are the two legs of the other right triangle, with the dotted line as the hypotenuse again. These are the two similar triangles, or polygons, in this problem.

The tree is 100 feet tall, so the answer is C.
Since it is only 80 feet from the house, it would definitely hit the house if it fell directly towards it.

✧ *Compare Area and Volume of Similar Figures* ✧

Look back to the bottom of page 35. The sides of the second triangle are 50 times the length of the sides of the first. With this number, you might conclude that the area of the second triangle would be 50 times larger than the smaller triangle.

Once you do the arithmetic, however, you realize that this assumption is incorrect. Since the formula for the area of a triangle is A = $\frac{1}{2}bh$, you actually have:

Area of Smaller Triangle	Area of Larger Triangle
$A = \frac{1}{2}bh$	$A = \frac{1}{2}bh$
$A = \frac{1}{2} \times 1 \times 1$	$A = \frac{1}{2} \times 50 \times 50$
A = 0.5 square units	A = 1,250 square units

Since the legs of a right triangle are perpendicular, their lengths are the base and height of the triangle.

The area of the larger triangle is much more than 50 times greater than the area of the smaller triangle. It is 2,500, or 50^2, times as great. To compare the areas of two similar figures, square the ratio formed by comparing a pair of corresponding sides. This is stated in the formula:

$$\text{Ratio of the Areas of Similar Figures} = \left(\frac{\text{length of a side in the first figure}}{\text{length of the corresponding side in the second figure}}\right)^2$$

The following problem shows you what happens when you compare the volumes of prisms whose faces are similar polygons. Approach this problem the same way as the right triangle. Use the volume formula on both shapes and then find the ratio.

A cardboard box in the shape of a rectangular prism has the width and length of 2 cm and a height of 4 cm. A second similar box has dimensions twice the size of the first. What is the ratio of the volume of the first box to the volume of the second box?

A 1:2
B 1:4
C 1:8
D 1:16

If the second box has dimensions twice the size of the first, then the dimensions of the boxes are:

First Box	Second Box
2 cm by 2 cm by 4 cm	4 cm by 4 cm by 8 cm

The formula for the volume of a rectangular prism is $V = lwh$. Comparing the volumes of the two boxes, you get

Volume of First Box	Volume of Second Box
$V = lwh$	$V = lwh$
$V = 2 \times 2 \times 4$	$V = 4 \times 4 \times 8$
$V = 16 \text{ cm}^3$	$V = 128 \text{ cm}^3$

The ratio of the volumes is 16:128. Dividing each number by 16, you have 1:8 which is choice C. Since each edge in the larger box is 2 times as long as the corresponding edge in the smaller box, the volume of the larger box is 2^3, or 8 times as great.

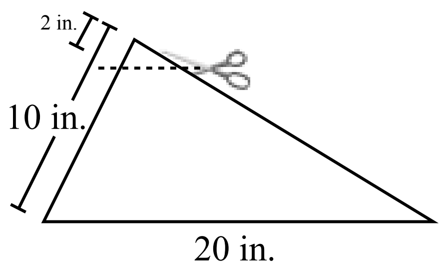
Therefore, while the ratio of areas is the *square* of the ratio of two corresponding sides for polygons, the ratio of volumes is the *cube* of the ratio of two corresponding sides for prisms and other 3-dimensional figures with faces that are similar polygons. In formula form, you can write:

$$\text{Ratios of the Volumes of Similar Solids} = \left(\frac{\text{length of edge in the first solid}}{\text{length of corresponding edge in the second solid}}\right)^3$$

Sample Questions for Content Domain III

This section has some sample questions for you to try. After you have answered all of the questions, check your answers in the “Answers to the Content Domain III Sample Questions” section that follows. This section will give you the correct answer to each question and show you the appropriate steps for solving the problem.

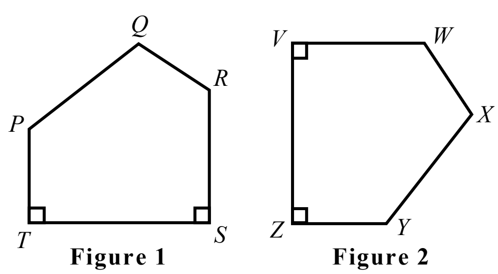
1 **Amy cut the top off of a triangle as shown in the figure below. The cut was parallel to the base and 2 inches from the top as measured along the 10 inch side.**



How long is the cut if the base of the triangle was 20 inches?

A 4 inches
B 5 inches
C 6 inches
D 10 inches

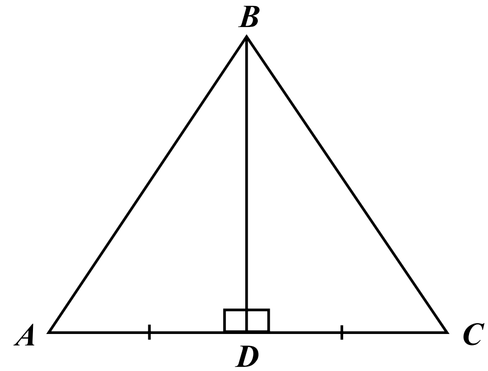
2 **In the diagram below, Figures 1 and 2 are congruent.**



Which segment in Figure 2 is congruent to segment $\overline{PT}$?

A $\overline{ZV}$
B $\overline{YZ}$
C $\overline{WX}$
D $\overline{VW}$

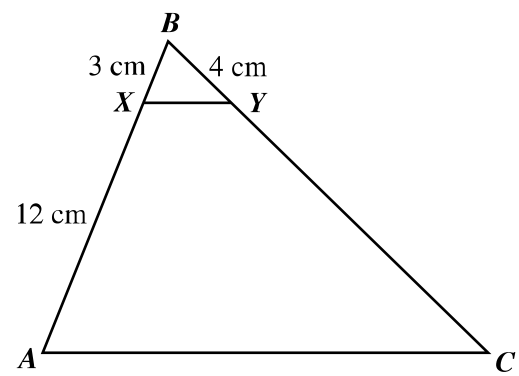
3 **Greg drew and labeled the triangles shown below.**



Which theorem proves $\Delta ABD \cong \Delta CBD$?

A Side–Side–Side (SSS)
B Side–Angle–Side (SAS)
C Angle–Side–Angle (ASA)
D Hypotenuse–Leg (HL)

4 **In the figure below, $\overline{XY} \parallel \overline{AC}$.**



What is the length of $\overline{BC}$?

A 13 cm
B 15 cm
C 16 cm
D 20 cm

Answers to the Content Domain III Sample Questions

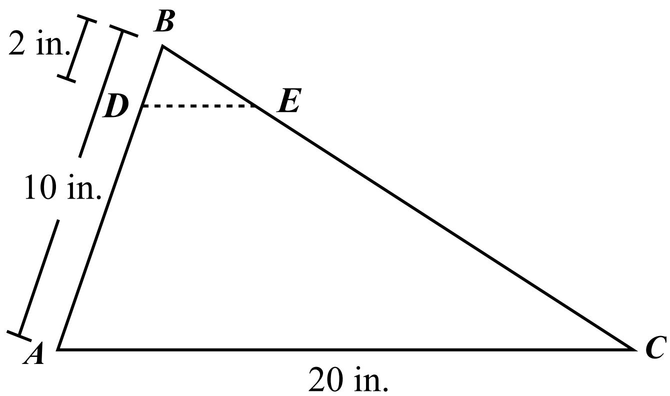
1. Answer: **A** Standard: *Solve problems involving similar polygons*

Cutting across the top of the triangle creates two similar triangles, or polygons. To find the length of the cut, you need to set up a proportion as follows:

$$\frac{\text{side in smaller triangle}}{\text{corresponding side in larger triangle}} = \frac{\text{base of smaller triangle}}{\text{base of larger triangle}}$$

Assigning labels for each point might help you understand the proportion. The figure below shows the two triangles with points labeled. Notice that side $\overline{DB}$ corresponds to side $\overline{AB}$ and side $\overline{DE}$ corresponds to side $\overline{AC}$. You should be able to see that $\triangle ABC \sim \triangle DBE$. Using these points, you can rewrite the above proportion to read:

$$\frac{DB}{AB} = \frac{DE}{AC}$$



Substituting the lengths of the sides, you get the following proportion:

$$\frac{2}{10} = \frac{DE}{20}$$

$$(20)(2) = (10)(DE)$$

$$40 = (10)(DE)$$

$$\frac{40}{10} = \frac{(10)(DE)}{10}$$

$$4 = DE$$

The cut was four inches across, choice **A**.

2. Answer: **B** Standard: *Identify congruent figures*

There are two right angles in Figure 1. One segment is between the two right angles. Two line segments come up from the two right angles. In Figure 1, $\overline{PT}$ is the shorter of these two segments. In Figure 2, then, you must find the shorter side that corresponds to $\overline{PT}$. This shorter segment is $\overline{ZY}$ or $\overline{YZ}$, choice **B**.

Another approach is to take Figure 1, rotate it 90˚ clockwise, and then flip the figure along its horizontal axis. This would give you Figure 2, and you would see that $\overline{PT}$ corresponds to $\overline{YZ}$.

3. Answer: **B** Standard: *Identify congruent triangles using basic congruence theorems*

The key here is to understand that both triangles share side *BD*. Since they are right triangles, and you know that $\overline{AD} \cong \overline{DC}$, you could look for the LL Theorem because you know that both legs are congruent. However, the LL Theorem is not one of the choices listed. Even so, using the right angles that both triangles have, you have two sides and the angle between them. This is the SAS Theorem, choice **B**.

4. Answer: **D** Standard: *Solve problems involving similar polygons*

There are two ways to answer this problem. You can find $\overline{YC}$, and then add it to $\overline{BY}$, or you can just find $\overline{BC}$. Since the second approach eliminates an intermediate step, searching for $\overline{BC}$ is a simpler approach.

To set up a proportion, you have $\Delta XBY \sim \Delta ABC$.

This states that the small triangle at the top is similar to the entire large triangle. Since corresponding sides of similar figures are in proportion, the following ratios can be set equal to one another.

$$\frac{BX}{BA} = \frac{BY}{BC}$$

$$\frac{3}{3+12} = \frac{4}{BC}$$

$$\frac{3}{15} = \frac{4}{BC}$$

$$4 \times 15 = 3 \times BC$$

$$60 = 3 \times BC$$

$$\frac{60}{3} = \frac{3 \times BC}{3}$$

$$20 = BC$$

This is choice **D**.

Content Domain IV: Polygons and Circles

A LOOK AT CONTENT DOMAIN IV

Test questions in this content domain will measure your understanding of polygons and circles. Your answers to the questions will help show how well you can perform on the following standards:

- Classify basic triangles and polygons
- State and use three common angle theorems
- State and apply properties to different quadrilaterals
- Apply the concept of inequality to segments and angles in triangles
- Test quadrilateral properties involving diagonals and line of symmetry
- Use the Pythagorean Theorem and its converse
- State the properties of two special right triangles
- Identify sine, cosine, and tangent values for right triangles
- Solve problems using sine, cosine, and tangent ratios
- Identify parts and properties of circles
- Solve problems using various relationships associated with circles

There are many different geometric shapes, and they all have specific names. Learning the names of shapes is the focus of Content Domain IV. To do this, you will use the vocabulary of the last two domains to define the different terms. Words like *parallel* (from Domain II) and *congruent* (from Domain III) will be found in many definitions, such as the definition of a rhombus.

Rhombus: a four-sided polygon with all four sides congruent and opposite sides parallel

In addition to using terms you have already seen, there will also be new ideas introduced. This is especially true of the standards dealing with circles. You might want to use a small notebook to create your own list of “Shapes and their Definitions.” A list like this will help you study and learn the different shapes.

Spotlight on the Standards

✦ *Classify Basic Triangles and Polygons* ✦

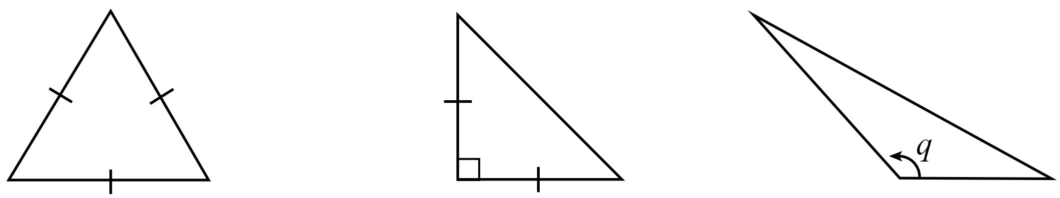
The word *triangle* describes a three-sided polygon. To be more specific, mathematicians created several different terms to describe various kinds of triangles. Triangles can be classified by their angles or sides.

Different Types of Triangles by Angles

Acute. An acute triangle has three interior angles that are all less than 90 degrees. Triangle 1 below is an example of an acute triangle.

Right. A right triangle—like Triangle 2 below—has one 90-degree angle as an interior angle.

Obtuse. An obtuse triangle has one interior angle greater than 90 degrees. This is $\angle q$ in obtuse Triangle 3.



Triangle 1 Triangle 2 Triangle 3

Using the angle definitions is one way to describe these three triangles. You can also use the lengths of their sides to describe these triangles.

Different Types of Triangles by Sides

Equilateral. As the prefix *equi*- suggests, an equilateral triangle has three sides of equal length. Triangle 1 is an example of an equilateral triangle. Note that an acute triangle may or may not be equilateral.

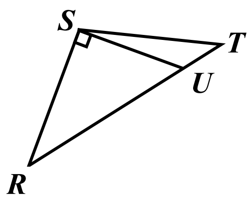
Isosceles. The old phrase "two out of three ain't bad" can be used to remember the number of equal sides in an isosceles triangle. At least two sides of an isosceles triangle are of equal length, like Triangle 2. For this reason, an equilateral triangle is also an isosceles triangle.

Scalene. No sides are equal in a scalene triangle. Triangle 3 is an example of a scalene triangle.

As you can see, triangles can be described in more than one way. You can have combinations, too, such as an *isosceles obtuse triangle* or *isosceles right triangle.* Not every combination works, but you can give an accurate description of triangles by using terms for both the angles and sides.

This question includes words to clearly describe the diagram.

In the figure below, $\overline{RS} \cong \overline{ST}$ and $\angle RSU$ is a right angle. What is the MOST specific way to describe ΔRST?



A right isosceles
B right scalene
C obtuse isosceles
D acute isosceles

You know that two out of three sides of ΔRST are congruent since $\overline{RS} \cong \overline{ST}$. This makes ΔRST an isosceles triangle, so choice B can be eliminated. In the figure, $m\angle RSU$ is 90°, and this means that $m\angle RST$ is greater than 90° as shown below.

$$m\angle RST = m\angle RSU + m\angle UST$$

$$m\angle RST = 90° + m\angle UST$$

Angles greater than 90° are obtuse, so ΔRST can best be described as an obtuse isosceles triangle, choice C.

✦ *State and Use Three Common Angle Theorems* ✦

The first angle theorem is called the **Interior Sum Theorem**. It states:
The sum of the measures of the three interior angles of a triangle always equals 180°.

The theorem is useful if you have a problem where two of the three angles in a triangle are given. You can use the Interior Sum Theorem to find the third angle, and this fact will often point you toward the right answer.

If you combine the Interior Sum Theorem with the description of triangles from the previous standards, you can answer a question like the following:

Triangle *ABC* is an equilateral triangle. What is another way to describe this triangle?

A right
B obtuse
C acute
D scalene

To answer this question, you need to know that the angles opposite congruent sides of a triangle are congruent. Since all three sides of an equilateral triangle are congruent, then all three interior angles of the triangle must be equal as well. Calling this angle *g*, we know that

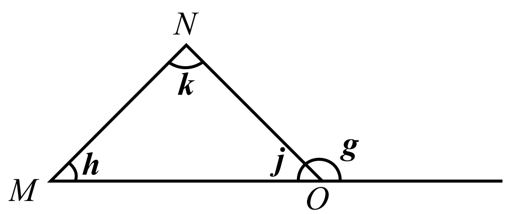
$$g + g + g = 180° \qquad \text{Interior Sum Theorem}$$
$$3g = 180°$$
$$\frac{3g}{3} = \frac{180°}{3}$$
$$g = 60°$$

Every angle inside the equilateral triangle is 60°, which is acute. The correct answer is therefore C.

STRATEGY BOX –
Apply the Triangle Properties

Applying the Congruent Sides–Congruent Angles Theorem to an isosceles triangle, you know that two angles inside the triangle will be congruent to each other since two sides of the triangle are congruent. For a scalene triangle, all three interior angles must be unequal since all sides of a scalene triangle are not equal.

If the side of a triangle is extended, an angle adjacent to the interior angle is formed. This is an exterior angle of the triangle. In the figure, $\angle g$ is the exterior angle formed when side $\overline{MO}$ is extended.



Since $\angle g$ and $\angle j$ form a linear pair, these two angles are supplementary.

$$\angle g + \angle j = 180°$$

From the Interior Sum Theorem, you know that

$$m\angle j + m\angle h + m\angle k = 180°$$

This can be rewritten as

$$m\angle j = 180° - m\angle h - m\angle k$$

Substituting this value of $m\angle j$ into the original equation (about supplementary angles), you have:

$$m\angle g + (180° - m\angle h - m\angle k) = 180°$$
$$m\angle g + 180° - 180° - m\angle h - m\angle k = 180° - 180°$$
$$m\angle \mathrm{g} - m\angle h - m\angle k = 0°$$
$$m\angle g = m\angle h + m\angle k$$

The terms convex and concave are sometimes used to describe polygons. In a **convex** polygon, any line segment connecting two interior points always stays entirely inside the polygon. Think of a common red stop sign. This is a convex polygon. **Concave** is simply a polygon that is not convex. Imagine a fork. If you draw a segment from one tine to another, this line travels outside the fork, making it concave. A concave polygon contains at least one reflex angle.

This leads to the **Exterior Angle Theorem**:

The exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles.

The third theorem in this standard refers to other polygons, not triangles. However, it is based on the Interior Sum Theorem, since convex polygons can be broken down into sets of triangles.

For a convex polygon with x sides, the sum of the interior angles is given by the formula $S = (180)(x – 2)$.

You can see this formula works for a triangle ($x = 3$), since:

$$S = (180)(x - 2)$$
$$S = (180)(3 - 2)$$
$$S = (180)(1) = 180°$$

An octagon has eight sides, so a convex octagon has an interior angle sum of 1,080°:

$$S = (180)(x - 2)$$
$$S = (180)(8 - 2)$$
$$S = (180)(6)$$
$$S = 1080°$$

STRATEGY BOX – Find the Missing Angles

The three angle formulas here can help you find the measure of angles not directly given in a problem. Quite often, finding the measure of these missing angles is the key to answering the problem correctly.

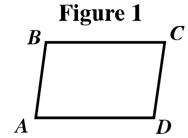
✦ *State and Apply Properties to Different Quadrilaterals* ✦

Any polygon that has four sides is a **quadrilateral**. The following figures are all quadrilaterals, but they have other attributes.

Special Kinds of Quadrilaterals

1. **Parallelogram.** In a parallelogram, the opposite sides are parallel to each other. Figures 1, 2, 3, and 4 are all parallelograms.

Figure 1

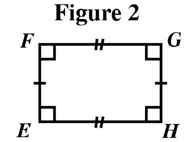


$\overline{AB} \parallel \overline{CD}$

$\overline{BC} \parallel \overline{AD}$

2. **Rectangle.** A rectangle is a parallelogram with four interior right angles. Opposite sides of a rectangle are congruent in length, and adjacent sides of a rectangle are perpendicular to each other. Figures 2 and 3 fit the definition of a rectangle.

Figure 2

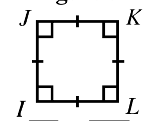


$\overline{EF} \parallel \overline{GH}, \overline{FG} \parallel \overline{EH}$

$\overline{EF} \cong \overline{GH}, \overline{FG} \cong \overline{EH}$

Square. A square is a rectangle with all sides congruent in length. Figure 3 is the only square. Since a square is a rectangle, it is also a parallelogram.

Figure 3

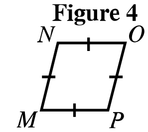


$\overline{IJ} \parallel \overline{KL}$

$\overline{JK} \parallel \overline{IL}$

3. **Rhombus.** A rhombus has four sides equal in length. However, the interior angles of a rhombus are undefined. Figures 3 and 4 are both rhombi. Note that since all the sides of a square are congruent, a square is also a rhombus.

Figure 4

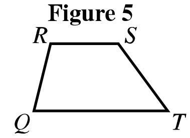


$\overline{MN} \parallel \overline{OP}$

$\overline{NO} \parallel \overline{MP}$

4. **Trapezoid.** A trapezoid has one set of opposite sides parallel. The two parallel sides are called bases, while the non-parallel sides are the legs. Figure 5 is a trapezoid.

Figure 5



$\overline{RS} \parallel \overline{QT}$

$\overline{RS}$ and $\overline{QT}$ are bases

$\overline{RQ}$ and $\overline{ST}$ are legs

All five shapes are quadrilaterals, and some shapes can be described in more than one way. Be sure you know these shapes so that you can answer a question like:

A square is a quadrilateral that is NOT also a

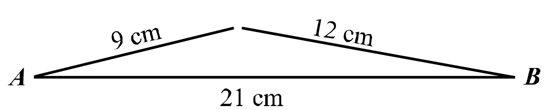
A trapezoid
B rectangle
C rhombus
D parallelogram

A trapezoid has only one pair of sides parallel, and a square has both opposite sides parallel. A is the correct answer.

✦ *Apply the Concept of Inequality to Segments and Angles in Triangles* ✦

If someone asked you to imagine a triangle with sides 9 cm, 21 cm, and 12 cm long, you would probably be able to do so. It seems plausible that a triangle could have those lengths.

If you sat down with some paper and a ruler and actually attempted to draw a triangle with sides 9 cm, 21 cm, and 12 cm long, you would find that there is a difference between what can be imagined and what can actually be created. The problem is that the two shorter sides, 9 cm and 12 cm, add together to equal the length of the third side, 21 cm. There is no way three interior angles can be created. To see this on paper, draw a line 21 cm across. Call it $\overline{AB}$. Now draw a 9-cm line from A and a 12-cm line from B. See if there's any way these two lines can meet without increasing their length.



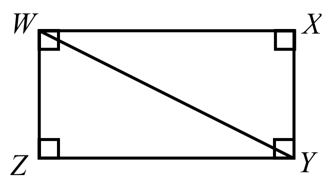
A theorem combining inequality and triangles can be stated:

In any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

The same theorem is not true of angles. You can have one angle in a triangle that exceeds the sum of the other two angles. However, keep in mind that all three angles must always add up to 180° (Interior Sum Theorem).

✦ *Test Quadrilateral Properties Involving Diagonals and Line of Symmetry* ✦

There are two diagonals in any quadrilateral. They are created by drawing a segment from one vertex to the opposite, nonconsecutive vertex. What's interesting is that any time you draw a diagonal in a quadrilateral, you create two triangles. Look at the rectangle that follows and see how diagonal $\overline{WY}$ — connecting nonconsecutive vertices W and Y—creates two triangles, ΔWZY and ΔYXW.



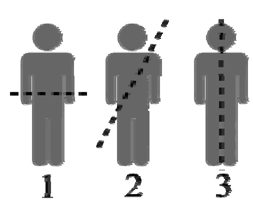
If you think triangles ΔWZY and ΔYXW are congruent, you have good eyes. The two triangles are congruent. This leads to something you should remember for the ***Geometry* EOCT**:

When you draw a diagonal in a parallelogram, you create two congruent triangles.

> If you flip and then rotate ΔWZY mentally, you can see that it is congruent to ΔYXW.

This is not always true for the other quadrilaterals such as trapezoids, but it is *always* true for squares, rectangles, and rhombi since they are parallelograms. Using a diagonal to create two congruent triangles is a key idea that may help you answer a question about quadrilaterals on the exam.

The human figure below will be used to illustrate a line of symmetry—the other main concept in this standard. To do this, the figure will have to be shown in three different ways.



> You can use a mirror to test a line of symmetry. If you place a mirror right along a line of symmetry, the reflection created in the mirror should join with the half being reflected to create the picture of the original figure. You can see how this would work with the Figure 3, but not the other two.

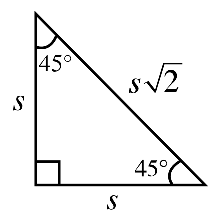
A **line of symmetry** divides a figure into two congruent halves. The dotted line of symmetry in Figure 1 creates two halves, a top and a bottom. However, the halves are not identical in size and shape. The same problem occurs in Figure 2: the end result is two parts that are not congruent. However, the dotted line of symmetry in Figure 3 does show two halves that are congruent.

✴ Use the Pythagorean Theorem and its Converse ✴

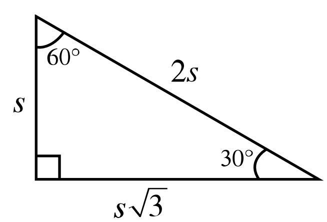
The Pythagorean Theorem and its converse are covered in Content Domain V of the ***Algebra I*** **EOCT** under the standard "Apply the Pythagorean Theorem to Problems." A review of this material on pages 76 and 77 will give you the knowledge needed to answer Pythagorean Theorem-related problems that might appear on the ***Geometry*** **EOCT**.

✴ State the Properties of Two Special Right Triangles ✴

If you know the length of two sides of any right triangle, you can use the Pythagorean Theorem to determine the length of the third side. However, for two special right triangles, you only need the length of one side to determine the lengths of the other two sides. These two special triangles are the 45°-45°-90° right triangle and the 30°-60°-90° right triangle. For these two triangles, the ratio of the lengths of all three sides is always the same. Here is an illustration showing the ratios.



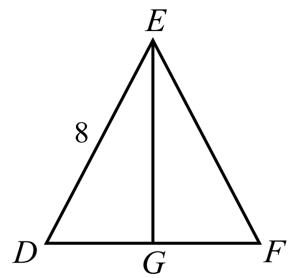
For a 45°- 45°-90° right triangle,
if the legs are length s, then the hypotenuse is length $s\sqrt{2}$.



For a 30°-60°-90° right triangle,
if the side opposite the 30° angle is s,
then the hypotenuse is $2s$ and the side opposite the 60° angle is $s\sqrt{3}$.

The question below uses a special right triangle, but it is up to you to recognize the special right triangle on your own, since the problem does not mention it specifically.

ΔDEF is an equilateral triangle. $\overline{EG}$ bisects $\angle DEF$. If $DE = 8$, what is the length of EG?



A 4
B $4\sqrt{2}$
C $4\sqrt{3}$
D $8\sqrt{2}$

In an equilateral triangle, all angles have measures equal to 60°. (You learned this from an earlier problem on pages 47 and 48.) If $\overline{EG}$ bisects $\angle DEF$, it cuts it in half, and half of 60° is 30°. Therefore, ΔEGD is a 30°-60°-90° right triangle, and the length of the hypotenuse is 8.

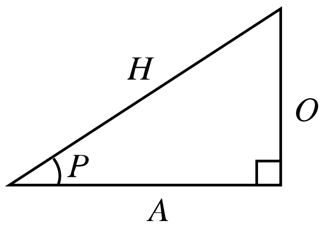
The side opposite the 30° angle is half the length of the hypotenuse, so $\frac{8}{2} = 4$, the length of $\overline{DG}$. $\overline{EG}$ is the side opposite the 60° angle and has length $s\sqrt{3}$. Since $s = 4$, then $EG = 4\sqrt{3}$. This is answer choice C.

✴ *Identify Sine, Cosine, and Tangent Values for Right Triangles* ✴

Another way to find the lengths of the sides of right triangles is to use the sine, cosine, or tangent ratio. These ratios will be defined on the next page.

In any right triangle, there are three angles—one right angle and two acute angles. If an acute angle in one right triangle is congruent to an acute angle in another right triangle, the two triangles are similar. Since the value of the sine, cosine, or tangent of an angle is determined by the angle, it does not depend on the size of the triangle.

The sine, cosine, and tangent ratios for either acute angle are defined as follows:



$$\text{sine } \angle P = \frac{\text{length of opposite side}}{\text{length of hypotenuse}} = \frac{O}{H}$$

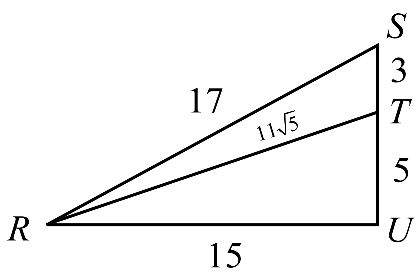
$$\text{cosine } \angle P = \frac{\text{length of adjacent leg}}{\text{length of hypotenuse}} = \frac{A}{H}$$

$$\text{tangent } \angle P = \frac{\text{length of opposite side}}{\text{length of adjacent leg}} = \frac{O}{A}$$

Some students confuse the concepts of sine and cosine. One way to remember is to think of cosine as closer to the angle, since cosine is $\frac{\text{adjacent}}{\text{hypotenuse}}$. If you can remember *cosine* as "close"-ine, you should remember that it takes the closer, adjacent side and places it above the hypotenuse.

If you know what each term means and how to apply the ratios, you should be able to answer a problem like this one:

Which angle in the figure has a tangent equal to $\frac{1}{3}$?



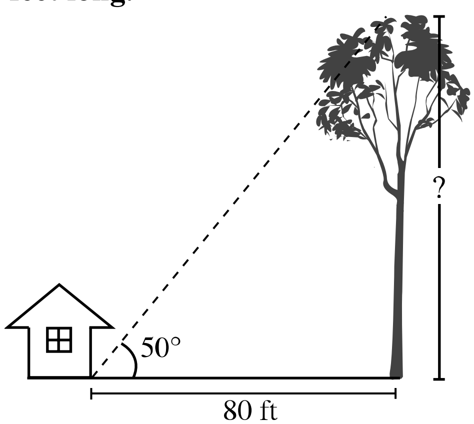
A $\angle URT$
B $\angle SRU$
C $\angle RSU$
D $\angle RTU$

The tangent ratio is found by placing the opposite side over the adjacent side. You need to find two segments that equal the tangent ratio of $\frac{1}{3}$. $TU = 5$, $RU = 15$, and $\frac{5}{15} = \frac{1}{3}$. These two segments are the opposite and adjacent sides of $\angle URT$, choice A.

✷ *Solve Problems Using Sine, Cosine, and Tangent Ratios* ✷

The previous standard defined the sine, cosine, and tangent ratios. This standard extends these concepts to solving real-world problems using these trigonometric ratios. For example, the problem on page 39 in Content Domain III concerns a tree falling directly towards a house. The problem below has been modified to show how a sine, cosine, or tangent ratio can be used to answer a similar question.

There is a large tree 80 feet from a house. The owners are worried that the tree might hit their house if it fell and want to estimate the height of the tree. In the figure below, when the sun's angle of elevation is $50°$, the tree casts a shadow 80 feet long.



$\sin 50° \approx 0.77$
$\cos 50° \approx 0.64$
$\tan 50° \approx 1.2$

What is the approximate height of the tree?

A 51 ft
B 62 ft
C 67 ft
D 96 ft

If the tree falls directly toward the owners' house, the height of the tree determines whether or not it will hit the house. In the diagram, the tree and its shadow represent the legs of a right triangle. The tree is the side *opposite* the $50°$ angle, and its shadow is the leg *adjacent* to this angle. Since the sides opposite and adjacent to the $50°$ angle are involved, you should use the tangent ratio.

$$\text{tangent of angle} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 50° = \frac{\text{height of tree}}{80}$$

$$1.20 \approx \frac{\text{height of tree}}{80}$$

$$(1.20)(80) \approx \frac{\text{height of tree}}{80}(80)$$

$$96 \approx \text{height of tree}$$

To solve this problem, you need the numerical value for the tangent of $50°$. On the EOCT, decimal approximations for the sine, cosine, and tangent of an angle will be given in a box located near the question.

The 96-foot tree would definitely hit the house 80 feet away if it fell directly towards the house. Note that options A and B are the answers if you incorrectly used the cosine or sine ratios, and option D uses the reciprocal of the tangent ratio as shown below:

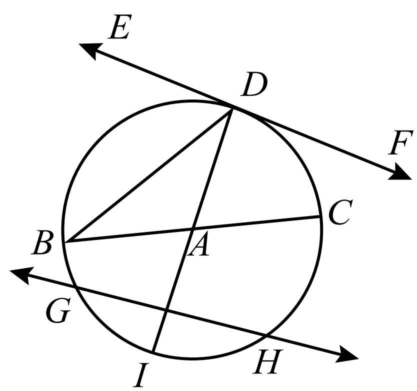
$$\tan 50° = \frac{80}{\text{height of tree}}$$

✴ *Identify Parts and Properties of Circles* ✴

Previous domains involved angles and polygons. Content Domain IV requires that you identify and define circles and the segments, lines, and angles associated with them. You will be asked to apply geometric relationships to solve problems involving these parts.

A **circle** is defined as the set of all points in a plane that are equidistant from a given point. This given point is the **center** of the circle, and circles are named using this center point. The circle shown in the figure below is named circle *A*.

A **radius** of a circle is a segment that has one endpoint on the circle and the other endpoint at the center of the circle. All radii in a circle are congruent. A segment that has both endpoints on the circle is a **chord** of the circle. A chord that passes through the center of a circle is a diameter. A **diameter** is twice the length of a radius of the circle. In the figure below, radii shown are $\overline{AD}$, $\overline{AC}$, $\overline{AI}$, and $\overline{AB}$. Chords are $\overline{BD}$, $\overline{GH}$, $\overline{BC}$, and $\overline{DI}$. Since $\overline{BC}$ and $\overline{DI}$ contain the center of the circle, these two chords are also diameters.



There are many other terms associated with circles. The list on page 57 defines some of the more common parts and relationships.

Common Terms Associated with Circles

Tangent. In a plane, a line that intersects a circle at exactly one point is tangent to the circle. The point of intersection is a point of tangency. In the figure, line *EF* is tangent to circle *A* at point *D*, the point of tangency. If two circles intersect at exactly one point, the circles are tangent to each other.

Secant. A line that intersects a circle at exactly two points is a secant of the circle. Any line that contains a chord of the circle is a secant of that circle. Line *GH* intersects circle *A* at exactly two points and is a secant of circle *A*.

Note that line *GH* contains chord $\overline{GH}$.

Central angle. A central angle is an angle whose vertex is at the center of a circle. The sides of a central angle contain radii of the circle. $\angle DAC$ and $\angle DAB$ are examples of central angles.

Arc. An arc is an unbroken piece of a circle. It has two endpoints on the circle and consists of these endpoints and all points along the circle from one endpoint to the other. Arcs are named by their endpoints with the arc symbol placed above them. Two arcs are associated with each central angle of measure less than 180°. The endpoints of each arc are where the sides of the central angle intersect with the circle. These endpoints and all points of the circle in the interior of the central angle form a minor arc of the circle. These same endpoints and all points of the circle in the exterior of the angle form a major arc of the circle. The two arcs formed by the endpoints of a diameter are semicircles. Each semicircle is exactly half a circle. Major arcs and semicircles must be named using three points—the endpoints and one other point on the arc inserted between these endpoints. In the figure, arcs $\overset{\frown}{DC}$ and $\overset{\frown}{GH}$ are minor arcs; arcs $\overset{\frown}{BDC}$ and $\overset{\frown}{BHC}$ are semicircles; and arcs $\overset{\frown}{BDH}$ and $\overset{\frown}{IDC}$ are major arcs.

The measure of a minor arc is the measure of its central angle. The measure of a semicircle is 180°. The measure of a major arc is the difference between 360° and the measure of its associated minor arc.

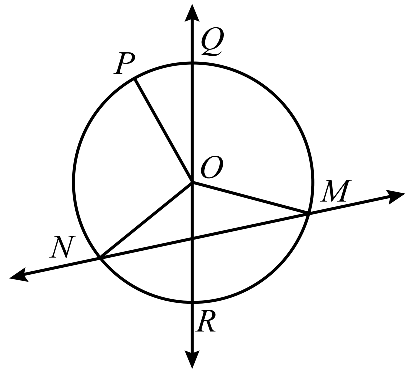
Concentric circle. Two coplanar circles are concentric if they have the same center but different radii. Circles on a dartboard are an example of concentric circles.

Inscribed angle. An angle is inscribed in a circle if its vertex is on the circle and its sides contain chords of the circle. $\angle BDI$ is an inscribed angle because its vertex, *D*, is a point on the circle, and the sides of $\angle BDI$ contain chords $\overline{BD}$ and $\overline{DI}$ of the circle.

There are more terms that can be used to describe circles and their parts. Consult your textbook for a review of the others. Problems in this standard focus on how well you know the meanings of these terms and can recognize the parts in a figure.

Practice the vocabulary words on page 57 by answering the following question:

In the figure below, which term BEST describes $\overleftrightarrow{MN}$ in relation to circle *O*?



A chord
B radius
C secant
D tangent

$\overline{MN}$ has its endpoints on the circle, so you might be tempted to choose option A. However, note that the question uses the line symbol above *MN*. Since line *MN* intersects circle *O* in exactly two points (and contains chord *MN*), it is a secant of circle *O*. Choice C is the correct answer.

✴ *Solve Problems Using Various Relationships Associated with Circles* ✴

The many theorems associated with the relationships between a circle and its parts are too numerous to list in this study guide. However, you will be asked to apply these geometric relationships when solving problems in this standard. Consult your textbook for a review of these theorems. The list below is only a partial list of relationships that are summarized in theorems you should know. Use it as a guide to lead you to other related theorems in your textbook.

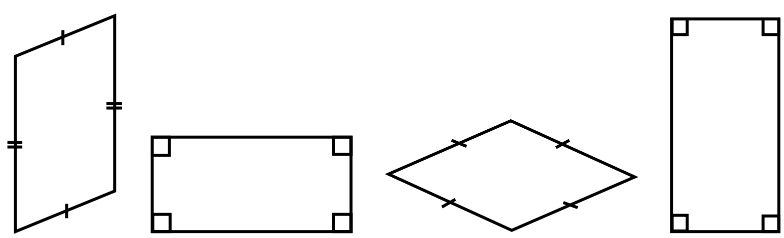
Relationships between a Circle and the Parts of a Circle

1 Central angles and arcs
2 Inscribed angles and arcs
3 Angles formed by tangents, secants, and chords
4 Arcs and Chords
5 Properties of tangents

Sample Questions for Content Domain IV

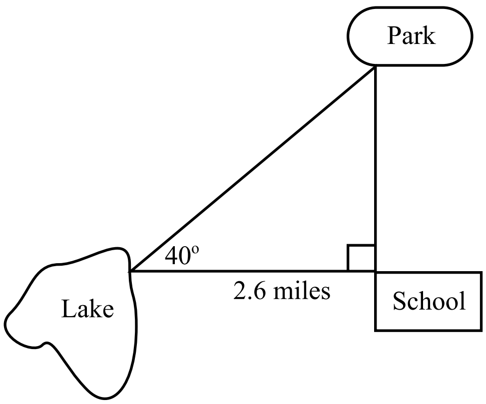
This section has some sample questions for you to try. After you have answered all of the questions, check your answers in the “Answers to the Content Domain IV Sample Questions” section that follows. This section will give you the correct answer to each question and show you the appropriate steps for solving the problem.

1 **What is the MOST descriptive classification for the figures shown below?**



A parallelograms
B rectangles
C rhombi
D trapezoids

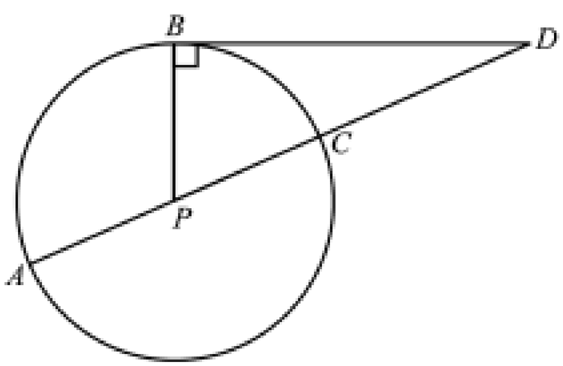
2 **A school located due south of a park is 2.6 miles due east of a lake, as shown on the map below.**



Which expression represents the distance between the lake and the park?

A $2.6\cos 40°$
B $2.6\tan 40°$
C $\dfrac{2.6}{\cos 40°}$
D $\dfrac{2.6}{\tan 40°}$

3 **The radius of circle *P*, shown below, is 6. $\overline{BD}$ is tangent to circle *P* at *B*.**



If $BD = 8$, what is the measure of $\overline{CD}$?

A 4
B 6
C 14
D 16

4 **If quadrilateral *ABCD* is a rhombus, which statement is NOT always true?**

A $\overline{AB} \cong \overline{CD}$
B $\overline{AB} \cong \overline{BC}$
C $\overline{AD}$ is parallel to $\overline{BC}$
D $\overline{AD}$ is perpendicular to $\overline{DC}$

Answers to the Content Domain IV Sample Questions

1. Answer: **A** Standard: *State and apply properties to different quadrilaterals*

All four figures are quadrilaterals, but this is not an answer choice. Two of the figures have four right angles. These two are rectangles, which are special parallelograms, but the other two are not. Therefore, choice **B** is incorrect. The third figure with four congruent sides is the only rhombus, so choice **C** is not the answer. However, a rhombus is also a special parallelogram. Because both pairs of opposite sides are congruent in the first figure, it is a parallelogram. Trapezoids are quadrilaterals with exactly one pair of opposite sides parallel and can never be parallelograms. Therefore, answer **D** is incorrect. The property shared by all four figures is that both pairs of opposite sides are parallel so they are all parallelograms. Choice **A** is the correct answer.

2. Answer: **C** Standard: *Identify and use sine, cosine, and tangent ratios for right triangles to solve application problems*

On the map, the distance between the lake and the park represents the hypotenuse of the right triangle. In relation to the 40° angle, the length of the adjacent side is known and the hypotenuse, h, is the unknown. Therefore, the cosine ratio should be used to solve this problem.

$$\text{cosine of an angle} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 40^\circ = \frac{2.6}{h}$$

$$(\cos 40^\circ)h = \left(\frac{2.6}{h}\right)h$$

$$(\cos 40^\circ)h = 2.6$$

$$\frac{(\cos 40^\circ)h}{(\cos 40^\circ)} = \frac{2.6}{(\cos 40^\circ)}$$

$$h = \frac{2.6}{(\cos 40^\circ)}$$

$$\text{distance between lake and park} = \frac{2.6}{\cos 40^\circ}$$

This is choice **C**.

3. Answer: **A** Standard: *Solve problems using various relationships associated with circles*

Since the radius of the circle is 6, you know that both *BP* and *PC* are 6. Combining *BP* = 6 with the fact that *BD* = 8 (also given in the question), you can see that you have the two sides of the right triangle. To solve for the hypotenuse $\overline{PD}$, then, requires the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$(BP)^2 + (BD)^2 = (PD)^2$$
$$6^2 + 8^2 = (PD)^2$$
$$36 + 64 = (PD)^2$$
$$100 = (PD)^2$$
$$\sqrt{100} = \sqrt{(PD)^2}$$
$$10 = PD$$

The length *PD* is the sum of the lengths of *PC* and *CD*. Now you know two of these three lengths.

$$PD = PC + CD$$
$$10 = 6 + CD \qquad \text{Since the radius equals 6}$$
$$10 - 6 = 6 - 6 + CD$$
$$4 = CD$$

Choice **A** is 4.

4. Answer: **D** Standard: *Test quadrilateral properties including diagonals and line of symmetry*

It is fitting to end this domain with a quote from the beginning. On page 45, the definition of a rhombus is given as "a four-sided figure with all four sides congruent and opposite sides parallel." It would be helpful to draw and label a rhombus before you answer this question. On your figure, you should have marked opposite sides congruent and consecutive sides congruent, so choices **A** and **B** are always true in rhombus *ABCD*. Since a rhombus is a parallelogram, choice **C** is always true. Choice **D** is the only choice not eliminated and is the correct answer. Consecutive sides of a rhombus *could* be perpendicular, but this does not *always* have to be true.