Content Domain V: Perimeter, Area, and Volume

A Look at Content Domain V

Test questions in this content domain will measure your understanding of perimeter, area, and volume. Your answers to the questions will help show how well you can perform on the following standards:

- ✖ Define perimeter, area, and volume and associated units of measurement
- ✖ Find the perimeter of various two-dimensional shapes
- ✖ Find the area of various two-dimensional shapes
- ✖ Find the area (lateral, total, and surface) of various three-dimensional shapes
- ✖ Find the volume of certain three-dimensional solids
- ✖ Solve problems involving perimeter, area, and volume

Can you dig a hole with a fish? Can you stop a moving train with a loaf of bread? Can you find the area of a rectangle with the formula for the circumference of a circle?

The answer to all these questions is "no." Only the final question is relevant to the ***Geometry*** **EOCT**, but it illustrates how you need to use the right tool (or formula) to get the correct result. Using the wrong formula on a geometry problem is as ineffective as using a fish to dig a hole in the ground. It just won't work.

Content Domain V requires you to apply the formulas for perimeter, area, surface area, and volume to plane and solid geometric figures. Some, but not all, of these formulas will be listed at the beginning of the ***Geometry*** **EOCT**, but the formulas are not identified by geometric shape. You must be able to recognize the formula and apply it to the appropriate figure. To prepare for the **EOCT**, make a list of the formulas in this study guide, memorize these formulas, and use them to answer the practice questions.

Spotlight on the Standards

✖ Define Perimeter, Area, and Volume and Associated Units of Measurement ✖

Visualize a rectangular parking space for a car. It is approximately 8 feet long and 6 feet wide. If you walk along the outer edges of this space, you are walking around its perimeter. The perimeter of a polygon is the sum of the lengths of its sides. Since the two lengths (l) are equal to each other and the two widths (w) are equal to each other in a rectangle, the formula for perimeter, P, of a rectangle is:

$$P = 2(l + w) \text{ or } P = 2l + 2w$$

The perimeter of the parking space is:

P = 2(8 feet + 6 feet)
P = 2(14 feet)
P = 28 feet

Distance measures length, a one-dimensional unit. Perimeters of figures are labeled in units associated with length, such as inches, centimeters, feet, miles, kilometers, etc.

The area of a polygon is the number of square units contained in its interior. The area of a rectangle, A, is found by multiplying its length times width: $A = lw$. Apply this formula to find the area of the parking space:

$A = lw$ Formula for area of a rectangle
A = (8 feet)(6 feet)
A = 48 square feet = 48 ft^2

Area is measured in *square* units because two dimensions are multiplied to calculate area. Algebraically, this makes sense because you are multiplying 8 feet by 6 feet and (feet) x (feet) = $(\text{feet})^2$, or squared feet. Also, area measures the interior of a plane figure, and planes have two dimensions—*length* and *width*. This extra dimension accounts for the squared value of all units of measurement used in labeling areas.

If a rectangular box 5 feet high is put over the parking space, the interior of this box is the volume of the box. Volume is the measure of the interior space of a three-dimensional solid. Three dimensions (length, width, and height) are multiplied to calculate volume, and the units associated with volume are *cubic* units. The formula for volume, V, of a rectangular prism is $V = lwh$. To calculate the volume of the box over the parking space, apply the formula:

$V = lwh$ Formula for volume of a rectangular prism
V = (8 feet)(6 feet)(5 feet)
V = 240 cubic feet = 240 ft^3

Volume is the measure of the space inside of a three-dimensional solid. In the example, you multiplied a third dimension (height) by the area of the parking space. Height affected the unit of measure—changing it from *square feet* (area covering only length and width) to *cubic feet* (covering length, width, and height).

Consider the following question:

Kevin recently bought a poster for his room. Which unit of measure is MOST appropriate to describe the size of the poster?

A cubic meters
B square meters
C square kilometers
D kilometers

A poster has length and width and will cover a flat (plane) surface, so its size would be measured by finding the area. Since area is measured in square units, choices A and D can be eliminated. Of the two remaining choices, B is most appropriate for the size of a poster because a kilometer is more than half a mile.

STRATEGY BOX – Know the Appropriate Units

Perimeter measures length in *one*-dimensional units such as *meters*.
Area measures flat, *two*-dimensional surfaces in square units such as $meters^2$.
Volume measures interior space of *three*-dimensional solids in cubic units such as $meters^3$.

✖ *Find the Perimeter of Various Two-dimensional Shapes* ✖

In the previous content domain, you learned the names and definitions of many different geometric shapes. All of these figures have perimeters, and the formulas you can use to find these perimeters are listed below.

Perimeter Formulas for Common Geometric Shapes

Planar Figure **Formula**

△ **Triangle** P = side + side + side
This is the basic formula. For an equilateral triangle, the formula is $P = 3s$ since all sides are congruent.

▭ **Rectangle** P = 2(length + width) or $P = 2(l + w)$ or $P = 2l + 2w$
Since opposite sides are congruent, you need to only add the length and width once and then multiply by 2 in order to take the other two sides into account.

□ **Square and** ▱ **Rhombus** P = 4(side) or $P = 4s$

▱ **Parallelogram** P = 2(side + adjacent side) or $P = 2(a + b)$
In a parallelogram, opposite sides are congruent. This makes the perimeter formula for a parallelogram the same as the one for a rectangle.

⏢ **Trapezoid** $P = \text{base}_1 + \text{base}_2 + \text{leg}_1 + \text{leg}_2$
For a trapezoid, you have to add the lengths of the four sides.

◯ **Circle** $C = 2\pi(\text{radius})$ or $C = 2\pi r$ or $C = \pi d$
The perimeter of a circle is called its **circumference**, and it is denoted by a capital C instead of P. Pi—noted by the symbol π—is an irrational number usually approximated at 3.14.

◜ **Arc length** $\text{Arc Length} = \dfrac{\text{measure of the arc}}{360^\circ} \times C$

Arc length is the length of an arc on the circumference of a circle. It is a fractional part of the circumference. The measure of an arc in degrees is the measure of its central angle. Since there are 360° in a circle, the fraction is determined by dividing the measure of the arc by 360°.

These formulas should enable you to answer any perimeter problem you might encounter. Keep in mind, though, that perimeter is just the distance around a figure. You can also find the perimeter of any polygon by adding the lengths of its sides.

✖ *Find the Area of Various Two-dimensional Shapes* ✖

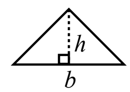
The previous standard related to perimeter. This one relates to area.

Here are the corresponding area formulas for all the shapes discussed in the last standard.

Area Formulas for Common Geometric Shapes

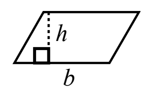
<u>Planar Figure</u> **<u>Formula</u>**

Triangle $A = \frac{1}{2}$(base)(height) or $A = \frac{1}{2}bh$



Any side of a triangle can be its base. The height is the length of the perpendicular segment from the opposite vertex to a line containing the base.

Rectangle A = (length)(width) or $A = lw$

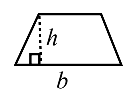


Parallelogram A = (base)(height) or $A = bh$

Any side of a parallelogram can be its base. The height is the length of the perpendicular segment from the opposite side to a line containing the base.

Square A = (side)(side) or $A = s^2$

Trapezoid $A = \frac{1}{2}(\text{base}_1 + \text{base}_2)(\text{height})$ or $A = \frac{1}{2}(b_1 + b_2)h$



The bases of a trapezoid are the parallel sides. The height is the length of the perpendicular segment whose endpoints are on opposite bases.

Circle $A = \pi$(radius)(radius) or $A = \pi r^2$

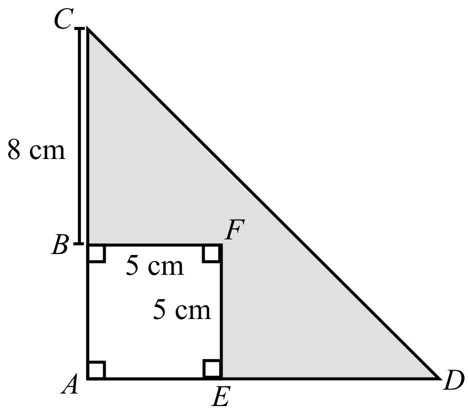
Arc sector $A_{\text{sector}} = \dfrac{\text{measure of the arc}}{360^{\circ}} \times A_{\text{whole}}$

A sector of a circle is a region bounded by two radii and an arc of the circle. The area of a sector is a fractional part of the area of the circle. This fraction is determined by dividing the measure of the arc by 360°.

Since a rhombus is also a parallelogram, the formula for the area of a rhombus is the same as the formula for area of a parallelogram.

Once you understand the formulas on page 67, you can apply the correct one. Try answering this question:

If ΔACD is an isosceles triangle, what is the area of the shaded region below?



A 25 cm^2
B 52 cm^2
C 59.5 cm^2
D 84.5 cm^2

There are two figures in this problem—a triangle and a square. (The unshaded figure is a square because it has four right angles and $\overline{BF} \cong \overline{FE}$.) To find the area of the shaded region, subtract the area of the square from the area of the triangle.

Area of $BCDEF$ = Area of ΔACD – Area of square $ABFE$

Area of $BCDEF = \frac{1}{2}bh - s^2$

Since $AB + BC = AC$ and each side of the square has length 5, then:

$5 + 8 = AC$

$13 = AC$

Since ΔACD is isosceles, then $AC = AD = 13$. The legs of a right triangle are the lengths of its base and height. Substituting into the formula for the area of the region:

Area of $BCDEF = \frac{1}{2}bh - s^2$

Area of $BCDEF = (\frac{1}{2})(13)(13) - 5^2$

Area of $BCDEF = 84.5 - 25$

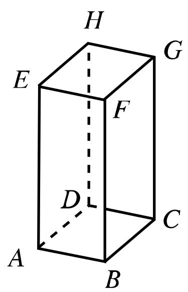
Area of $BCDEF = 59.5$ cm^2

This is answer choice C.

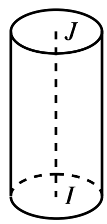
✖ Find the Area (Lateral, Total, and Surface) of Various Three-dimensional Shapes ✖

As you saw with the parking space example on page 63, adding height to a two-dimensional figure creates a three-dimensional figure. There are many three-dimensional figures, but only a few of the shapes will appear on the ***Geometry*** **EOCT**.

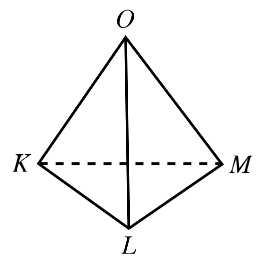
Here are four basic three-dimensional shapes:



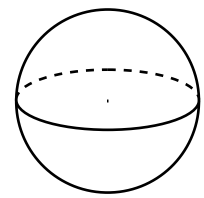
rectangular prism



circular cylinder



triangular pyramid



sphere

The terms ***altitude*** and ***height*** are interchangeable.

Prisms are three-dimensional solids with sides (faces) that are polygons. The two bases of a prism are congruent faces that lie in parallel planes. In the prism above, the bases are *ABCD* and *EFGH*. The other faces are named lateral faces, and if these are all rectangles, the prism is a right prism. The segments joining corresponding vertices of the bases are called lateral edges. In the figure, $\overline{EA}$, $\overline{FB}$, $\overline{GC}$, and $\overline{HD}$ are lateral edges. The height, or altitude, of a prism is the perpendicular distance between its bases. In a right prism, the height is the length of a lateral edge.

A **circular cylinder** has bases that are congruent circles that lie in parallel planes. If the segment joining the center of its bases is perpendicular to the bases, the cylinder is a right cylinder, and this segment is its height.

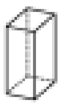
Pyramids have only one base and lateral faces that are triangles with a common vertex. The height of a pyramid is the perpendicular distance between the base and the common vertex. The intersection of two lateral faces is a lateral edge, and if the lateral edges are all congruent, the pyramid is a regular pyramid. The slant height of a regular pyramid is the altitude of any lateral face.

A **sphere** is the set of all points in space equidistant from a given point called the center. The radius of a sphere is the segment with endpoints at the center and on the sphere.

The **lateral area** of a prism or pyramid is the sum of the areas of its lateral faces. The **lateral area** of a cylinder is the area of its curved lateral surface. The **total surface area** of each of these solids is the sum of its base area and lateral area.

The formulas below can be used to find lateral and surface areas of the geometric solids described on page 69. These areas are measured in square units (units^2).

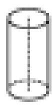
Area Formulas for Some Three-dimensional Shapes



Right Prism

Lateral Area: (perimeter of base polygon)(height)

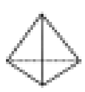
Total Area: (lateral area) + 2(area of base polygon)



Right Circular Cylinder

Lateral Area: (circumference of base circle)(height)
$= 2\pi r$(height) or $2\pi rh$, *when* h = height

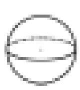
Total Area: (lateral area) + 2(area of base circle)
$= 2\pi rh + 2(\pi r^2)$
$= 2\pi r(h + r)$



Regular Pyramid

Lateral Area: $\frac{1}{2}$(perimeter of base polygon)(slant height)

Total Area: (lateral area) + (area of base polygon)



Sphere

Surface Area: $4\pi r^2$

Spheres have no lateral edges, so there is only one type of area (surface area).

✖ *Find the Volume of Certain Three-dimensional Solids* ✖

As stated previously, volume is the measure of the interior space of a three-dimensional solid. Volume is measured in cubic units (units^3).

Use the volume formulas in the table on the next page to calculate the volumes of geometric solids that might appear on the ***Geometry*** **EOCT.**

Volume Formulas for Some Three-dimensional Solids

Shape	Volume Formula
Prism (rectangular)	(length of base)(width of base)(height of prism) lwh
Cylinder	(area of the base)(height) $\pi r^2 h$
Pyramid	$\frac{1}{3}$(area of base)(height) = $\frac{1}{3}Bh$
Sphere	$\frac{4}{3}\pi r^3$

✖ *Solve Problems Involving Perimeter, Area, and Volume* ✖

Now that you have all the relevant formulas, it's time to see how these formulas can be applied to real-world situations. Questions in this standard will require you to understand a real-world situation, select the appropriate formula, and then use that formula to find the answer to the question.

Identify the appropriate formula and apply it to this question:

Rosa is packing a set of wooden blocks into a rectangular box. Each wooden block is a 1 cm cube. If the dimensions of the box (a rectangular right prism) are 12 cm by 6 cm by 8 cm, how many wooden blocks can the box hold?

A 144
B 576
C 1152
D 2304

To determine the number of wooden blocks that can fit into the box, you first need to find the volume of the box. The first step can be done using the volume formula for a right prism.

Volume = lwh
Volume = (12 cm)(6 cm)(8 cm)
Volume = 576 cm^3

Each cube has a volume of 1 cm^3, so 576 of them will fit into the box. This is choice B.

Sample Questions for Content Domain V

This section has some sample questions for you to try. After you have answered all of the questions, check your answers in the "Answers to the Content Domain V Sample Questions" section that follows. This section will give you the correct answer to each question and show you the appropriate steps for solving the problem.

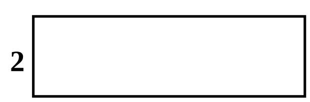
1 **How many square inches of paint would be on a sphere with a 12-inch diameter?**

A 48π in.^{2}
B 144π in.^{2}
C 288π in.^{2}
D 576π in.^{2}

2 **Which of the following units is MOST appropriate to express the distance around a small park ?**

A square feet
B yards
C cubic meters
D millimeters

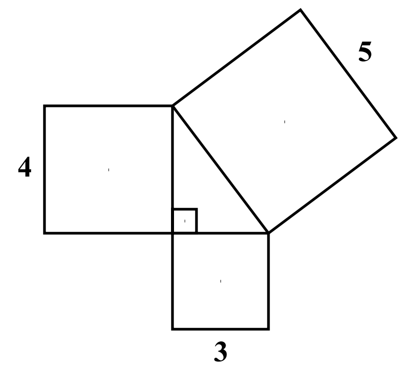
3 **The area of the rectangle below is 18 square units.**



What is the perimeter of the rectangle?

A 11 units
B 16 units
C 20 units
D 22 units

4 **The figure below is formed with three squares on the sides of a right triangle.**



What is the area of the entire figure in square units?

A 44
B 50
C 56
D 62

Answers to the Content Domain V Sample Questions

1. Answer: **B** Standard: *Find the surface area of various three-dimensional solids*
To find the amount of paint on a sphere (in square inches), you would find the surface area. Use the surface area formula on page 68: surface area of sphere = $4\pi r^2$.

The question states that the diameter of the sphere is 12 inches. Since a radius is always half the diameter, the radius of the sphere is 6 inches. Since $r = 6$, you have the following steps on the next page.

Surface area of sphere = $4\pi r^2$
Surface area of sphere = $4\pi(6^2)$
Surface area of sphere = $4\pi(36)$
Surface area of sphere = 144π square inches, or 144π in^2. This is answer **B**.

2. Answer: **B** Standard: *Define perimeter, area, and volume and associated units of measurement*

The distance around the park is the perimeter—a linear measurement. Therefore, choices A and C can be eliminated because they apply to measurements of area and volume. Even though choice D is a linear unit of measurement, it is so small that it would result in a perimeter with a very large numerical value. Of the two linear units, choice B is MOST appropriate for this situation.

3. Answer: **D** Standard: *Find the perimeter of various two-dimensional shapes*

Since the area and width of the rectangle are given, substitute these two values into the formula for area of a rectangle to find the missing length.

$$A = lw$$
$$18 = 2l$$
$$\frac{18}{2} = \frac{2l}{2}$$
$$9 = l$$

To calculate the perimeter of the rectangle, substitute the values for length and width into the appropriate formula.

$P = 2(l + w)$
$P = 2(2 + 9)$
$P = 2(11)$
$P = 22$, answer choice **D**.

4. Answer: **C** Standard: *Find the area of various two-dimensional shapes*

The figure consists of three squares enclosing a right triangle. The sides of the three squares are given, and since all sides of a square are equal, you also know that the right triangle has two legs of length 3 and 4 and a hypotenuse of 5. Since the triangle's legs meet at the right angle, these values can be used as the base and height of the triangle.

Total Area = $\text{Area}_{\text{square1}} + \text{Area}_{\text{square2}} + \text{Area}_{\text{square3}} + \text{Area}_{\text{triangle}}$

The formula for the area of a square is $A = s^2$, and since the three squares have sides of 3, 4, and 5 respectively, you have the following equation.

Total Area = $3^2 + 4^2 + 5^2 + (\frac{1}{2})bh$

Total Area = $9 + 16 + 25 + (\frac{1}{2})(3)(4)$

Total Area = $50 + 6$
Total Area = 56
This is choice **C**.

Content Domain VI: Coordinate, Transformational, and Three-dimensional Geometry

A LOOK AT CONTENT DOMAIN VI

Test questions in this content domain will measure your understanding of visual aspects of geometry. Your answers to the questions will help show how well you can perform on the following standards:

- Use visualization skills to interpret different figures
- Identify common polyhedrons and associated terms
- Find images that have been reflected, translated, or rotated
- Find images of figures under dilations
- Examine basic properties of various transformations
- Examine symmetry, similarity, and congruence of figures
- Graph ordered pairs in a coordinate plane
- Apply the distance and midpoint formulas
- Find the slope of a line, write an equation of a line, and graph linear equations
- Find the point of intersection for two lines
- Use coordinate geometry to explore properties of geometric figures

There is a visual element to each of the three predominant themes—coordinate, transformational, and three-dimensional geometry—in Content Domain VI. The coordinate geometry standards place geometric figures on an *xy*-plane. The transformational problems bring back the idea of similarity you learned in Content Domain III. Finally, the three-dimensional problems use the basic figures discussed in the previous domain (prisms, pyramids, cylinders, and spheres), as well as a few others. Knowing what these shapes look like under different circumstances will help you answer these questions.

Spotlight on the Standards

◆ *Use Visualization Skills to Interpret Different Figures* ◆

This standard might ask you to describe cross sections of three-dimensional figures, identify a locus of points, or use properties of projections. A cross section of a geometric solid is the intersection of a plane and the solid. A locus of points is the set of all points that satisfy given conditions.

For example, take an orange and slice it in half right down the middle perpendicular to the stem line. In geometric terms, you have taken a sphere (the orange) and bisected it through its centerpoint using a plane. Looking directly at the inside of the half-orange, you no longer see a sphere. Instead, you see a circle. The cut (or intersecting plane) has shown that a cross-section view of a sphere yields a circle.

Use your visualization skills to answer this question:

The bases of a right prism are regular hexagons. A plane perpendicular to the bases passes though the interior of the prism. Which could be the shape of a cross section where the plane intersects the prism?

A triangle
B rectangle
C trapezoid
D hexagon

If you selected choice D, read the question again. The plane would have to be *parallel* to the bases to produce a cross section with this shape.

> **STRATEGY BOX –Picture the Figure**
> To answer questions involving cross sections or other cuts, first picture the original figure in your mind. Then make the proper cut mentally and determine what the new shape looks like. Remember that a cross section of a three-dimensional figure will usually yield a two-dimensional figure of some kind.

Since the plane is perpendicular to the bases, it intersects them in congruent, parallel segments. The plane intersects two lateral faces of the prism in two segments that are congruent and parallel. Because the prism is a right prism, consecutive sides of these four segments are perpendicular. Therefore, the cross section is a rectangle. Choice B is the correct answer.

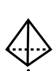
◆ *Identify Common Polyhedrons and Associated Terms* ◆

Polygon is a generic term used to describe any "many-sided figure." The term *polygon* can refer to a wide variety of two-dimensional shapes. Similarly, a **polyhedron** is a three-dimensional figure that encloses a single region of space. Each face of a polyhedron is a polygon.

Prisms and pyramids are both polyhedrons. Here are two more polyhedrons that might show up on questions in this standard.

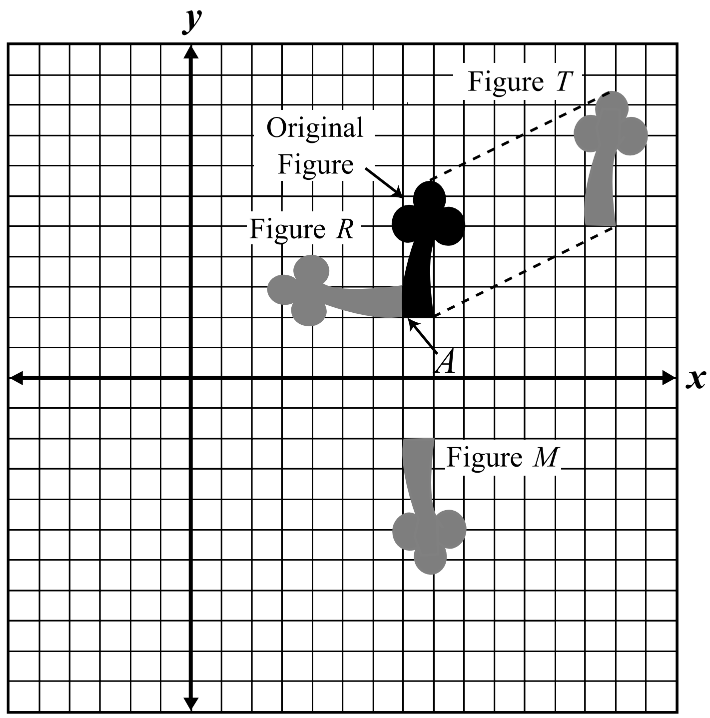
Some Common Polyhedrons

 **Cube.** A cube is a hexahedron with six congruent square faces. Therefore, all edges of a cube are also congruent.

 **Tetrahedron.** A tetrahedron has four faces and all are congruent equilateral triangles.

◆ *Find Images That Have Been Reflected, Translated, or Rotated* ◆

A three-leaf clover isn't the type of geometric shape you will often encounter on the ***Geometry* EOCT**. Even so, one can be used to illustrate three basic transformations: reflections, rotations, and translations.



The original figure is shown in black on the *xy*-plane. To **reflect** this figure, it is flipped over a line to create a mirror image of the original figure. In this case, the *x*-axis acts as a line of symmetry, creating the gray version (Figure *M*) of the clover below the original one. (For more about lines of symmetry, see page 51 in Content Domain IV.) If you placed a mirror along the *x*-axis, the reflection in this mirror would exactly resemble the reflected image in gray.

Rotations involve spinning the original figure by n degrees. The rotation is done around a fixed point like Point A in the example. Since there is a pivot point, rotated images are often still touching the original figure, although this is not always true. Using Point A as a pivot, the clover has been rotated or "spun" counterclockwise 90 degrees to make Figure R.

Translations are similar to moving something along tracks. The object is picked up and moved to a new location. The clover is translated to Figure T. The dotted lines are there to show you the movement of the figure from the original place to the new location.

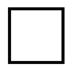
Reflections, rotations, and translations are three basic transformations in a plane. Remember that no matter what type of transformation is used, the figure retains its original size and shape.

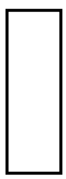
◆ *Find Images of Figures Under Dilations* ◆

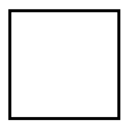
A dilation is a transformation that results in a similar, but not congruent, figure. (Similarity is covered in detail on page 35 in Content Domain III.)

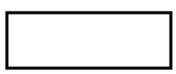
Therefore, "finding the dilated figures" means the same thing as "finding the similar figures." You learned what this means in Domain III, and you need to incorporate that knowledge with the visual facts on a question like the following:

Which of the following shapes is a dilation of Figure *W*?

Figure *W* 

A 

C 

B 

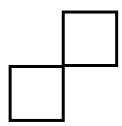
D 

Figure *W* is a square, so a dilation of Figure *W* will be a similar square. Choices A and B are rectangles, and choice D has two squares. The large square in choice C is correct.

◆ *Examine Basic Properties of Various Transformations* ◆

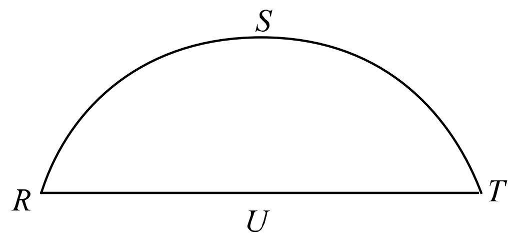
The last two standards addressed four different types of transformations and how they can affect a geometric shape. Problems in this standard assess your ability to recognize and identify when each of these four transformations are being used. The questions will typically feature a coordinate grid. Quite often this grid will have two different shapes on it, an original and a modified shape. Your task will be to determine which transformation—reflection, rotation, translation, or dilation—was used to create the modified figure.

How well you do on this standard will be determined by how well you understand the previous two standards. Make sure you can tell the difference between the four major types of transformations in order to correctly answer the questions for this standard.

◆ *Examine Symmetry, Similarity, and Congruence of Figures* ◆

The terms *symmetry*, *similarity*, and *congruency* should be familiar to you. These terms might be used in conjunction with some type of transformation in order to assess how well you understand the concepts they represent. Apply your skills to this question:

Look at the figure below.



If this figure were rotated clockwise 90 degrees around Point *T*, how many more lines of symmetry would be created in the new figure?

A 0
B 1
C 2
D an infinite number

This problem combines the idea of symmetry with the idea of rotation. In the original figure, $\overleftrightarrow{SU}$ is the only line of symmetry. It divides the figure into two congruent sectors.

If you rotate the figure clockwise 90 degrees, have you changed anything other than the location of the figure? You have not. Remember that rotation does not affect the size or shape of a figure. $\overleftrightarrow{SU}$ is still the only line of symmetry. Even though it was a vertical line in the original figure and a horizontal line in the rotated figure, no additional lines of symmetry were created. The correct answer is A.

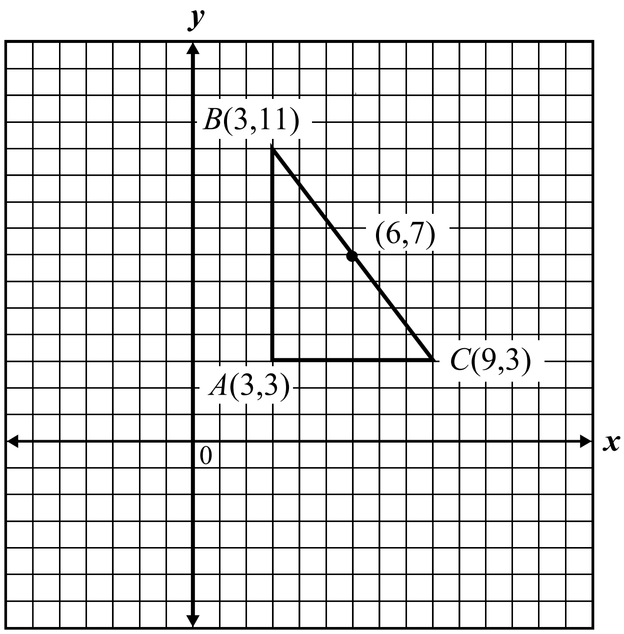
◆ *Graph Ordered Pairs in a Coordinate Plane* ◆

The questions here are the same as the ones presented in the ***Algebra I*** **EOCT** study guide section under Content Domain IV. Review the relevant Algebra I standard entitled "Graph Points on a Coordinate Plane" starting on page 58 to learn about this topic.

◆ *Apply the Distance and Midpoint Formulas* ◆

You already know how to measure distances with a ruler. This standard will explain how to find distances on a grid.

Look at the vertical segment *AB*.



To find the vertical distance from A to B, subtract the *y*-coordinates of these two points and take the absolute value of this difference.

$AB = |y_2 - y_1|$
$AB = |11 - 3|$
$AB = |8|$
$AB = 8$

To find the horizontal distance from *A* to *C*, subtract the *x*-coordinates of these two points and take the absolute value of this difference.

$AC = |x_2 - x_1|$
$AC = |9 - 3|$
$AC = |6|$
$AC = 6$

Finding the distance between Points *B* and *C* is not as straightforward, which is why the distance formula must be used. The distance formula is:

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance formula should look somewhat familiar to you. It is actually the Pythagorean Theorem in a coordinate grid context. Look at $\triangle ABC$. It is a right triangle with legs $\overline{AC}$ and $\overline{AB}$ hypotenuse $\overline{BC}$. The Pythagorean Theorem states

$$BC^2 = AC^2 + AB^2$$

$$BC^2 = \left(x_2 - x_1\right)^2 + \left(y_2 + y_1\right)^2$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The Distance Formula is derived using the Pythagorean Theorem. Substituting coordinates, you can calculate the length of $\overline{BC}$.

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BC = \sqrt{6^2 + 8^2}$$

$$BC = \sqrt{100}$$

$$BC = 10$$

The midpoint formula is much simpler. The midpoint of the segment can be found by averaging the *x*- and *y*-coordinates of the two endpoints. To find the midpoint of *BC*:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Midpoint} = \left(\frac{3+9}{2}, \frac{11+3}{2}\right)$$

$$\text{Midpoint} = \left(\frac{12}{2}, \frac{14}{2}\right)$$

$$\text{Midpoint} = (6, 7)$$

The midpoint of $\overline{BC}$ is at (6, 7).

◆ Find the Slope of a Line, Write an Equation of a Line and Graph Linear Equations ◆

Refer to pages 59 – 68 in the ***Algebra I*** **EOCT** Study Guide for a thorough review of how to find the slope of a line and graph linear equations. The standards include the following:

Define and calculate the slope of a line
Identify the slope and intercepts of a linear equation
Understand and use information on a linear equation graph
Identify specific lines on a graph

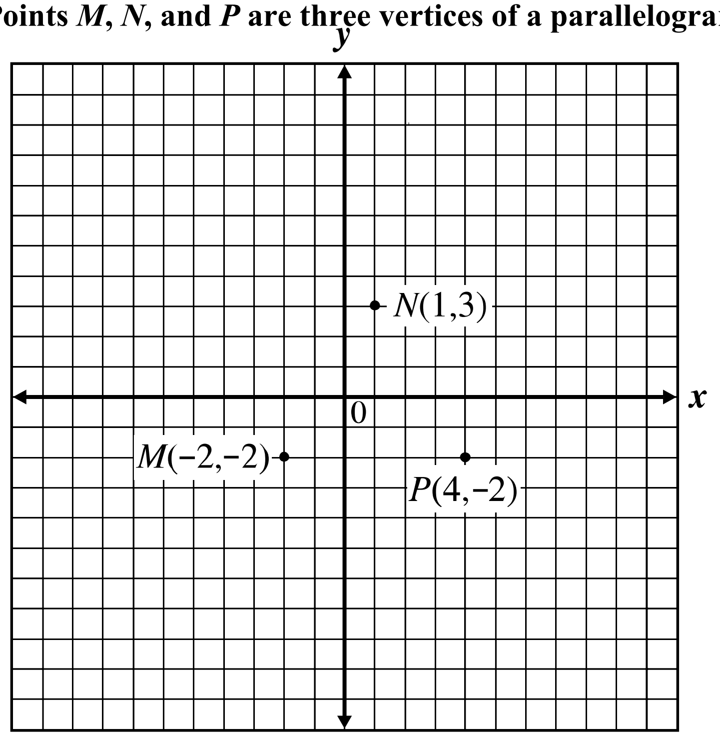
◆ Find the Point of Intersection for Two Lines ◆

Finding a point of intersection for two lines is the same process as solving a system of linear equations. This information can be found in Domain III of the ***Algebra I*** **EOCT** Study Guide on pages 41 – 44. The standard is entitled "Solve a System of Linear Equations."

◆ Use Coordinate Geometry to Explore Properties of Geometric Figures ◆

Problems in this standard assess how well you apply the definitions of geometric figures on coordinate grids. Quite often, this means you must find the coordinates of a missing point to complete the figure correctly.

Points *M*, *N*, and *P* are three vertices of a parallelogram.



Where should Point *O* be placed to complete parallelogram *MNOP*?

A (–6, 3)
B (3, –5)
C (7, 3)
D (–6, –6)

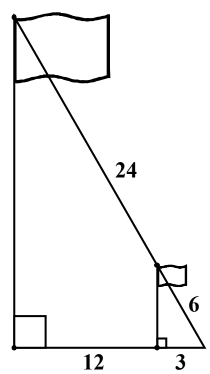
In a parallelogram, opposite sides are parallel and the same length. Segment *MP* has a length of 6.

Placing Point *O* somewhere with a *y*-coordinate of 3 would make $\overline{NO}$ horizontal, and then $\overline{NO} \parallel \overline{MP}$. This eliminates choices B and D. Choice A would give $\overline{NO}$ a length of 7, which is too long since *MP* = 6. However, placing a point at (7, 3) results in *NO* = 6, which makes *MNOP* a parallelogram with congruent and parallel opposite sides. The answer is C.

Sample Questions for Content Domain VI

This section has some sample questions for you to try. After you have answered all of the questions, check your answers in the "Answers to the Content Domain VI Sample Questions" section that follows. This section will give you the correct answer to each question and show you the appropriate steps for solving the problem.

1 **In the figure below, the smaller flagpole is a reduction of the larger.**



What is the scale factor of this dilation?

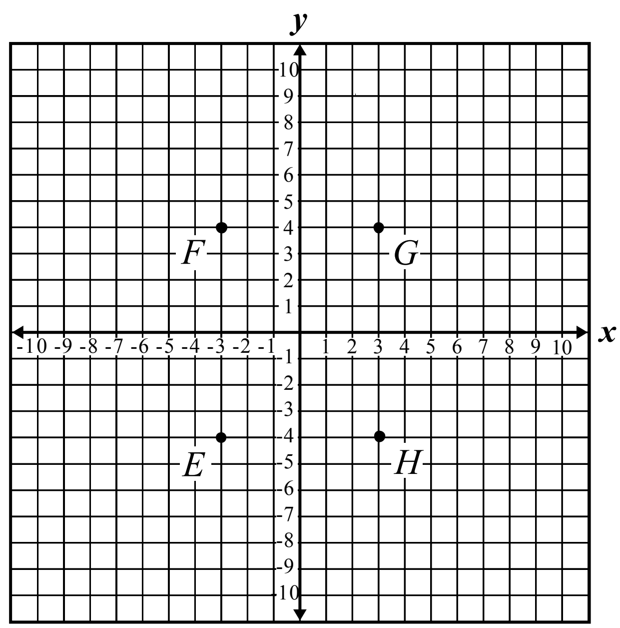
A $\frac{1}{8}$

B $\frac{1}{5}$

C $\frac{1}{4}$

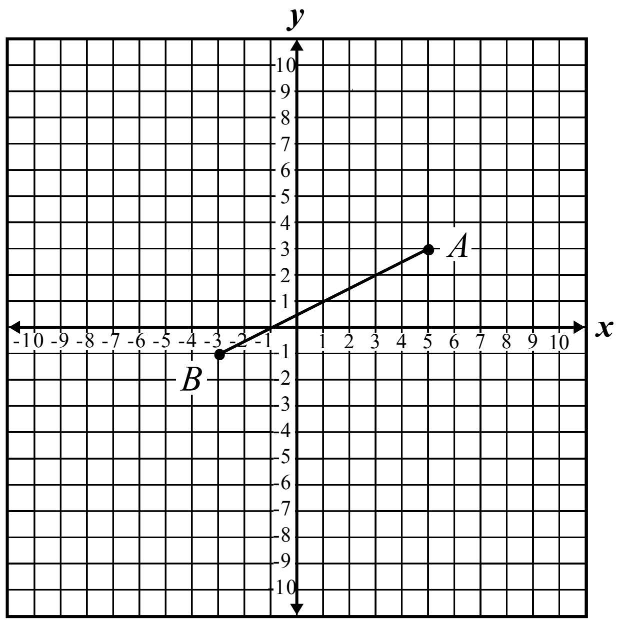
D $\frac{1}{2}$

2 **Which point on the grid below has the same y-coordinate as $P(3,4)$ and the same x-coordinate as $Q(-3,-4)$?**



A E
B F
C G
D H

3 **What are the coordinates of the midpoint of $\overline{AB}$ shown below?**



A $(-4,5)$

B $(-1,1)$

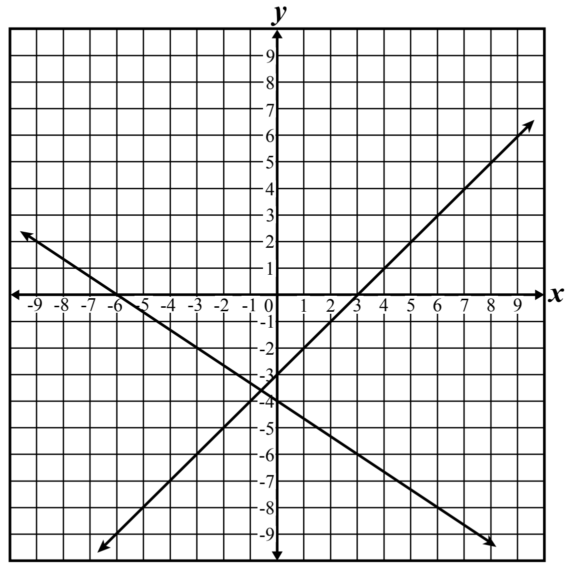
C $\left(\frac{1}{2},-\frac{1}{2}\right)$

D $(1,1)$

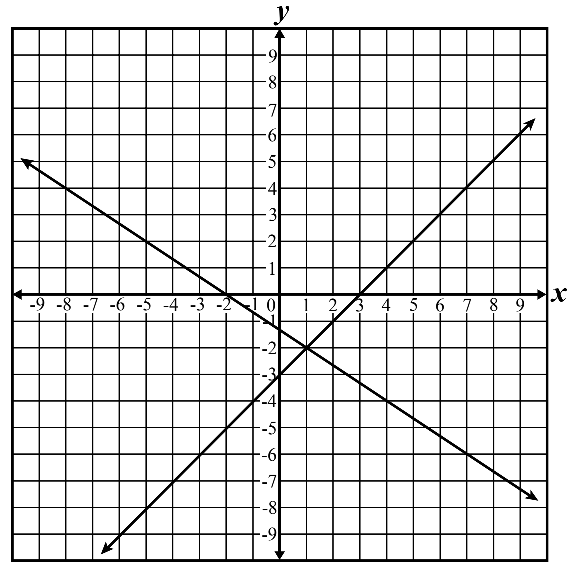
4 Which graph represents the solution to the system of equations below?

$$2x + 3y = -4$$
$$x - y = 3$$

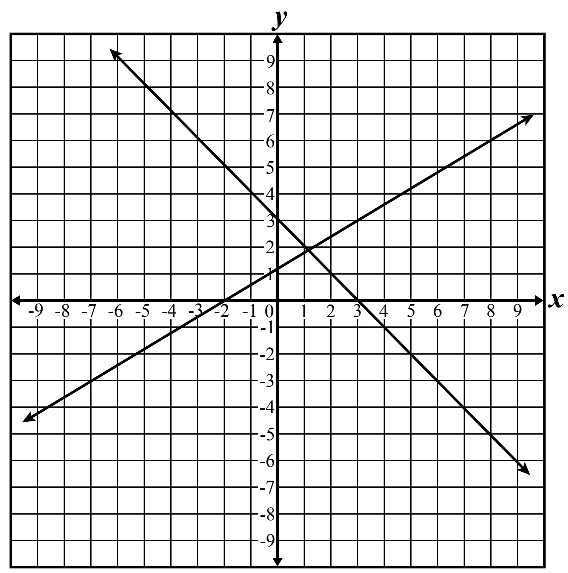
A



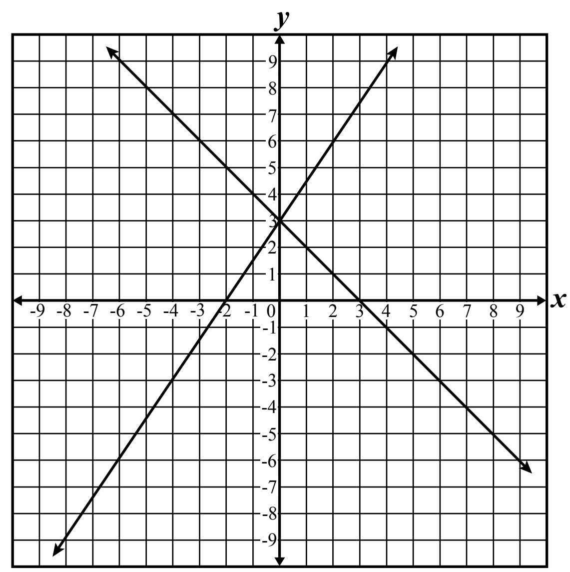
C



B



D



Answers to the Content Domain VI Sample Questions

1. Answer: **B** Standard: *Examine symmetry, similarity, and congruence of figures*
To find the scale factor of the dilated figures, place one set of corresponding sides into fraction form. The hypotenuse of the smaller flagpole is 6. The hypotenuse for the larger flagpole is 30, since 24 + 6 = 30. Placing these values into a ratio gives you $\frac{6}{30}$.

Simplifying the fraction gives you the reduced ratio of $\frac{1}{5}$, choice **B**.

2. Answer: **B** Standard: *Graph ordered pairs in a coordinate plane*
This question can be broken into two parts. First, the *y*-coordinate of Point *P* is 4. Second, the *x*-coordinate of Point *Q* is –3. Therefore, the coordinates of the point that would satisfy the question's requirements would be (–3, 4), or Point *F*. The correct choice is answer **B**.

3. Answer: **D** Standard: *Apply the distance and midpoint formulas*
The coordinates of *A* are (5, 3) and *B* are (–3, –1). Placing these into the midpoint formula gives you the following.

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Midpoint} = \left(\frac{5+(-3)}{2}, \frac{3+(-1)}{2}\right)$$

$$\text{Midpoint} = \left(\frac{2}{2}, \frac{2}{2}\right)$$

Midpoint = (1, 1) This is answer choice **D**.

4. Answer: **C** Standard: *Find the point of intersection for two lines*
You are given two equations, $2x + 3y = -4$ and $x - y = 3$. The solution of the system of equations is the point of intersection. One way to answer this question is to graph the two lines and find where they intersect.

Answer choice **C** shows the two lines graphed correctly.

Appendix A

EOCT Sample Overall Study Plan Sheet

Here is a sample of what an OVERALL study plan might look like. You can use the Blank Overall Study Plan Sheet in Appendix B or create your own.

Materials/Resources I May Need When I Study:
(You can look back at page 2 for ideas.)

1. *This study guide*
2. *Pens*
3. *Highlighter*
4. *Notebook*
5. *Dictionary*
6. *Calculator*
7. *Geometry textbook*

Possible Study Locations:

- First Choice: The library
- Second Choice: My room
- Third Choice: My mom's office

Overall Study Goals:

1. *Read and work through the entire study guide*
2. *Answer the sample questions and study the answers*
3. *Do additional practice with a geometry book*

Number of Weeks I Will Study: *6 weeks*

Number of Days a Week I Will Study: *5 days a week*

Best Study Times for Me:

- Week Days: *7:00 p.m. – 9:00 p.m.*
- Saturday: *9:00 a.m. – 11:00 a.m.*
- Sunday: *2:00 p.m. – 4:00 p.m.*

Appendix B

EOCT Blank Overall Study Plan Sheet

Materials/Resources I May Need When I Study:
(You can look back at page 2 for ideas.)

1. ______________________________________

2. ______________________________________

3. ______________________________________

4. ______________________________________

5. ______________________________________

6. ______________________________________

7. ______________________________________

Possible Study Locations:

- First Choice: ______________________________
- Second Choice: ____________________________
- Third Choice: _____________________________

Overall Study Goals:

1. ______________________________________

2. ______________________________________

3. ______________________________________

Number of Weeks I Will Study: ____

Number of Days a Week I Will Study: _______________________________________

Best Study Times for Me:

- Week Days: _____________________________________
- Saturday: ______________________________________
- Sunday: _______________________________________

Appendix C

EOCT Sample Daily Study Plan Sheet

Here is a sample of what a DAILY study plan might look like. You can use the Blank Daily Study Plan Sheet in Appendix D or create your own.

Materials I May Need Today:
1. *Study Guide*
2. *Pen*
3. *Notebook*
4. *Calculator*

Today's Study Location: *the desk in my room*

Study Time Today: *From 7:00 p.m. to 8:00 p.m. with a short break at 7:30 p.m.*
(Be sure to consider how long you can actively study in one sitting. Can you sit for 20 minutes? 30 minutes? An hour? If you say you will study for three hours, but get restless after 40 minutes, anything beyond 40 minutes may not be productive – you will most likely fidget and daydream your time away. "Doing time" at your desk doesn't count for real studying.)

If I start to get tired or lose focus today, I will: *do some sit-ups.*

Today's Study Goals and Accomplishments: (Be specific. Include things like number of pages, sections, or standards. The more specific you are, the better able you will be to tell if you reached your goals. Keep it REALISTIC. You will retain more if you study small "chunks" or blocks of material at a time.)

Study Task	*Completed*	*Needs more work*	*Needs more information*
1. Review what I learned last time	X		
2. Study the first standard in Content Domain II	X		
3. Study the second standard in Content Domain II		X	

What I learned today:
1. *Reviewed geometric shapes*
2. *Learned geometric terms*
3. *Learned the meanings of right, acute, obtuse, and straight angles*

Today's reward for meeting my study goals: *Eating some popcorn*

Appendix D

EOCT Blank Daily Study Plan Sheet

Materials I May Need Today:

1. ____
2. ____
3. ____
4. ____

Today's Study Location: ____

Study Time Today: ____
(Be sure to consider how long you can actively study in one sitting. Can you sit for 20 minutes? 30 minutes? An hour? If you say you will study for three hours, but get restless after 40 minutes, anything beyond 40 minutes may not be productive – you will most likely fidget and daydream your time away. "Doing time" at your desk doesn't count for real studying.)

If I start to get tired or lose focus today, I will: ______________________________

Today's Study Goals and Accomplishments: (Be specific. Include things like number of pages, sections, or standards. The more specific you are, the better able you will be to tell if you reached your goals. Keep it REALISTIC. You will retain more if you study small "chunks" or blocks of material at a time.)

Study Task	*Completed*	*Needs more work*	*Needs more information*
1.			
2.			
3.			
4.			
5.			

What I learned today:

1. ____
2. ____
3. ____

Today's reward for meeting my study goals: ______________________________