GRADE

8

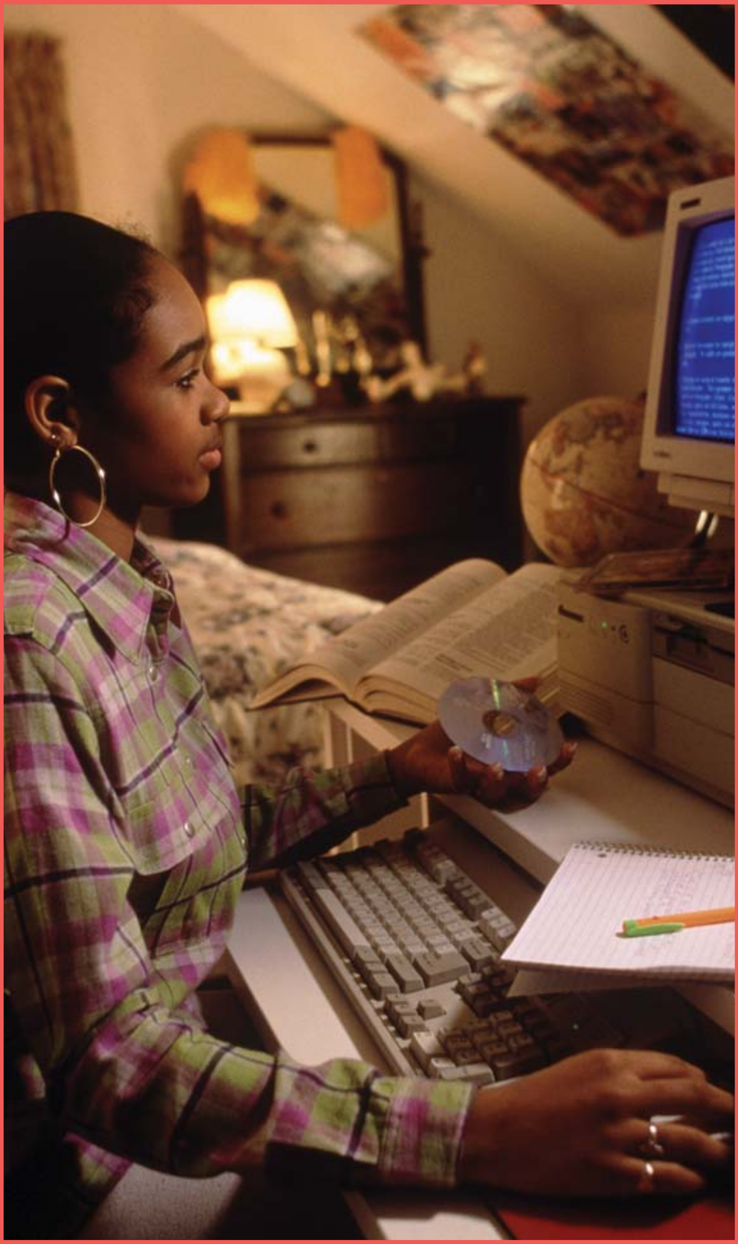
Revised 2007

TAKS STUDY GUIDE

Texas Assessment of Knowledge and Skills

Mathematics

A Student and Family Guide



Revised Based on TEKS Refinements

TAKS STUDY GUIDE

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Mathematics

A Student and Family Guide

A Letter from the Deputy Associate Commissioner for Student Assessment

Dear Student and Parent:

The Texas Assessment of Knowledge and Skills (TAKS) is a comprehensive testing program for public school students in grades 3–11. TAKS, including TAKS (Accommodated) and Linguistically Accommodated Testing (LAT), has replaced the Texas Assessment of Academic Skills (TAAS) and is designed to measure to what extent a student has learned, understood, and is able to apply the important concepts and skills expected at each tested grade level. In addition, the test can provide valuable feedback to students, parents, and schools about student progress from grade to grade.

Students are tested in mathematics in grades 3–11; reading in grades 3–9; writing in grades 4 and 7; English language arts in grades 10 and 11; science in grades 5, 8, 10, and 11; and social studies in grades 8, 10, and 11. Every TAKS test is directly linked to the Texas Essential Knowledge and Skills (TEKS) curriculum. The TEKS is the state-mandated curriculum for Texas public school students. Essential knowledge and skills taught at each grade build upon the material learned in previous grades. By developing the academic skills specified in the TEKS, students can build a strong foundation for future success.

The Texas Education Agency has developed this study guide to help students strengthen the TEKS-based skills that are taught in class and tested on TAKS. The guide is designed for students to use on their own or for students and families to work through together. Concepts are presented in a variety of ways that will help students review the information and skills they need to be successful on TAKS. Every guide includes explanations, practice questions, detailed answer keys, and student activities. At the end of this study guide is an evaluation form for you to complete and mail back when you have finished the guide. Your comments will help us improve future versions of this guide.

There are a number of resources available for students and families who would like more information about the TAKS testing program. Information booklets are available for every TAKS subject and grade. Brochures are also available that explain the Student Success Initiative promotion requirements and the new graduation requirements for eleventh-grade students. To obtain copies of these resources or to learn more about the testing program, please contact your school or visit the Texas Education Agency website at www.tea.state.tx.us/student.assessment.

Texas is proud of the progress our students have made as they strive to reach their academic goals. We hope the study guides will help foster student learning, growth, and success in all of the TAKS subject areas.

Sincerely,

Gloria Zyskowski

Gloria Zyskowski
Deputy Associate Commissioner for Student Assessment
Texas Education Agency

Contents

Mathematics

MATHEMATICS

INTRODUCTION

What Is This Book?

This is a study guide to help you strengthen the skills tested on the Grade 8 Texas Assessment of Knowledge and Skills (TAKS). TAKS is a state-developed test administered with no time limit. It is designed to provide an accurate measure of learning in Texas schools.

By acquiring all the skills taught in eighth grade, you will be better prepared to succeed on the Grade 8 TAKS and during the next school year.

What Are Objectives?

Objectives are goals for the knowledge and skills that students should achieve. The specific goals for instruction in Texas schools were provided by the Texas Essential Knowledge and Skills (TEKS). The objectives for TAKS were developed based on the TEKS.

How Is This Book Organized?

This study guide is divided into the six objectives tested on TAKS. A statement at the beginning of each objective lists the mathematics skills you need to acquire. The study guide covers a large amount of material. You should not expect to complete it all at once. It may be best to work through one objective at a time.

Each objective is organized into review sections and a practice section. The review sections present examples and explanations of the mathematics skills for each objective. The practice sections feature mathematics problems that are similar to the ones used on the TAKS test.

How Can I Use This Book?

First look at your Confidential Student Report. This is the report the school gave you that shows your TAKS scores. This report will tell you which TAKS subject-area test(s) you passed and which one(s) you did not pass. Use your report to determine which skills need improvement. Once you know which skills need to be improved, you can read through the instructions and examples that support those skills. You may also choose to work through all the sections. Pace yourself as you work through the study guide. Work in short sessions. If you become frustrated, stop and start again later.

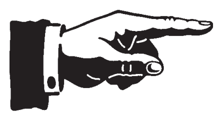
What Are the Helpful Features of This Study Guide?

- There are several words in this study guide that are important for you to understand. These words are boldfaced in the text and are defined when they are introduced. Locate the boldfaced words and review the definitions.
- Examples are contained inside shaded boxes.
- Each objective has "Try It" problems based on the examples in the review sections.
- A Grade 8 Mathematics Chart is included on pages 8–9 and also as a tear-out page in the back of the book. This chart includes useful mathematics information. The tear-out Mathematics Chart in the back of the book also provides both a metric and a customary ruler to help solve problems requiring measurement of length.
- Look for the following features in the margin:

  Ms. Mathematics provides important instructional information for a topic.

Detective Data offers a question that will help remind you of the appropriate approach to a problem.

Do you see that . . . points to a significant sentence in the instruction.



How Should the "Try It" Problems Be Used?

"Try It" problems are found throughout the review sections of the mathematics study guide. These problems provide an opportunity for you to practice skills that have just been covered in the instruction. Each "Try It" problem features lines for your responses. The answers to the "Try It" problems are found immediately following each problem.

While completing a "Try It" problem, cover up the answer portion with a sheet of paper. Then check the answer.

What Kinds of Practice Questions Are in the Study Guide?

The mathematics study guide contains questions similar to those found on the Grade 8 TAKS test. There are two types of questions in the mathematics study guide.

- Multiple-Choice Questions: Most of the practice questions are multiple choice with four answer choices. These questions present a mathematics problem using numbers, symbols, words, a table, a diagram, or a combination of these. Read each problem carefully. If there is a table or diagram, study it. You should read each answer choice carefully before choosing the best answer.
- Griddable Questions: Some practice questions use a seven-column answer grid like those used on the Grade 8 TAKS test.

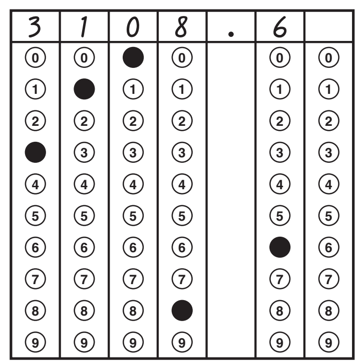
How Do You Use an Answer Grid?

The answer grid contains seven columns, including columns for two decimal places: tenths and hundredths.

Suppose 3,108.6 is the answer to a problem. First write the number in the blank spaces. Be sure to use the correct place value. For example, 3 is in the thousands place, 1 is in the hundreds place, 0 is in the tens place, 8 is in the ones place, and 6 is in the tenths place.

Then fill in the correct bubble under each digit. Notice that if there is a zero in the answer, you need to fill in the bubble for the zero.

The grid shows 3,108.6 correctly entered. The zero in the tens place is bubbled in because it is part of the answer. It is not necessary to bubble in the zero in the hundredths place, because this zero will not affect the value of the correct answer.



Where Can Correct Answers to the Practice Questions Be Found?

The answers to the practice questions are in the answer key at the back of this book (pages 158–168). Each question includes a reference to the page number in the answer key for the answer to the problem. The answer key explains the correct answer, and it also includes some explanations for incorrect answers. After you answer the practice questions, you can check your answers.

If you still do not understand the correct answer after reading the answer explanations, ask a friend, family member, or teacher for help. Even if you have chosen the correct answer, it is a good idea to read the answer explanation because it may help you better understand why the answer is correct.

Texas Assessment of Knowledge and Skills

Grade 8
Mathematics Chart

LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

Metric and customary rulers can be found on the tear-out Mathematics Chart in the back of this book.

Grade 8 Mathematics Chart

Perimeter	square	$P = 4s$
	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	square	$A = s^2$
	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	circle	$A = \pi r^2$
P represents the Perimeter of the Base of a three-dimensional figure.		
B represents the Area of the Base of a three-dimensional figure.		
Surface Area	cube (total)	$S = 6s^2$
	prism (lateral)	$S = Ph$
	prism (total)	$S = Ph + 2B$
	pyramid (lateral)	$S = \frac{1}{2}Pl$
	pyramid (total)	$S = \frac{1}{2}Pl + B$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
Volume	prism	$V = Bh$
	cylinder	$V = Bh$
	pyramid	$V = \frac{1}{3}Bh$
	cone	$V = \frac{1}{3}Bh$
	sphere	$V = \frac{4}{3}\pi r^3$
Pi	π	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Simple Interest Formula		$I = prt$

TX-03300139

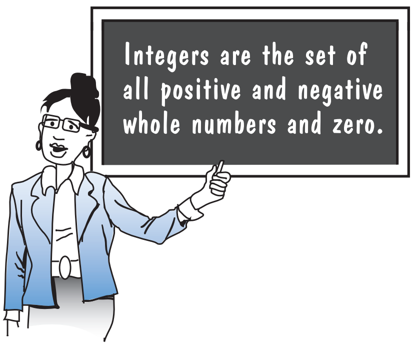
Objective 1

The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

For this objective you should be able to

- understand that different forms of numbers are appropriate for different situations; and
- select and use appropriate operations to solve problems and justify solutions.

What Are Rational Numbers?



Rational numbers are numbers that can be written as the ratio of two integers where zero is not the denominator. A ratio can be expressed as a fraction.

The fraction $\frac{-2}{3}$ is an example of a rational number; it is the ratio of two integers. This rational number could also be written as $-\frac{2}{3}$.

Rational numbers include any real number that can be written as a fraction. Integers, percents, and some decimals are rational numbers.

Type of Number	Example	As a Ratio of Two Integers
An integer	-9	$\frac{-9}{1}$, or $-\frac{9}{1}$
A decimal number that terminates or forms a repeating pattern	0.5 $0.\overline{3}$	$\frac{5}{10}$, or $\frac{1}{2}$ $\frac{1}{3}$
A percent	25%	$\frac{25}{100}$, or $\frac{1}{4}$

How Do You Select the Appropriate Form of a Rational Number to Solve Problems?

When you solve problems that involve rational numbers, you may need to convert the numbers from one form to another. For example, to find a percent of change, convert the percent to a decimal before multiplying. In a problem that includes both fractions and decimals, it may be helpful to convert all the numbers to either fractions or decimals.

At a restaurant Agnes ordered food that cost \$14.89. The tax on her bill was 7%. She gave the waiter a 15% tip on the total bill, including tax. Write an expression that can be used to represent the tip Agnes left.

- First rewrite each of the percents as a decimal.

$$7\% = 0.07$$

$$15\% = 0.15$$

- Represent the 7% tax on Agnes's bill. Multiply 0.07 by the cost of the food.

$$0.07 \cdot \$14.89$$

- Represent the total bill, including tax. Add the tax to the cost of the food.

$$\$14.89 + (0.07 \cdot \$14.89)$$

- Represent the 15% tip Agnes left. Multiply 0.15 by the total bill, including tax.

$$0.15[\$14.89 + (0.07 \cdot \$14.89)]$$

The expression $0.15[\$14.89 + (0.07 \cdot \$14.89)]$ can be used to represent the tip Agnes left.

What Are Irrational Numbers?

Irrational numbers are numbers that cannot be written as the ratio of two integers. An irrational number cannot be expressed precisely in decimal form because the decimal does not terminate or form a repeating pattern.

- The number $\sqrt{2}$ is an irrational number. No matter how many decimal places $\sqrt{2}$ is extended, it is still only an approximation because $\sqrt{2}$ cannot be expressed precisely in decimal form.

  The decimal 1.41 is an approximation of $\sqrt{2}$.

  $$1.41 \cdot 1.41 = 1.9881$$

  The number 1.9881 is close to 2.

  The decimal 1.414213562 is a better approximation of $\sqrt{2}$.

  $$1.414213562 \cdot 1.414213562 = 1.999999999$$

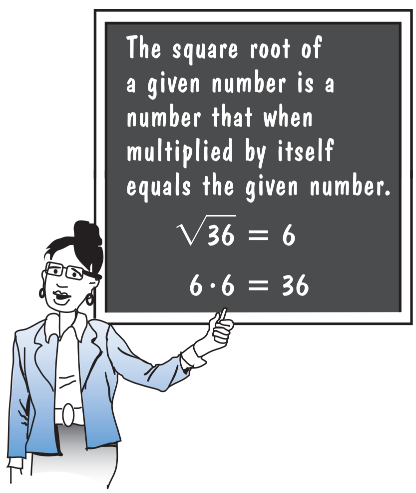
  The number 1.999999999 is closer to 2.

- The number pi, π, is also an irrational number. Here is an estimate of π expressed to 20 decimal places.

  $$3.14159265358979323846$$

  The value of π has been calculated to millions of digits by computers. It is not a repeating decimal. There is no pattern to its digits.

  Since π cannot be expressed precisely in decimal form, π is an irrational number.



Objective 1

How Do You Estimate the Value of an Irrational Number?

One way to estimate the value of an irrational number is to find two consecutive rational numbers with the value of the irrational number between them.

Approximate the value of $\sqrt{32}$.

Find a pair of consecutive integers. The first integer squared should be less than 32. The second integer squared should be greater than 32. The value of $\sqrt{32}$ will be between these two consecutive integers.

Since $5^2 = 25$ and $25 < 32$, the value of $\sqrt{32}$ is greater than 5.

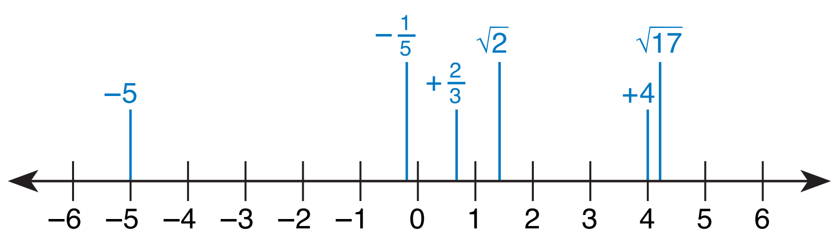
Since $6^2 = 36$ and $36 > 32$, the value of $\sqrt{32}$ is less than 6.

The value of $\sqrt{32}$ is between 5 and 6.

Any decimal number between 5 and 6 is an approximate value of $\sqrt{32}$.

What Is a Real Number?

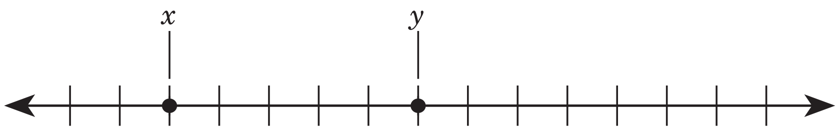
A **real number** is any rational or irrational number. The set of real numbers can be graphed on a number line. Every point on the number line has a real number associated with it.

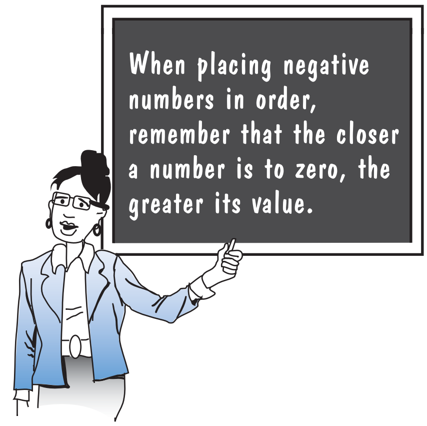


How Do You Compare and Order Rational Numbers?

A number line can help you compare and order rational numbers. On a number line, positive numbers are to the right of 0, and negative numbers are to the left of 0.

Look at this number line with the rational numbers x and y represented.





Use these guidelines to compare and order rational numbers on a number line.

- If x is to the left of y on the number line, then $x < y$.
- If y is to the right of x on the number line, then $y > x$.

The rational numbers $-6\frac{1}{2}$, -3, and 2.7 are graphed on the number line below.

You can use the number line to see that $-6\frac{1}{2} < -3$ and $-3 < 2.7$.

$$-6\frac{1}{2} < -3 < 2.7$$

Objective 1

You can also compare and order rational numbers without a number line. If the rational numbers you are comparing are in different forms, use these guidelines to convert them into the same form.

Conversion	Guideline	Example
Fraction to a decimal	• Divide the numerator by the denominator.	$\frac{3}{4} = 4\overline{)3.00}$ = 0.75
Decimal less than 1 to a fraction	• Use the smallest place value, the one farthest to the right, to determine the denominator of the fraction. • Use the digits to the right of the decimal point to determine the numerator of the fraction.	$0.35 = \frac{35}{100}$
Decimal greater than 1 to a mixed number	• Use the digits to the left of the decimal point as the whole-number part of the mixed number. • Convert the digits to the right of the decimal point to a fraction.	$3.28 = 3\frac{28}{100}$
Decimal to a percent	• Move the decimal point two places to the right. • Put a percent sign after the number.	$0.45 = 45\%$
Percent to a decimal	• Move the decimal point two places to the left. • Drop the percent sign.	$3.5\% = 0.035$
Fraction to a percent	• First convert the fraction to a decimal. • Then convert the decimal to a percent.	$\frac{1}{5} = 0.20 = 20\%$
Percent to a fraction	• Express the percent as a fraction with a denominator of 100. • If the percent is greater than 100%, it may be expressed as a mixed number.	$35\% = \frac{35}{100}$ $125\% = \frac{125}{100} = 1\frac{25}{100}$

Objective 1

If the rational numbers you wish to compare are in the same form, use these rules to compare and order them.

- Order decimal numbers by comparing the digits in each place value from left to right.

  If the decimal numbers do not have the same number of decimal places, write zeros behind the last digit to the right of the decimal point. This does not change the value of a decimal. For example, 3.51 is equal to 3.5100.

- If two fractions have the same denominator, compare their numerators.

  For example, $\frac{5}{23} < \frac{8}{23}$ because $5 < 8$.

- If two fractions do not have the same denominator, find a common denominator and then compare the numerators.

  Compare $\frac{2}{3}$ and $\frac{5}{11}$. Since $\frac{2}{3}$ is equal to $\frac{22}{33}$ and $\frac{5}{11}$ is equal to $\frac{15}{33}$, compare $\frac{22}{33}$ and $\frac{15}{33}$. Since $22 > 15$, then $\frac{22}{33} > \frac{15}{33}$ and $\frac{2}{3} > \frac{5}{11}$.

Place this list of numbers in order from least to greatest.

$$0.35, \quad -4\tfrac{1}{2}, \quad -1, \quad \tfrac{3}{8}, \quad 30\%$$

- One way to help you order the numbers is to write them all in decimal form.

$$-4\frac{1}{2} = -4.5$$

$$\frac{3}{8} = 0.375$$

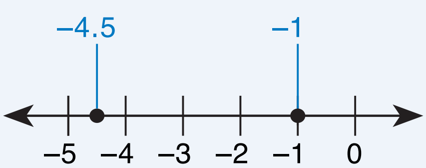
$$30\% = 0.30$$

- To compare the decimals, write each number to three decimal places.

$$0.350, \quad -4.500, \quad -1.000, \quad 0.375, \quad 0.300$$

- Order the negative numbers first.

  When graphed on a number line, -4.5 is to the left of -1.



$-4.500 < -1.000$

The number -4.500 is the smallest, so write it first.

The number -1.000 is the next smallest. Write it after -4.500.

- Order the three remaining numbers: 0.350, 0.375, and 0.300. They all have the same value, 3, in the tenths place. Look at the hundredths place: $0.3\underline{5}0$, $0.3\underline{7}5$, $0.3\underline{0}0$.

  Since $0 < 5$, then $0.300 < 0.350$.

  Since $5 < 7$, then $0.350 < 0.375$.

- List the numbers in order from least to greatest.

  $-4.500, \quad -1.000, \quad 0.300, \quad 0.350, \quad 0.375$

Written in their original form, the list of numbers in order from least to greatest is: $-4\frac{1}{2}, \quad -1, \quad 30\%, \quad 0.35, \quad \frac{3}{8}$.

Try It

For five days last winter, Clara's science class recorded the lowest outdoor temperatures in degrees Celsius. The results are shown below.

Lowest Outdoor Temperatures

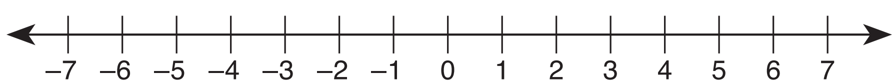
Day	Lowest Temperature (°C)
Monday	3
Tuesday	−2
Wednesday	−1
Thursday	5
Friday	−4

List these temperatures in order from warmest to coldest. List the days in order from warmest to coldest.

One way to solve this problem is to graph the temperatures on a

______________________________.

Place a mark on the number line below for each temperature.



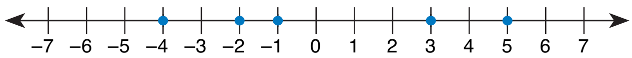
List the temperatures in order from warmest to coldest:

_______°C, _______°C, _______°C, _______°C, and _______°C.

Use the table to match the temperatures to their corresponding days.

The days listed in order from warmest to coldest are __________, __________, __________, __________, and __________.

One way to solve this problem is to graph the temperatures on a number line.



The temperatures listed in order from warmest to coldest are 5°C, 3°C, −1°C, −2°C, and −4°C. The days listed in order from warmest to coldest are Thursday, Monday, Wednesday, Tuesday, and Friday.

How Do You Solve Problems Involving Rational and Irrational Numbers?

You solve problems involving rational and irrational numbers in the same way you solve any other problem. First understand the problem. Identify the quantities involved and the relationships between them. Write an equation that can be used to find the answer. Solve the equation and then check your answer to see whether it is reasonable.

At the grocery store bananas cost \$0.55 for 2 pounds, and melons cost \$0.19 per pound. James bought 3 pounds of bananas and a melon that weighed 4.77 pounds. What was the total cost of the fruit he bought?

- Find the amount that James spent on bananas.

  Write a proportion, which is a statement that shows two ratios are equal.

  Find two ratios that compare the number of pounds of bananas purchased to the cost of the bananas. The price of 2 pounds of bananas is \$0.55, and James bought 3 pounds of bananas for x.

  $$\frac{\text{pounds}}{\text{cost}} \qquad \frac{2}{0.55} = \frac{3}{x}$$

  To solve the proportion for x, find the cross products and divide by 2.

  $$2x = 3 \cdot 0.55$$
  $$2x = 1.65$$
  $$x = 0.825$$

  Round 0.825 to the nearest cent. James paid \$0.83 for the bananas.

- Find the cost of a melon.

  Multiply \$0.19, the cost per pound for melons, by 4.77, the weight of the melon James bought.

  $$0.19 \cdot 4.77 = 0.9063$$

  Round 0.9063 to the nearest cent. James paid \$0.91 for the melon.

- Find the total cost of the fruit James bought.

  $$0.83 + 0.91 = 1.74$$

James paid a total of \$1.74 for the bananas and the melon.

The area of a square is 12 square centimeters. Find a reasonable estimate of the length of a side of the square.

- Use the formula for the area of a square.

$$A = s^2$$

- Substitute 12 into the formula for A, the area of the square.

$$12 = s^2$$

- Solve for s. Take the square root of both sides of the equation.

$$\sqrt{12} = \sqrt{s^2}$$
$$\sqrt{12} = s$$

- To approximate the value of $\sqrt{12}$, find two consecutive integers with the value of $\sqrt{12}$ between them. The first integer squared should be less than 12, and the second integer squared should be greater than 12.

$$3^2 = 9 \text{ and } 4^2 = 16$$
$$\sqrt{9} < \sqrt{12} < \sqrt{16}$$
$$3 < \sqrt{12} < 4$$

Since $\sqrt{12}$ is between 3 and 4, any number between 3 and 4 is a reasonable estimate of the length of a side of the square.

Try It

Darren has a circular piece of paper that covers an area of 157 square inches. What is the approximate radius in inches of the piece of paper?

The formula for the area of a circle is ________________.

Substitute ________ for A, the area of the circle.

$$\underline{\qquad\qquad} = \pi r^2$$

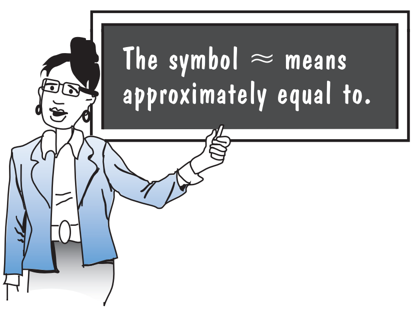
To solve the equation, divide both sides of the equation by

_________. Use 3.14 as an estimate of the value of π.

$$\frac{\square}{3.14} \approx \frac{3.14\,r^2}{3.14}$$

$$\underline{\qquad\qquad} \approx r^2$$

$$r \approx \sqrt{\underline{\qquad\quad}}$$



To approximate the value of $\sqrt{\_\_\_\_\_\_}$, find two consecutive integers with the value of $\sqrt{\_\_\_\_\_\_}$ between them.

The first integer squared should be less than ________, and the second integer squared should be greater than ________.

Since ________ $= 49$ and ________ $= 64$, the value of $\sqrt{\_\_\_\_\_\_}$ is between ________ and ________.

An answer slightly larger than _______ would be reasonable.

The radius of the paper would be slightly larger than ________ inches.

The formula for the area of a circle is $A = \pi r^2$. Substitute 157 for A, the area of the circle: $157 = \pi r^2$. To solve the equation, divide by π.

$$\frac{157}{3.14} \approx \frac{3.14r^2}{3.14}$$
$$50 \approx r^2$$
$$r \approx \sqrt{50}$$

To approximate the value of $\sqrt{50}$, find two consecutive integers with the value of $\sqrt{50}$ between them. The first integer squared should be less than 50, and the second integer squared should be greater than 50. Since $7^2 = 49$ and $8^2 = 64$, the value of $\sqrt{50}$ is between 7 and 8. An answer slightly larger than 7 would be reasonable. The radius of the paper would be slightly larger than 7 inches.

How Do You Determine Whether the Answer to a Problem Is Reasonable?

One way to determine whether the answer to a problem is reasonable is to estimate the solution and see how big or small the answer should be. Then compare your estimate to the answer you calculated. The estimate and your calculation should be close to each other.

You can estimate an answer by rounding all the numbers in a problem before doing any calculations. Then perform the operations with the rounded numbers. Think about how rounding the numbers before calculating will affect the answer. Determine whether the exact answer should be greater or less than your estimate.

Tom measured a rectangular box. The dimensions were 5.8 inches, 8.1 inches, and 3.9 inches. He then calculated the box's volume to be about 19,000 cubic inches. Is 19,000 in.^{3} a reasonable value for the volume of the box?

To decide whether Tom's answer is reasonable, estimate the volume.

- The volume of a rectangular prism (Tom's box) is $V = lwh$.
- One way to estimate the volume is by rounding each dimension to the nearest whole number.

  5.8 rounds to 6

  8.1 rounds to 8

  3.9 rounds to 4
- The volume of the box is approximately $6 \cdot 8 \cdot 4$, or about $48 \cdot 4$. This is close to $50 \cdot 4$, which is 200 in.^{3} The answer should be close to 200 in.^{3}

Tom's calculation of 19,000 in.^{3} is not close to the estimate of 200 in.^{3}, so it is not a reasonable value for the volume of the box.

Try It

Gloria knows that the radius of a circular table is between 9 and 10 inches. Is it reasonable to say that the circumference of the table is about 60 inches?

The formula for the circumference of a circle is

$C =$ ________________________ .

Find the smallest and largest possible values for the circumference by substituting first _______ and then _______ for the radius of the table in the circumference formula.

$C \approx 2($_______$)($_______$); C \approx$ _______

$C \approx 2($_______$)($_______$); C \approx$ _______

The circumference of the table is between _______ and _______ inches, so 60 inches __________ a reasonable estimate of the circumference of the table.

The formula for the circumference of a circle is $C = 2\pi r$. Find the smallest and largest possible values for the circumference by substituting first 9 and then 10 for the radius of the table in the circumference formula.

$$C \approx 2(3.14)(9);\ C \approx 56.52$$

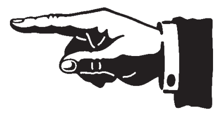
$$C \approx 2(3.14)(10);\ C \approx 62.8$$

The circumference of the table is between 56.52 and 62.8 inches, so 60 inches is a reasonable estimate of the circumference of the table.

What Is Scientific Notation?

Scientific notation is a way of expressing numbers using powers of 10. Using scientific notation helps keep track of decimal places in very large or very small numbers and makes it easier to do arithmetic with them.

When a number is expressed in scientific notation, it is written as the product of a factor and a power of 10. The factor must be a number that is equal to or greater than 1 but less than 10. Look at these two examples.



Do you see that . . .

Written in scientific notation, 2,300,000,000 is 2.3×10^9.

Written in scientific notation, 0.00000000034 is 3.4×10^{-10}.

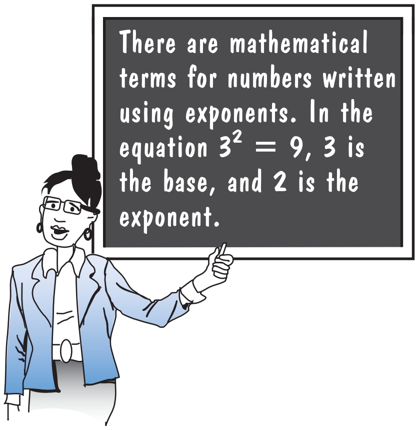
Powers of 10

$10^1 = 10$	$10^{-1} = \frac{1}{10} = 0.1$
$10^2 = 10 \cdot 10 = 100$	$10^{-2} = \frac{1}{100} = 0.01$
$10^3 = 10 \cdot 10 \cdot 10 = 1{,}000$	$10^{-3} = \frac{1}{1{,}000} = 0.001$
$10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$	$10^{-4} = \frac{1}{10{,}000} = 0.0001$

Objective 1

How Do You Convert Between Scientific and Standard Notation?

Numbers written in regular place-value form are in **standard notation**. For example, the numbers 34,285 and 5.7 are both in standard notation.



Scientific to Standard Notation

To change a number from scientific to standard notation, move the decimal point the number of places shown in the exponent of 10.

- If the exponent of 10 is positive, the number in standard notation will be greater than or equal to 10. Move the decimal point to the right.

$$2.35 \times 10^5 = 235{,}000.$$

1 2 3 4 5

- If the exponent of 10 is negative, the number in standard notation will be less than 1. Move the decimal point to the left.

$$7.05 \times 10^{-5} = 0.0000705$$

5 4 3 2 1

Standard to Scientific Notation

To change a number from standard to scientific notation, move the decimal point until the number is greater than or equal to 1 and less than 10. The exponent of 10 is the number of places you moved the decimal point.

- If the number is greater than or equal to 10, move the decimal point to the left. Make the exponent positive.

$$5{,}200{,}000. = 5.2 \times 10^6$$

6 5 4 3 2 1

- If the number is less than 1, move the decimal point to the right. Make the exponent negative.

$$0.000000164 = 1.64 \times 10^{-7}$$

1 2 3 4 5 6 7

Write 17,800 in scientific notation.

- The number 17,800 is greater than 10. Move the decimal point to the left until you have a factor that is greater than or equal to 1 but less than 10.
- Move the decimal point four places to the left so the factor is 1.78.
- Since the decimal point was moved four places to the left, the exponent of 10 is 4, so write 10^4.

The number 17,800 written in scientific notation is 1.78×10^4.

Write 0.000063 in scientific notation.

- The number 0.000063 is less than 1. Move the decimal point to the right until you have a factor that is equal to or greater than 1 but less than 10.
- Move the decimal point five places to the right so the factor is 6.3.
- Since the decimal point was moved five places to the right, the exponent of 10 is -5, so write 10^{-5}.

The number 0.000063 written in scientific notation is 6.3×10^{-5}.

Write 1.41×10^4 in standard notation.

- The exponent of 10 is 4. Move the decimal point four places to the right.
- The factor 1.41 does not have four digits to the right of the decimal point. Write two zeros to the right of the hundredths place in order to have four places.

$$1.41 = 1.4100$$

- Move the decimal point four places to the right to get the number 14,100.

The expression 1.41×10^4 written in standard notation is 14,100.

Write 3.9×10^{-3} in standard notation.

- The exponent of 10 is -3. Move the decimal point three places to the left.
- The factor 3.9 does not have three digits to the left of the decimal point. Write two zeros to the left of the ones place in order to have three places.

$$3.9 = 003.9$$

- Move the decimal point three places to the left to get the number 0.0039.

The expression 3.9×10^{-3} written in standard notation is 0.0039.

Try It

The speed of light written in scientific notation is 2.99793×10^8 meters per second. Express the speed of light in standard notation.

The exponent of 10 is ________.

Move the decimal point ________ places to the ________.

In standard notation the speed of light is ______________________ meters per second.

The exponent of 10 is 8. Move the decimal point eight places to the right. In standard notation the speed of light is 299,793,000 meters per second.

Try It

A microscopic cell measures 0.0038 centimeter in diameter. Express the diameter of the cell in scientific notation.

To obtain a number equal to or greater than _________ but less than _________, move the decimal point _________ places to the _____________.

The exponent of 10 is _________.

In scientific notation the diameter of the cell is _____________ cm.

To obtain a number equal to or greater than 1 but less than 10, move the decimal point three places to the right. The exponent of 10 is -3. In scientific notation the diameter of the cell is 3.8×10^{-3} cm.

Now practice what you've learned.

Question 1

Brian's science teacher told him that he needed a score of at least 82% on his next quiz to earn a passing grade in the class. Which fraction represents a quiz score high enough for Brian to earn a passing grade?

A $\frac{15}{20}$

B $\frac{22}{25}$

C $\frac{80}{100}$

D $\frac{39}{50}$

Answer Key: page 158

Question 2

Which list shows the following rational numbers in order from greatest to least?

$$0.29,\ -5\frac{1}{2},\ -1.5,\ \frac{3}{5},\ 45\%$$

A $\frac{3}{5},\ 45\%,\ 0.29,\ -1.5,\ -5\frac{1}{2}$

B $-5\frac{1}{2},\ -1.5,\ \frac{3}{5},\ 45\%,\ 0.29$

C $45\%,\ 0.29,\ \frac{3}{5},\ -5\frac{1}{2},\ -1.5$

D $\frac{3}{5},\ 0.29,\ 45\%,\ -5\frac{1}{2},\ -1.5$

Answer Key: page 158

Question 3

Five friends went out to lunch together. The bill for all their meals was \$42.25 before tax. The tax was 4% of the bill. The friends split the total bill, including tax, evenly. Which expression can be used to find the amount of tax that each person paid?

A $(42.25 + 0.04) \div 5$

B $(42.25 \cdot 0.04) \cdot 5$

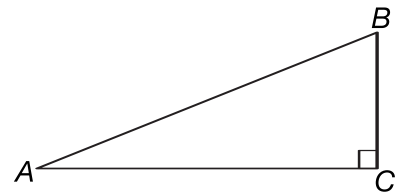
C $(42.25 \div 0.04) \cdot 5$

D $(42.25 \cdot 0.04) \div 5$

Answer Key: page 158

Question 4

In the triangle below, the length of side AB is $\sqrt{29}$ inches. What is the approximate value of $\sqrt{29}$?



A Between 4 and 5

B Between 3 and 4

C Between 5 and 6

D Between 6 and 7

Answer Key: page 158

Question 5

The distance from Earth to the moon is approximately 384,000 kilometers. Which of these numbers shows the approximate distance in kilometers from Earth to the moon in scientific notation?

A 3.84×10^{6}

B 3.84×10^{5}

C 3.84×10^{-6}

D 3.84×10^{-5}

Answer Key: page 158

Question 6

At the local grocery store, beans cost $0.88 per pound, and bread costs $1.48 for 2 loaves. Which equation can be used to find t, the total cost if Don buys 1.19 pounds of beans and 1 loaf of bread?

A $t = (1.19 + 0.88) + (1.48 + 2)$

B $t = (1.19 \cdot 0.88) + (2 \cdot 1.48)$

C $t = (1.19 \cdot 0.88) + (1.48 \div 2)$

D $t = (1.19 \cdot 0.88) + (2 \div 1.48)$

Answer Key: page 158

Question 7

A car-rental company advertises a price of $230.93 to rent a car for 7 days. At that rate, what would it cost in dollars and cents to rent a car for 10 days?

Record your answer and fill in the bubbles. Be sure to use the correct place value.

				.		
0	0	0	0		0	0
1	1	1	1		1	1
2	2	2	2		2	2
3	3	3	3		3	3
4	4	4	4		4	4
5	5	5	5		5	5
6	6	6	6		6	6
7	7	7	7		7	7
8	8	8	8		8	8
9	9	9	9		9	9

Answer Key: page 158

Question 8

A phone company charges $29.50 per month for the first 1,000 minutes of local calls and $0.04 a minute for any local calls beyond 1,000 minutes. Last month Carol made 1,215 minutes of local calls. What was her total cost for local calls last month?

A $29.50

B $48.60

C $38.10

D $78.10

Answer Key: page 159

Question 9

Mario wrote four checks in the following amounts: $8.39, $12.22, $11.48, and $19.02. He estimated the total of the checks he had written by adding 8, 12, 11, and 19. Which of these statements best describes the actual total of the checks?

A Less than the estimate, because the values Mario added were all less than the actual amounts

B Greater than the estimate, because the values Mario added were all less than the actual amounts

C Less than the estimate, because the values Mario added were all greater than the actual amounts

D Greater than the estimate, because the values Mario added were all greater than the actual amounts

Answer Key: page 159

Question 10

Barb cut out a circular piece of stained glass with a diameter of 8.1 centimeters. She then calculated the area of the piece of glass. Which is a reasonable value for Barb's calculation of the area of the piece of glass?

A $206\ \text{cm}^2$

B $824\ \text{cm}^2$

C $50\ \text{cm}^2$

D $25\ \text{cm}^2$

Answer Key: page 159

Question 11

Albert earns $78 for working 6 hours. Which equation could be used to find the number of dollars, d, that Albert earns in 9 hours?

A $\frac{78}{d} = \frac{9}{6}$

B $78d = 6 \cdot 9$

C $6 + 9 = 78 + d$

D $6d = 78 \cdot 9$

Answer Key: page 159

Objective 2

The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

For this objective you should be able to

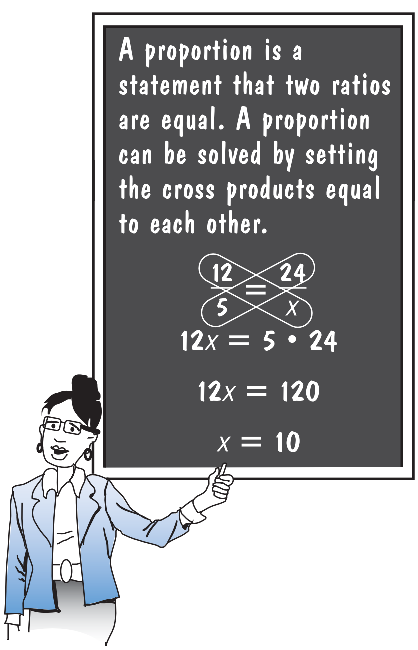
- identify proportional or non-proportional linear relationships in problem situations and solve problems;
- make connections among various representations of a numerical relationship; and
- use graphs, tables, and algebraic representations to make predictions and solve problems.

What Is a Proportional Relationship?

A **ratio** is a comparison of two quantities. A **proportion** is a statement that two ratios are equal. There are many real-life problems that involve proportional relationships. For example, you can use proportions when converting units of measurement. You also can use proportions to solve problems involving percent and rates.

To solve problems that involve proportional relationships, follow these guidelines.

- Identify the ratios to be compared. Be certain to compare the corresponding quantities in the same order.
- Write a proportion using the two ratios.
- Solve the proportion.



How many inches are in 7.5 feet?

- The number of inches in 7.5 feet is proportional to the number of inches in 1 foot. Let x represent the number of inches in 7.5 feet. Write a proportion.

$$\frac{\text{inches}}{\text{feet}} \quad \frac{x}{7.5} = \frac{12}{1}$$

- Solve the proportion.

$$\frac{x}{7.5} = \frac{12}{1}$$

$$1 \cdot x = 7.5 \cdot 12$$

$$x = 90$$

There are 90 inches in 7.5 feet.

Objective 2

In some problems you may be asked whether two ratios form a proportion.

A survey of 100 drivers in Dallas found that 52 drove cars, and the remaining people surveyed drove other types of vehicles. A second survey of 250 drivers found that 127 drove cars. Are these two survey results proportional?

To determine whether the two survey results are proportional, compare the ratios of the number of people who drove cars to the total number of people surveyed. If the two ratios are equal, the results are proportional.

- In the first survey the ratio of car drivers to the number of people surveyed is $\frac{52}{100}$. In the second survey the ratio of car drivers to the number of people surveyed is $\frac{127}{250}$.
- The relationship is proportional if the two ratios are equal.

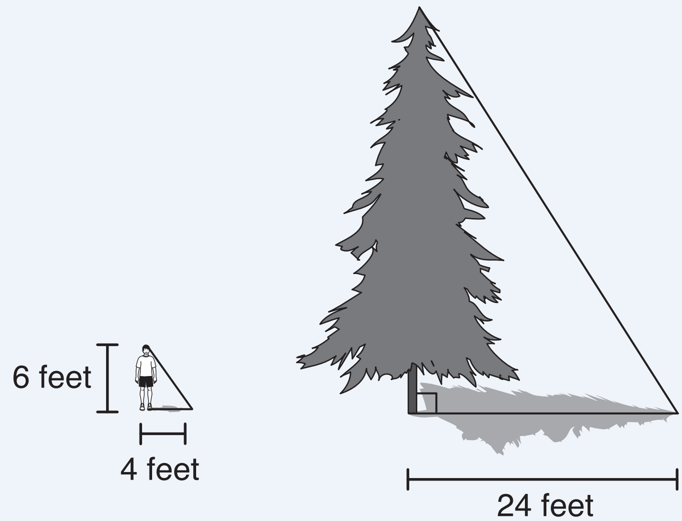
$$\frac{52}{100} \stackrel{?}{=} \frac{127}{250}$$

- Compare the cross products. Are they equal?

$$52 \cdot 250 \stackrel{?}{=} 100 \cdot 127$$
$$13{,}000 \neq 12{,}700$$

Since the cross products are not equal, the ratios are not equal. This means that the ratios do not form a true proportion.

Julio is 6 feet tall. While standing in the sun, he measured his shadow. It was 4 feet long. At the same time the tree near him cast a shadow 24 feet long. What is the height of the tree in feet?



- Both Julio and the tree form a right angle with the ground. The triangles involving Julio and the tree, pictured above, are similar.
- Because the right triangles are similar, this problem involves a proportional relationship. The ratio of Julio's height to his shadow's length is equal to the ratio of the tree's height to its shadow's length.
- Write a proportion. Set the two corresponding ratios of height to shadow length equal to each other. Let x equal the height of the tree in feet.

$$\frac{\text{height}}{\text{shadow}} \quad \frac{6}{4} = \frac{x}{24}$$

- Solve the proportion using cross products.

$$6 \cdot 24 = 4x$$

$$144 = 4x$$

$$\frac{144}{4} = \frac{4x}{4}$$

$$36 = x$$

The tree is 36 feet tall.

Try It

Gina knows that 2 out of every 3 of her relatives live in Texas. If Gina has 48 relatives, how many live in Texas?

This problem involves a ______________ relationship.

Let n represent the total number of Gina's relatives who ______________________.

One ratio comparing the number of Gina's relatives who live in Texas to the total number of her relatives is $\frac{\square}{\square}$.

Another ratio that compares the number of Gina's relatives who live in Texas to the total number of her relatives is $\frac{n}{\square}$.

Write a ______________, a statement that two ratios are ________________.

$$\frac{\square}{\square} = \frac{n}{\square}$$

Solve the proportion by setting the ____________________ equal to each other.

$$2 \cdot \underline{\qquad\quad} = \underline{\qquad\quad}\, n$$

$$\underline{\qquad\quad} = 3n$$

$$\frac{\square}{\square} = \frac{3n}{\square}$$

$$\underline{\qquad\quad} = n$$

Gina has _________ relatives who live in Texas.

This problem involves a proportional relationship. Let n represent the total number of Gina's relatives who live in Texas. One ratio comparing the number of Gina's relatives who live in Texas to the total number of her relatives is $\frac{2}{3}$. Another ratio that compares the number of Gina's relatives who live in Texas to the total number of her relatives is $\frac{n}{48}$. Write a proportion, a statement that two ratios are equal.

$$\frac{2}{3} = \frac{n}{48}$$

Solve the proportion by setting the cross products equal to each other.

$$2 \cdot 48 = 3n$$

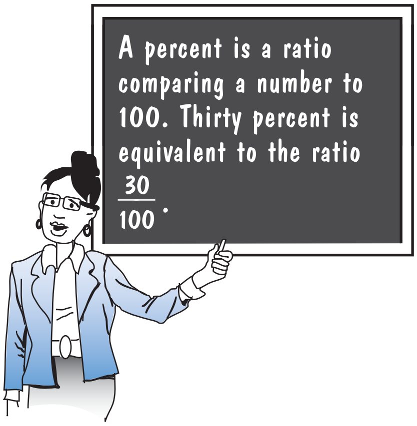
$$96 = 3n$$

$$\frac{96}{3} = \frac{3n}{3}$$

$$32 = n$$

Gina has 32 relatives who live in Texas.

Objective 2



Problems that involve percent can also be solved using proportions.

Mr. Cruz asked the seniors at a high school where they would like to go on a class trip. Of those surveyed, 65 percent voted for Washington, D.C. If there are 320 seniors, how many voted for Washington, D.C.?

First write a proportion that can be used to find n, the number of seniors who voted for Washington, D.C.

- Of those surveyed, 65 percent voted for Washington, D.C. A percent is a ratio that compares a number to 100. Write a ratio that compares 65 to 100.

$$\frac{65}{100}$$

- There are 320 seniors, and n of those seniors voted for Washington, D.C. Write a ratio that compares n to 320.

$$\frac{n}{320}$$

- Write a proportion, a statement that two ratios are equal.

$$\frac{65}{100} = \frac{n}{320}$$

Solve the proportion. Set the cross products equal to each other.

$$65 \cdot 320 = 100n$$

$$20{,}800 = 100n$$

$$\frac{20{,}800}{100} = \frac{100n}{100}$$

$$208 = n$$

So 208 seniors voted to go to Washington, D.C., on a class trip.

Try It

A can of mixed nuts contains 4 ounces of walnuts, 2 ounces of pecans, and 10 ounces of peanuts. What percent of the total weight of nuts in the can is made up of peanuts?

Let x represent the percent of the nuts' weight that is made up of peanuts.

The total weight of nuts in the can is

__________ + __________ + __________ = __________ ounces.

Write a proportion.

$$\frac{\text{weight of peanuts}}{\text{total weight}} \qquad \frac{\square}{\square} = \frac{x}{100}$$

Use cross products to solve the proportion.

$$\_\_\_\_ \cdot 100 = \_\_\_\_\, x$$

$$\_\_\_\_ = \_\_\_\_\, x$$

$$\_\_\_\_ = x$$

Peanuts make up ______% of the total weight of nuts in the can.

The total weight of nuts in the can is $4 + 2 + 10 = 16$ ounces. Write a proportion.

$$\frac{10}{16} = \frac{x}{100}$$

$$10 \cdot 100 = 16x$$

$$1{,}000 = 16x$$

$$62.5 = x$$

Peanuts make up 62.5% of the total weight of nuts in the can.

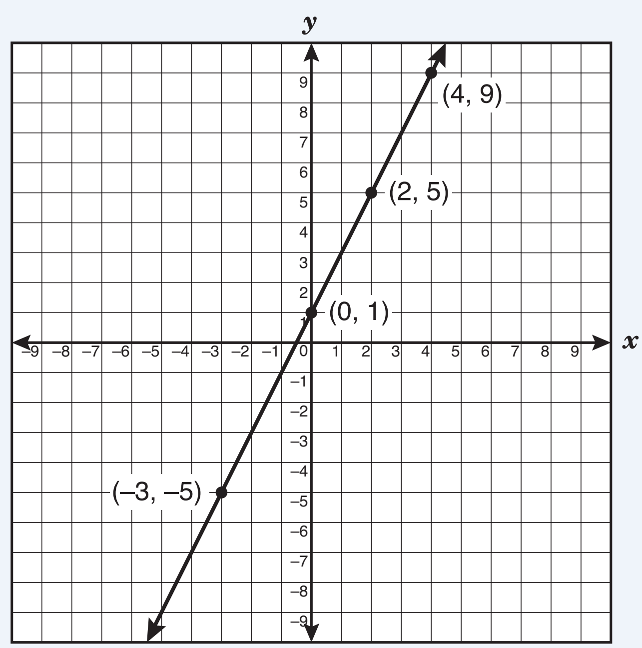
How Do You Compare Different Representations of a Relationship?

Sometimes you are asked to compare one representation of a relationship to another representation that is written in a different form. For example, you might be asked to compare a table to a graph or to find an equation to describe a relationship given in words.

The following table and graph are different representations of the same relationship.

x	y
–3	–5
0	1
2	5
4	9

Every ordered pair in the table is also a point on the line graphed below. Every point on the line fits the pattern shown in the table.



To see whether two different representations of a relationship are equivalent, follow these guidelines.

- See whether the pairs of related data from one representation fit the pattern for the other representation.
- Look for exceptions. Finding just one pair of values that works in both representations is not sufficient. Check as many pairs as possible.

When customers use a calling card, a phone company charges a 55-cent connection fee plus 3 cents for each minute the phone conversation lasts. The table below shows the cost of calls lasting 3, 5, 8, and 12 minutes.

Calling-Card Charges

Length of Call (minutes), x	Cost of Call (cents), y
3	64
5	70
8	79
12	91

Could you use the equation $y = 3x + 55$ to find y, the cost in cents of a calling-card telephone call lasting x minutes?

You are being asked whether two different representations of a relationship, a table and an equation, are equivalent.

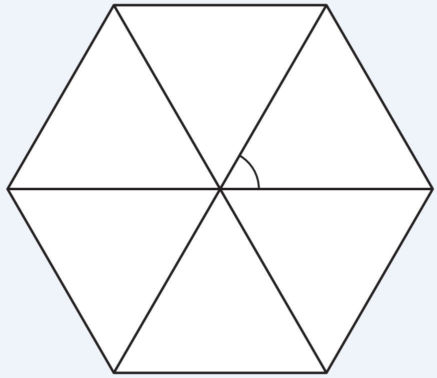
Substitute the values from the table for x and y and see whether they satisfy the equation $y = 3x + 55$.

x	$y = 3x + 55$	y	Yes/No
3	$64 \stackrel{?}{=} 3(3) + 55$ $64 \stackrel{?}{=} 9 + 55$ $64 = 64$	64	Yes
5	$70 \stackrel{?}{=} 3(5) + 55$ $70 \stackrel{?}{=} 15 + 55$ $70 = 70$	70	Yes
8	$79 \stackrel{?}{=} 3(8) + 55$ $79 \stackrel{?}{=} 24 + 55$ $79 = 79$	79	Yes
12	$91 \stackrel{?}{=} 3(12) + 55$ $91 \stackrel{?}{=} 36 + 55$ $91 = 91$	91	Yes

All the pairs of related numbers from the table fit the pattern for the equation.

If this pattern continues, the equation $y = 3x + 55$ could be used to find y, the cost in cents of a calling-card telephone call lasting x minutes.

To find the measures of the angles at the center of regular polygons, divide 360° by the number of sides in the polygon.



Does the table below show this relationship?

Number of Sides (n)	Angle Measure
4	90°
5	72°
6	60°
7	50°
8	45°

You are being asked whether two different representations of a relationship, a verbal description and a table, are equivalent.

Pick values from the table for the number of sides and the measure of the angle and see whether they satisfy the verbal description of this relationship.

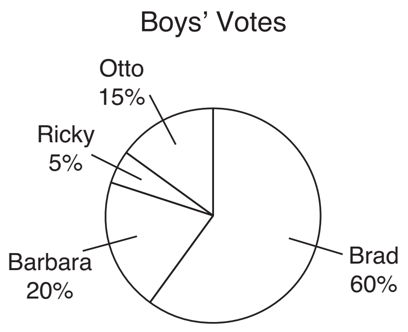
n	$\frac{360°}{n}$	Angle Measure	Yes/No
4	$\frac{360°}{4} = 90°$	90°	Yes
5	$\frac{360°}{5} = 72°$	72°	Yes
6	$\frac{360°}{6} = 60°$	60°	Yes
7	$\frac{360°}{7} = 51.42°$	50°	No
8	$\frac{360°}{8} = 45°$	45°	Yes

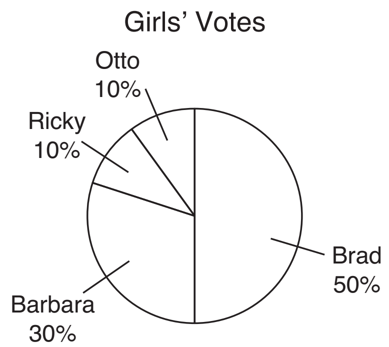
Not all the pairs of related numbers from the table fit the verbal description; there is one exception. The table's values do not match the verbal description of the relationship when the polygon has 7 sides.

The table does not match the verbal description of this relationship.

Try It

The circle graphs below show the percent of boys and girls in the junior class at Donley High School who voted for various candidates for class president.





Does the table below accurately represent the information in the graphs?

Junior Class President Election

Candidate	Boys' Votes	Girls' Votes
Brad	180	100
Barbara	60	60
Ricky	15	20
Otto	45	20

The two circle graphs give the voting data in the form of

____________________.

To compare the graphs to the table, convert the data in the table to

____________________.

To find what percent of the boys voted for Brad, ________________ the number of boys who voted for Brad by the total number of boys. Then convert the quotient to a percent.

_________ ÷ 300 = _________

_________ = _________%

According to the table, _________% of the boys voted for Brad.

Compare this value to the circle graph. The graph shows that _________% of the boys voted for Brad.

These numbers agree, but _________ of the pairs of numbers must agree if these two representations are equivalent.

Complete the table below by converting the numbers of votes listed in the original table to percents.

Junior Class President Election

Candidate	Boys' Votes	Girls' Votes
Brad	60%	50%
Barbara		
Ricky		
Otto		

All the data in the table ________________ with the data in the graphs. Therefore, the table accurately represents the information in the graphs.

The two circle graphs give the voting data in the form of percents. To compare the graphs to the table, convert the data in the table to percents. To find what percent of the boys voted for Brad, divide the number of boys who voted for Brad by the total number of boys.

$$180 \div 300 = 0.60$$

$$0.60 = 60\%$$

According to the table, 60% of the boys voted for Brad. The graph shows that 60% of the boys voted for Brad. These numbers agree, but all of the pairs of numbers must agree if these two representations are equivalent.

Junior Class President Election

Candidate	Boys' Votes	Girls' Votes
Brad	60%	50%
Barbara	20%	30%
Ricky	5%	10%
Otto	15%	10%

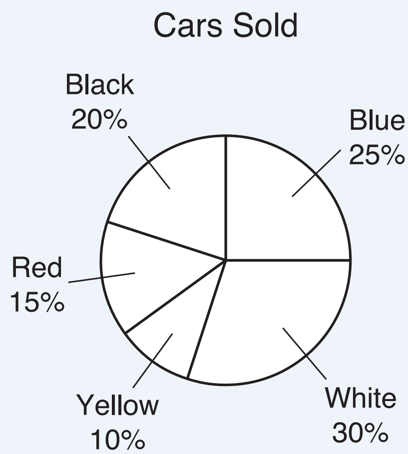
All the data in the table agree with the data in the graphs.

How Can Problems Be Solved Using Tables, Graphs, or Equations?

Problems that involve tables, graphs, and equations can be solved in the same way as other problems.

- Understand the problem. Identify the quantities that are involved and the relationships between those quantities.
- Write an equation you can use to solve the problem.
- Solve the equation. Answer the problem.
- See whether the answer you obtained to the problem is reasonable.

The graph shows the colors of cars a dealership sold in January.



The dealership expects to sell a total of 140 cars in February. If the number of red cars the dealership expects to sell in February is proportional to the number of red cars sold in January, how many red cars should the dealership expect to sell in February?

- Identify the percent of cars sold in January that were red. According to the graph, 15 percent of the cars sold in January were red.
- Use this percent to write a proportion that can be used to find the number of red cars that the dealership should expect to sell in February.

$$15\% = \frac{15}{100}$$
$$\frac{15}{100} = \frac{x}{140}$$
$$15 \cdot 140 = 100x$$
$$2{,}100 = 100x$$
$$x = 21$$

The dealership should expect to sell 21 red cars in February.

The table shows the total number of people who swam at a new neighborhood pool the first four days that it was open.

Pool Attendance

Day	Number of Swimmers
Monday	55
Tuesday	70
Wednesday	85
Thursday	100

The pool management schedules one lifeguard for each group of 40 or fewer swimmers. If the attendance pattern at the pool continues, how many lifeguards should be scheduled for Friday?

- First use the pattern of data in the table to predict Friday's attendance.

  From Monday to Tuesday, the attendance increased by 15 swimmers ($70 - 55 = 15$).

  From Tuesday to Wednesday, the attendance increased by 15 swimmers ($85 - 70 = 15$).

  From Wednesday to Thursday, the attendance increased by 15 swimmers ($100 - 85 = 15$).

  You can use this pattern to predict that Friday's attendance will be 15 swimmers greater than Thursday's attendance.

  $$100 + 15 = 115$$

  If the pattern continues, Friday's attendance should be 115 swimmers.

- For each group of 40 or fewer swimmers, the management schedules one lifeguard. Calculate the number of groups of 40 in 115 swimmers.

  $$115 \div 40 = 2.88$$

  There are 2 groups of 40 swimmers and 1 group of fewer than 40 swimmers. A lifeguard is needed for each group of 40 or fewer swimmers.

If the pattern continues, 3 lifeguards should be scheduled for Friday.

Try It

The table below describes an electric company's charges (c) in terms of the total number of kilowatt-hours of electricity (n) used during the month.

Residential Electric Billing Rates

Kilowatt-Hours Used per Month (n)	0–1,000	1,001–2,000	$> 2,000$
Amount Charged in Dollars (c)	$c = 30 + 0.05n$	$c = 35 + 0.04n$	$c = 50 + 0.03n$

The Wilson family used 1,650 kilowatt-hours of electricity last month. What was their electric bill for the month?

Since 1,650 is between 1,001 and 2,000, use the second formula in the table, ________________, to determine the Wilsons' electric bill for the month.

Substitute __________ for n in the formula.

$$c = 35 + 0.04n$$

$$c = 35 + 0.04(\_\_\_\_\_\_\_\_\_\_)$$

$$c = 35 + \_\_\_\_\_\_\_\_\_\_$$

$$c = \_\_\_\_\_\_\_\_\_\_$$

The Wilsons' electric bill for the month was $__________.

Since 1,650 is between 1,001 and 2,000, use the second formula in the table, $c = 35 + 0.04n$, to determine the Wilsons' electric bill for the month. Substitute 1,650 for n in the formula.

$$c = 35 + 0.04n$$

$$c = 35 + 0.04(1{,}650)$$

$$c = 35 + 66$$

$$c = 101$$

The Wilsons' electric bill for the month was $101.

How Can You Use an Algebraic Expression to Represent Any Term in a Sequence?

A **sequence** is a set of numbers written in a particular order. For example, 1, 7, 13, 19 is a sequence of four numbers. The number 1 is the first term in the sequence, 7 is the second term, 13 is the third term, and 19 is the fourth term.

What rule can be used to find the *n*th term in this sequence?

$$4, 8, 12, 16, \ldots$$

Look at the relationship between the terms in the sequence and their position in the sequence.

Position	1	2	3	4	. . .	n
Value of Term	4	8	12	16	. . .	?

The 1st term is $4 \cdot 1 = 4$.

The 2nd term is $4 \cdot 2 = 8$.

The 3rd term is $4 \cdot 3 = 12$.

The 4th term is $4 \cdot 4 = 16$.

Each term in this sequence is equal to 4 times its position number in the sequence.

Represent the relationship algebraically. The value of the *n*th term is $4 \cdot n$, or $4n$.

What is the 15th term in this sequence?

$$4, 7, 10, 13, \ldots$$

- First compare the position of a term to its value.

Position	Value of Term
1	4
2	7
3	10
4	13
n	?

- Look for a pattern that shows the relationship between a term's value and its position number.

  The 1st term is 4. Maybe the pattern is $4n$.

  The 2nd term is 7. Does the pattern $4n$ work for the second term? No. If n is 2, then $4n = 4(2) = 8$. The pattern does not work.

  Try a different multiple. Then add or subtract from the product to get the correct value. Try $3n + 1$.

  The rule works for the 1st term: $3(1) + 1 = 4$.

  The rule works for the 2nd term: $3(2) + 1 = 7$.

- Check to see whether the rule works for the next two terms in the sequence.

  The 3rd term in this sequence is 10 because $3(3) + 1 = 10$.

  The 4th term in this sequence is 13 because $3(4) + 1 = 13$.

  The nth term in the sequence is $3n + 1$.

- Find the value of the 15th term in the sequence. For the 15th term in the sequence, n is 15. Substitute 15 for n in the rule $3n + 1$.

$$3(15) + 1 = 45 + 1 = 46$$

The 15th term in the sequence is 46.

Objective 2

Look at this sequence of numbers.

$$-4, -6, -8, -10, -12, \ldots$$

Does the expression $-2(n + 1)$ show the relationship between any term and n, its position in the sequence?

Use the table to see whether this rule works for all the terms in the sequence.

Position	$-2(n + 1)$	Value of Term	Correct?
1	$-2(1 + 1) = -4$	-4	Yes
2	$-2(2 + 1) = -6$	-6	Yes
3	$-2(3 + 1) = -8$	-8	Yes
4	$-2(4 + 1) = -10$	-10	Yes
5	$-2(5 + 1) = -12$	-12	Yes

For the sequence $-4, -6, -8, -10, -12, \ldots$, the expression $-2(n + 1)$ shows the relationship between any term and n, its position in the sequence.

Try It

Which algebraic expression best describes the nth term in the sequence 3, 5, 7, 9, . . . , where n represents a term's position in the sequence?

$3n \qquad 2n + 1 \qquad n + 3$

Check each expression for the values of n given in the table. Identify the rule that produces the correct term in each case.

Position	Rule #1 $3n$	Rule #2 $2n + 1$	Rule #3 $n + 3$	Value of Term	Rule(s) Producing the Correct Value
1	$3(1) = 3$	$2(1) + 1$ $2 + 1 = 3$	$1 + 3 = 4$	3	Rule #1 Rule #2
2				5	
3				7	
4				9	

The expression ____________________ describes the nth term in the sequence because it is the only rule that works for all four numbers given.

Position	Rule #1 $3n$	Rule #2 $2n + 1$	Rule #3 $n + 3$	Value of Term	Rule(s) Producing the Correct Value
1	$3(1) = 3$	$2(1) + 1$ $2 + 1 = 3$	$1 + 3 = 4$	3	Rule #1 Rule #2
2	$3(2) = 6$	$2(2) + 1$ $4 + 1 = 5$	$2 + 3 = 5$	5	Rule #2 Rule #3
3	$3(3) = 9$	$2(3) + 1$ $6 + 1 = 7$	$3 + 3 = 6$	7	Rule #2
4	$3(4) = 12$	$2(4) + 1$ $8 + 1 = 9$	$4 + 3 = 7$	9	Rule #2

The expression $2n + 1$ describes the nth term in the sequence because it is the only rule that works for all four numbers given.

Now practice what you've learned.

Question 12

Sam can type 280 words in 8 minutes. If Sam continues to type at the same rate, which equation can be used to find n, the number of words he can type in half an hour?

A $\frac{280}{8} = \frac{n}{2}$

B $\frac{280}{8} = \frac{n}{30}$

C $\frac{280}{8} = \frac{30}{n}$

D Not Here

Answer Key: page 159

Question 13

Briana delivers newspapers. She can deliver 60 papers in 45 minutes. Which of these represents an equivalent rate of delivering newspapers?

A 30 papers in $\frac{1}{2}$ hour

B 75 papers in 1 hour

C 120 papers in $1\frac{1}{2}$ hours

D 100 papers in 1 hour

Answer Key: page 159

Question 14

At Austin Shoe Factory 5 pairs of shoes, on average, can be placed in shoe boxes every 3 minutes. At this rate, how many pairs of shoes can be placed in shoe boxes during 8 hours of work?

A 48

B 2,880

C 13.3

D 800

Answer Key: page 160

Question 15

At Cantor Middle School 78% of the students ride the bus to school. If 975 students ride the bus, how many students attend the school?

A 760

B 975

C 1,250

D 1,053

Answer Key: page 160

Question 16

The Williamson Lumber Company charges a fee of $25 for a lumber delivery plus an additional fee based on the number of pieces of lumber being delivered.

Williamson Lumber Company

Number of Pieces of Lumber	Additional Fee
20	$4.80
30	$7.20
40	$9.60
50	$12.00
100	$24.00

Which equation can be used to find the total cost in dollars, c, to deliver n pieces of lumber?

A $c = 0.48n + 25$

B $c = 0.24n + 25$

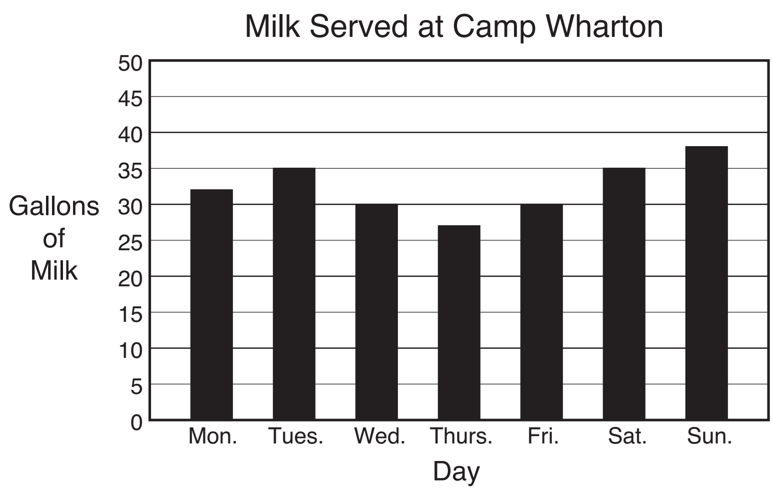
C $c = 2.5n + 4.80$

D $c = 0.24n \cdot 25$

Answer Key: page 160

Question 17

Camp Wharton serves milk to its campers at every meal. The graph below shows the number of gallons of milk that was served each day during one week.



Which table best represents the information in the graph?

A

Milk Served at Camp Wharton

Day	Milk (gallons)
Monday	32
Tuesday	35
Wednesday	30
Thursday	27
Friday	30
Saturday	35
Sunday	38

C

Milk Served at Camp Wharton

Day	Milk (gallons)
Monday	30
Tuesday	35
Wednesday	30
Thursday	25
Friday	30
Saturday	35
Sunday	40

B

Milk Served at Camp Wharton

Day	Milk (gallons)
Monday	35
Tuesday	35
Wednesday	30
Thursday	30
Friday	30
Saturday	35
Sunday	40

D

Milk Served at Camp Wharton

Day	Milk (gallons)
Monday	32
Tuesday	35
Wednesday	30
Thursday	27
Friday	30
Saturday	38
Sunday	35

Answer Key: page 161

Question 18

Linden Bank pays its customers interest on money kept in savings accounts. The table shows how much interest will be earned on $1,500 for different numbers of years the money is kept in the account.

Number of Years	2	3	4	5	11
Interest Earned	$195.00	$292.50	$390.00	$487.50	

Use the information in the table to determine how much interest in dollars and cents will be earned at Linden Bank in 11 years.

Record your answer and fill in the bubbles. Be sure to use the correct place value.

				.		
0	0	0	0		0	0
1	1	1	1		1	1
2	2	2	2		2	2
3	3	3	3		3	3
4	4	4	4		4	4
5	5	5	5		5	5
6	6	6	6		6	6
7	7	7	7		7	7
8	8	8	8		8	8
9	9	9	9		9	9

Answer Key: page 161

Question 19

A swim club charges its members a $25 annual membership fee plus $2 every time a member visits the pool. If Joanne spent a total of $365 last year in swim club charges, how many times did she visit the pool?

A 183

B 170

C 195

D 158

Answer Key: page 161

Question 20

Let n represent a term's position in a sequence. Which algebraic expression can be used to find the nth term of the sequence below?

$$2, 5, 8, 11, 14, \ldots$$

A $3n - 1$

B $2n$

C $3n + 2$

D $2n + 1$

Answer Key: page 161

Question 21

A sequence of numbers was formed using the rule $\frac{n+1}{3}$, where n represents the number's position in a sequence. Which sequence fits this rule?

A $\frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \frac{7}{3}, \frac{8}{3}, \ldots$

B $\frac{4}{3}, \frac{7}{3}, \frac{10}{3}, \frac{13}{3}, \frac{16}{3}, \ldots$

C $\frac{2}{3}, \frac{5}{3}, \frac{8}{3}, \frac{11}{3}, \frac{14}{3}, \ldots$

D $\frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \ldots$

Answer Key: page 161

Objective 3

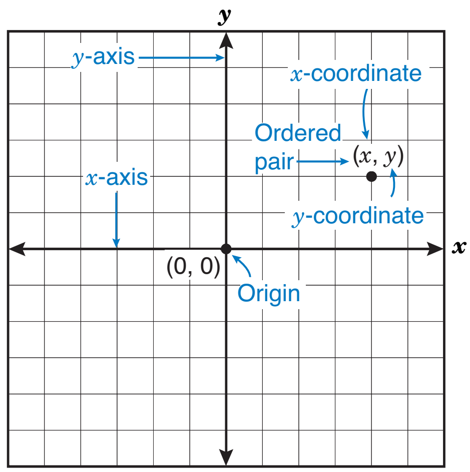
The student will demonstrate an understanding of geometry and spatial reasoning.

For this objective you should be able to

- use transformational geometry to develop spatial sense; and
- use geometry to model and describe the physical world.

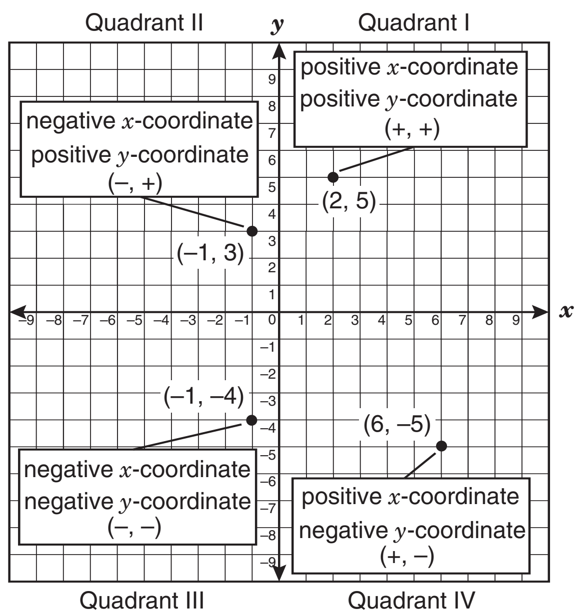
How Can You Locate and Name Points on a Coordinate Plane?

A **coordinate grid** is used to locate and name points on a plane. The coordinate grid is formed by two perpendicular number lines. A point is located by using an **ordered pair** of numbers. The two numbers that form the ordered pair are called **coordinates** of the point.



Do you see that . . .

The x-axis and y-axis divide the coordinate plane into 4 regions called **quadrants**. The quadrants are usually referred to by the Roman numerals I, II, III, and IV.



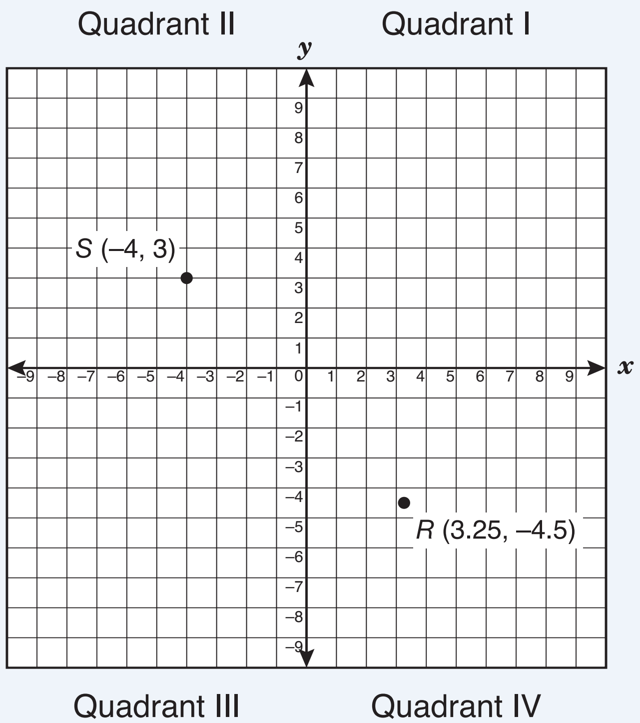
In which quadrants are points R (3.25, −4.5) and S (−4, 3) located?

For point R (3.25, −4.5):

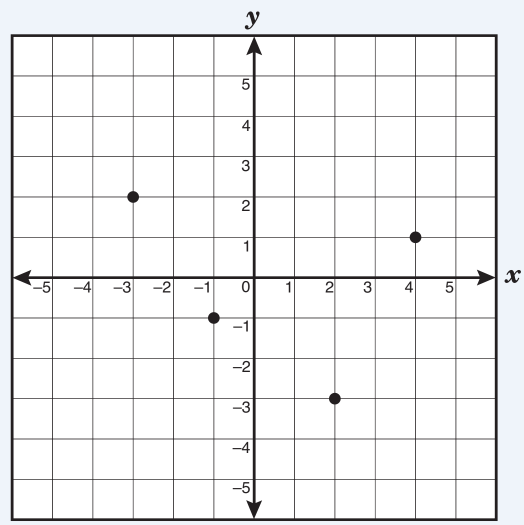
- The x-coordinate is 3.25, a positive value. The point is 3.25 units to the right of the origin, $\frac{1}{4}$ of the way between 3 and 4.
- The y-coordinate is −4.5, a negative value. The point is 4.5 units below the origin, halfway between −4 and −5.
- Point R (3.25, −4.5) is located in Quadrant IV because it has a positive x-coordinate and a negative y-coordinate.

For point S (−4, 3):

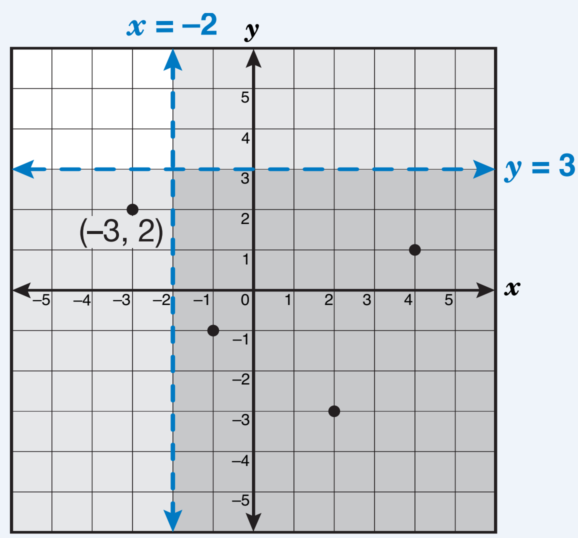
- The x-coordinate is −4, a negative value. The point is 4 units to the left of the origin.
- The y-coordinate is 3, a positive value. The point is 3 units above the origin.
- Point S (−4, 3) is located in Quadrant II because it has a negative x-coordinate and a positive y-coordinate.



Which point on the graph below does not meet the requirements $x > -2$ and $y < 3$?



- Draw a dashed line through $x = -2$. All points to the right of this line have an x-coordinate greater than -2.
- Draw a dashed line through $y = 3$. All points below this line have a y-coordinate less than 3.



Only the point with coordinates $(-3, 2)$ does not fall within the darker shaded region that satisfies the two inequalities.

Which of the three points below is on the line $y = 3x + 4$ and also in Quadrant II?

$A\ (-3, 5)$ $\quad B\ (-1, 1)$ $\quad C\ (1, 7)$

- Consider the coordinates of point A $(-3, 5)$. If x is replaced by -3 and y is replaced by 5, is the equation true?

$$y = 3x + 4$$
$$5 \stackrel{?}{=} 3(-3) + 4$$
$$5 \stackrel{?}{=} -9 + 4$$
$$5 \neq -5$$

No. Therefore, point $A\ (-3, 5)$ is not on the line $y = 3x + 4$.

- Consider the coordinates of point $B\ (-1, 1)$. If x is replaced by -1 and y is replaced by 1, is the equation true?

$$y = 3x + 4$$
$$1 \stackrel{?}{=} 3(-1) + 4$$
$$1 \stackrel{?}{=} -3 + 4$$
$$1 = 1$$

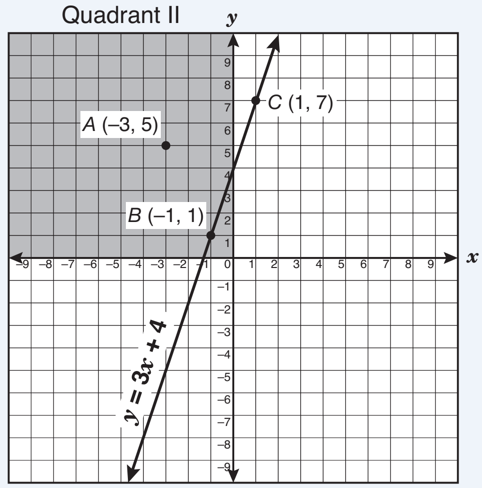
Yes. Therefore, point $B\ (-1, 1)$ is on the line $y = 3x + 4$.

Is point $B\ (-1, 1)$ in Quadrant II? Yes. It has a negative x-coordinate and a positive y-coordinate.

- Consider the coordinates of point $C\ (1, 7)$. If x is replaced by 1 and y is replaced by 7, is the equation true?

$$y = 3x + 4$$
$$7 \stackrel{?}{=} 3(1) + 4$$
$$7 \stackrel{?}{=} 3 + 4$$
$$7 = 7$$

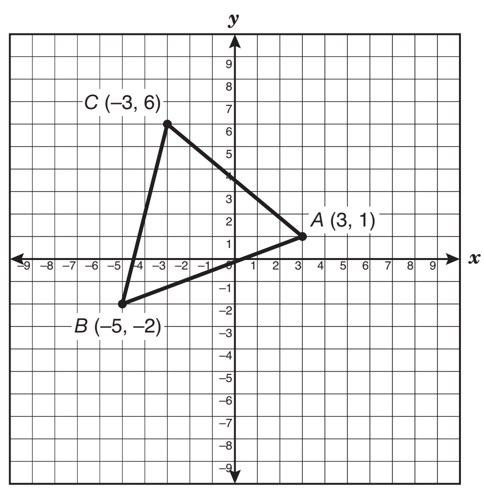
Yes. Therefore, point $C\ (1, 7)$ is on the line $y = 3x + 4$. Is point $C\ (1, 7)$ in Quadrant II? No. It has a positive x-coordinate and a positive y-coordinate, which means it is in Quadrant I.

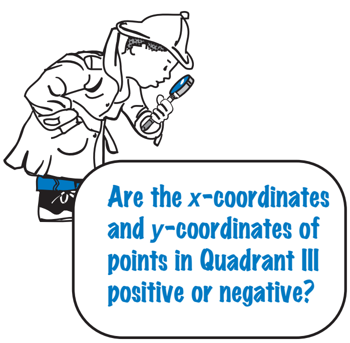


Only point B is on the line $y = 3x + 4$ and also in Quadrant II.

Try It

Which vertex of $\triangle ABC$ is in Quadrant III?





- For point A (3, 1)

  The x-coordinate is ________ , a positive value.

  The y-coordinate is ________ , a ________________ value.

  The point is located in Quadrant ________ because it has a positive x-coordinate and a positive y-coordinate.

- For point B (−5, −2)

  The x-coordinate is ________ , a negative value.

  The y-coordinate is ________ , a ________________ value.

The point is located in Quadrant ________ because it has a negative x-coordinate and a negative y-coordinate.

- For point C $(-3, 6)$

  The x-coordinate is ________ , a ______________ value.

  The y-coordinate is ________ , a ______________ value.

  The point is located in Quadrant ________ because it has a negative x-coordinate and a positive y-coordinate.

Point ________ is the only vertex of $\triangle ABC$ that is in Quadrant III.

For point A (3, 1)
The x-coordinate is 3, a positive value. The y-coordinate is 1, a positive value. The point is located in Quadrant I because it has a positive x-coordinate and a positive y-coordinate.

For point B $(-5, -2)$
The x-coordinate is -5, a negative value. The y-coordinate is -2, a negative value. The point is located in Quadrant III because it has a negative x-coordinate and a negative y-coordinate.

For point C $(-3, 6)$
The x-coordinate is -3, a negative value. The y-coordinate is 6, a positive value. The point is located in Quadrant II because it has a negative x-coordinate and a positive y-coordinate.

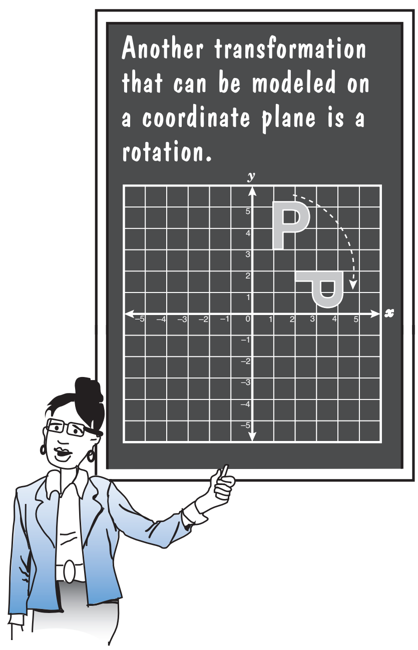
Point B is the only vertex of $\triangle ABC$ that is in Quadrant III.

How Can You Show Transformations on a Coordinate Plane?

Translations, reflections, and dilations can be modeled on a coordinate plane. A figure has been translated or reflected if it has been moved without changing its shape or size. A figure has been dilated if its size has been changed proportionally.

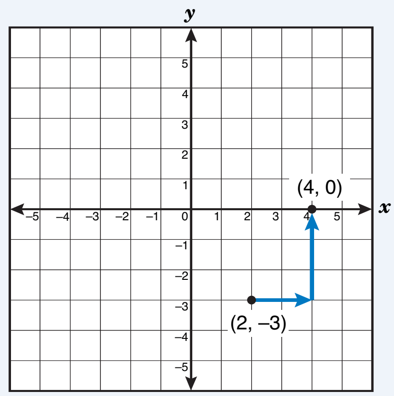
<u>Translations</u>

A **translation** of a figure is a movement of the figure along a line. It can be described by stating how many units to the left or right the figure is moved and how many units up or down it is moved. A figure and its translated image are always congruent.



Objective 3

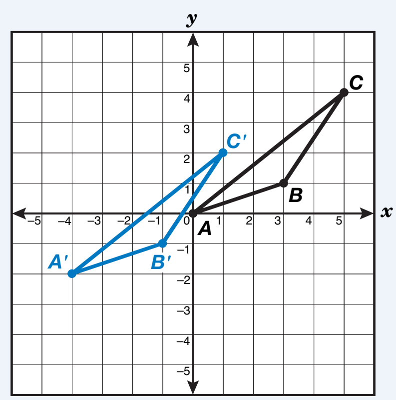
If the point (2, –3) is translated 2 units to the right and 3 units up, what are the coordinates of the new point?



- The x-coordinate increases by 2. Because $2 + 2 = 4$, the new x-coordinate is 4.
- The y-coordinate increases by 3. Because $-3 + 3 = 0$, the new y-coordinate is 0.

The coordinates of the new point are (4, 0).

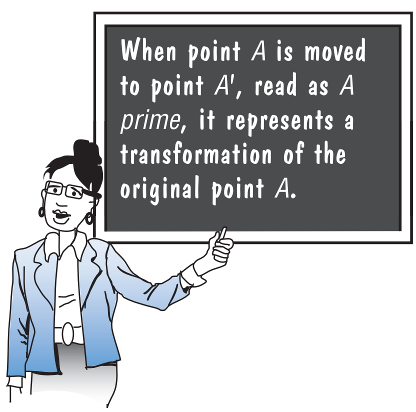
If $\triangle ABC$ is translated 4 units to the left and 2 units down, what will be the coordinates of the vertices of the translated triangle?



The vertices of $\triangle ABC$ are A (0, 0), B (3, 1), and C (5, 4).

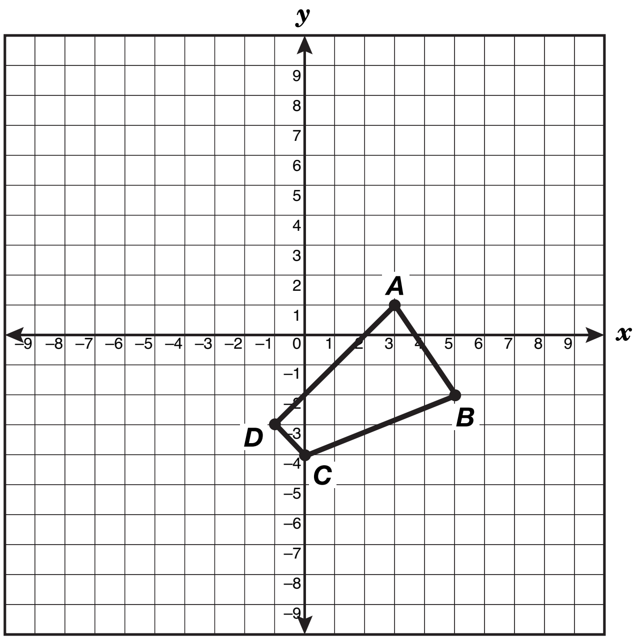
- The triangle is translated 4 units to the left, so subtract 4 from the x-coordinate of each vertex.
- The triangle is translated 2 units down, so subtract 2 from the y-coordinate of each vertex.

The vertices of $\triangle A'B'C'$ are A' $(-4, -2)$, B' $(-1, -1)$, and C' (1, 2).



Try It

Quadrilateral *ABCD* is translated to a new location. If point *A* is translated to (6, −1), what are the coordinates of points *B*′, *C*′, and *D*′?



The *x*-coordinate of point *A* was translated from 3 to 6.

Point *A* was moved ________ units to the right.

The *y*-coordinate of point *A* was translated from ________ to ________ .

Point *A* was moved ________ units ________________ .

Translate the remaining points ________ units to the ______________ and ________ units ______________ .

The coordinates of vertex *B*′ are (_______ , _______).

The coordinates of vertex *C*′ are (_______ , _______).

The coordinates of vertex *D*′ are (_______ , _______).

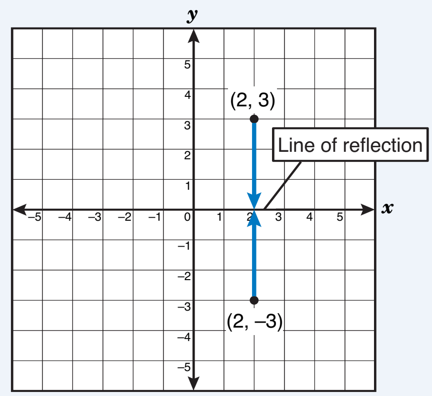
Point *A* was moved 3 units to the right. The *y*-coordinate of point *A* was translated from 1 to −1. Point *A* was moved 2 units down. Translate the remaining points 3 units to the right and 2 units down. The coordinates of vertex *B*′ are (8, −4). The coordinates of vertex *C*′ are (3, −6). The coordinates of vertex *D*′ are (2, −5).

Reflections

A **reflection** of a figure is its mirror image. A figure is reflected across a line called the **line of reflection**. The line of reflection serves as the mirror on which the figure is reflected. A figure and its reflected image are always congruent.

Each point of the reflected image is the same distance from the line of reflection as the corresponding point of the original figure, but it is on the opposite side of the line of reflection.

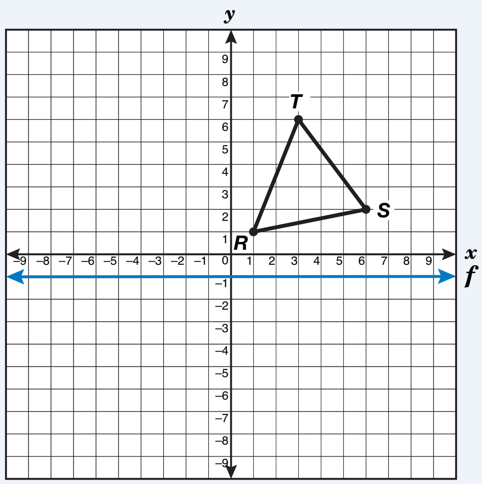
If the point (2, −3) is reflected across the *x*-axis, what will its new coordinates be?



- The *x*-coordinate of the point will be unchanged because the point is being reflected across the *x*-axis. The new point will have an *x*-coordinate of 2.
- The *y*-coordinate of the point is 3 units below the *x*-axis, so the *y*-coordinate of the new point will be 3 units above the *x*-axis. The new point will have a *y*-coordinate of 3.

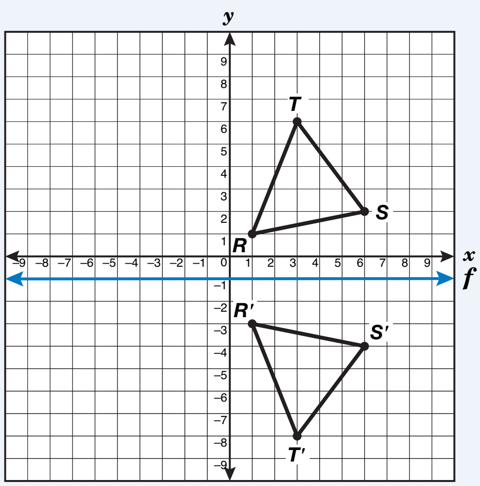
The coordinates of the new point will be (2, 3). The point (2, −3) and its image (2, 3) are equally distant from the line of reflection, the *x*-axis.

Triangle *RST* has vertices *R* (1, 1), *S* (6, 2), and *T* (3, 6). Find the coordinates of its reflection across line *f*.



Each vertex of the original triangle and the reflected triangle must be the same distance from line *f*.

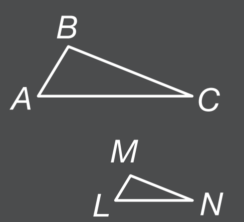
- Vertex *R* (1, 1) is 2 units above the line of reflection. The vertex of the reflection must be 2 units below the line. The coordinates of the reflected vertex *R′* are (1, −3).
- Vertex *S* (6, 2) is 3 units above the line of reflection. The vertex of the reflection must be 3 units below the line. The coordinates of the reflected vertex *S′* are (6, −4).
- Vertex *T* (3, 6) is 7 units above the line of reflection. The vertex of the reflection must be 7 units below the line. The coordinates of the reflected vertex *T′* are (3, −8).



The vertices of the reflected triangle are *R′* (1, −3), *S′* (6, −4), and *T′* (3, −8).

If two figures are similar, then they are the same shape but not necessarily the same size. The lengths of their corresponding sides are proportional. The measures of their corresponding angles are equal.

$\triangle LMN$ is similar to $\triangle ABC$.



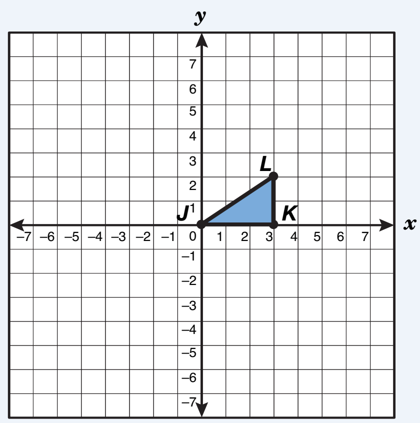
Dilations

A **dilation** is a proportional enlargement or reduction of a figure through a point called the center of dilation. The size of the enlargement or reduction is called the **scale factor** of the dilation.

- If the dilated image is larger than the original figure, then the scale factor > 1. This is called an **enlargement**.
- If the dilated image is smaller than the original figure, then the scale factor < 1. This is called a **reduction**.

A figure and its dilated image are always similar.

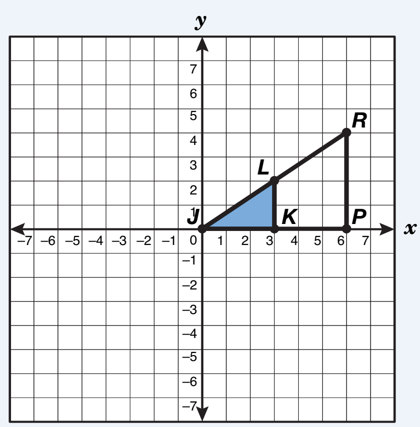
Triangle *JKL* is dilated by a scale factor of 2 using the origin as the center of dilation to form triangle *JPR*. The coordinates of *J* in both triangles remain at (0, 0). What are the coordinates of *P* and *R* in the dilation?



Do you see that . . .

If the triangle is dilated by a scale factor of 2, each of the new triangle's line segments will be twice as long as the original triangle's line segments.

- The new triangle is shown on the coordinate grid below.
- Notice that point *J* does not move. It remains at (0, 0) as the center of dilation.

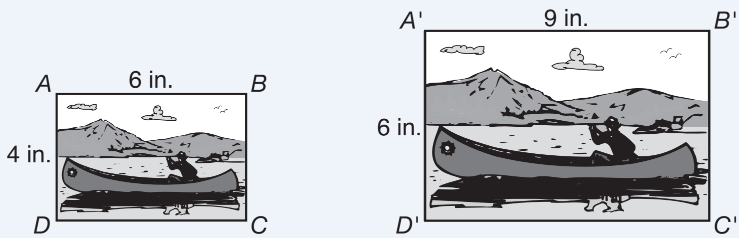


- $\overline{JP}$ will be twice as long as $\overline{JK}$. So the x-coordinate of point P will be 6, and the coordinates of point P will be (6, 0).
- Similarly, $\overline{PR}$ will be twice as long as $\overline{KL}$, so the coordinates of point R will be (6, 4).

The coordinates of $\triangle JPR$ will be J (0, 0), P (6, 0), and R (6, 4).

To find the scale factor of a dilation, compare the lengths of a pair of corresponding sides of the two figures. The ratio of their lengths is equal to the scale factor of the dilation.

If a 4 × 6 inch picture was enlarged so that the new picture measures 6 × 9 inch, by what scale factor was the picture enlarged?



- Write the ratios of the lengths of the corresponding sides of the dilated figure and the original figure.

$$\frac{\text{larger}}{\text{smaller}} \qquad \frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'D'}{CD} = \frac{A'D'}{AD}$$

- Substitute the known measures.

$$\frac{A'B'}{AB} = \frac{9}{6} = \frac{3}{2} \qquad \frac{B'C'}{BC} = \frac{6}{4} = \frac{3}{2}$$

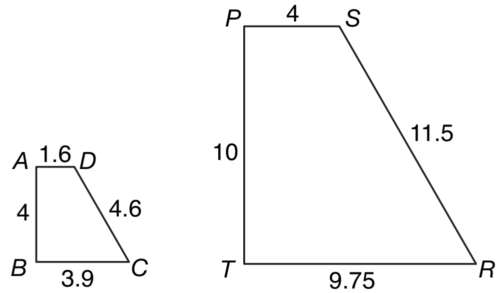
$$\frac{C'D'}{CD} = \frac{9}{6} = \frac{3}{2} \qquad \frac{A'D'}{AD} = \frac{6}{4} = \frac{3}{2}$$

The scale factor is $\frac{3}{2}$, or 1.5. Each dimension of the enlarged picture is 1.5 times its corresponding measure in the smaller picture.

Objective 3

Try It

Trapezoid *PTRS* is a dilation of trapezoid *ABCD*. What is the scale factor of the dilation?



To find the scale factor, find the ratios of corresponding sides.

$\overline{PT}$ corresponds to ________ .

$$\frac{\text{larger}}{\text{smaller}} \quad \frac{PT}{\square} = \frac{10}{\square}$$

Written as a decimal, the ratio of the lengths of these corresponding sides is ________ .

$\overline{PS}$ corresponds to ________ .

$$\frac{\text{larger}}{\text{smaller}} \quad \frac{PS}{\square} = \frac{\square}{\square}$$

Written as a decimal, the ratio of the lengths of these corresponding sides is ________ .

The scale factor of the dilation is ________ .

$\overline{PT}$ corresponds to $\overline{AB}$.

$$\frac{PT}{AB} = \frac{10}{4}$$

Written as a decimal, the ratio of the lengths of these corresponding sides is 2.5. $\overline{PS}$ corresponds to $\overline{AD}$.

$$\frac{PS}{AD} = \frac{4}{1.6}$$

Written as a decimal, the ratio of the lengths of these corresponding sides is 2.5. The scale factor of the dilation is 2.5.

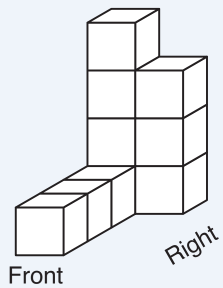
How Do You Recognize a 3-Dimensional Figure from Different Perspectives?

Given a drawing of a 3-dimensional figure, you should be able to recognize other drawings that represent the same 3-dimensional figure from a different perspective.

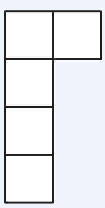
A 3-dimensional figure can be represented by drawing the figure from three different views: front, top, and side.

To recognize the 3-dimensional figure from different perspectives, you must visualize what the 3-dimensional figure would look like if you were seeing it from above, from one side, and from the front.

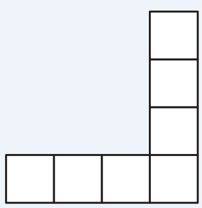
The 3-dimensional figure below is made up of equal-sized cubes. Can you visualize what it would look like from above, from the side, and from the front?



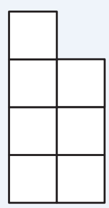
These are the top, side, and front views of the 3-dimensional figure.



Top view



Right-side view



Front view

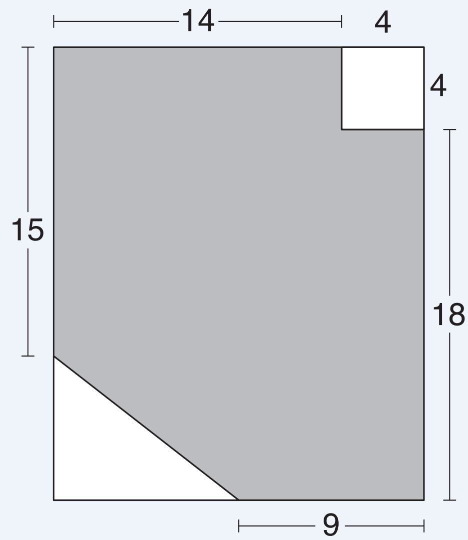
Objective 3

What Kinds of Problems Can You Solve with Geometry?

You can solve many types of problems using geometry, including problems involving these geometric concepts:

- the area or perimeter of figures;
- the measure of the sides or angles of polygons;
- the surface area and volume of 3-dimensional figures;
- the ratios of the sides of similar figures; and
- the relationship among the sides of a right triangle.

The shaded part of the rectangular room in the drawing below is to be carpeted. The given dimensions are in feet.



How many square feet of carpet will be required?

- The room is a rectangle with two pieces that will not be carpeted: a square in the upper right corner and a triangle in the lower left corner. Use subtraction to remove the area of the two unshaded parts from the area of the whole.
- The area of a rectangle is $A = lw$, where l stands for the length of the rectangle and w stands for the width.

  The length of the rectangle is $18 + 4 = 22$ ft.
  The width of the rectangle is $14 + 4 = 18$ ft.
  The area of the rectangle is $22 \cdot 18 = 396$ ft^2.
- The area of the square is $A = s^2$, where s stands for the length of a side of the square.

  The length of a side of the square is 4 ft.
  The area of the square is $4 \cdot 4 = 16$ square ft^2.
- The area of the triangle is $A = \frac{1}{2}bh$, where b stands for the length

of the base of the triangle and h stands for the height of the triangle.

The base of the triangle is $18 - 9 = 9$ ft.
The height of the triangle is $22 - 15 = 7$ ft.

The area of the triangle is $\frac{1}{2} \cdot 9 \cdot 7 = 31.5 \text{ ft}^2$.

- Subtract to find the area of the carpet needed to cover the floor.

$$396 - 16 - 31.5 = 348.5 \text{ ft}^2$$

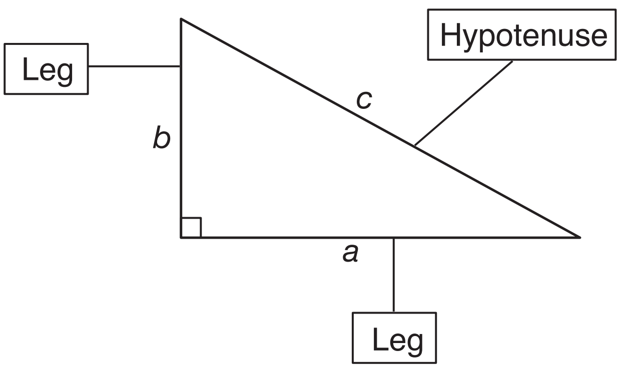
Covering the floor will require 348.5 square feet of carpet.

What Is the Pythagorean Theorem?

The **Pythagorean Theorem** is a relationship among the lengths of the sides of a right triangle. This relationship applies only to right triangles.

The sides of a right triangle have special names.

- The **hypotenuse** of a right triangle is the longest side of the triangle. The hypotenuse is always opposite the right angle in the triangle. In the diagram below, the length of the hypotenuse is represented by c.
- The **legs** of a right triangle are the two sides that form the right angle. In the diagram below, the lengths of the legs are represented by a and b.

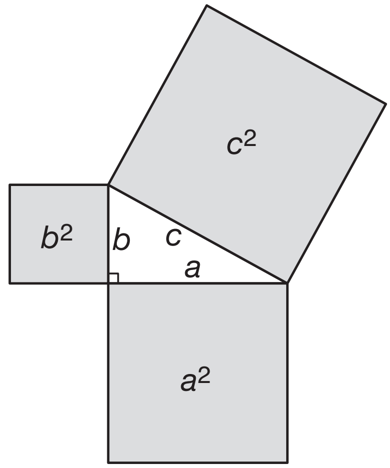


The Pythagorean Theorem can be stated algebraically or verbally, or interpreted with a geometric model.

Algebraic	Verbal
In any right triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$.	In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Objective 3

Geometric Model

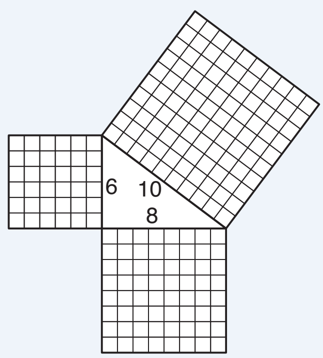


In the geometric model, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs.

You can use the Pythagorean Theorem to determine whether a triangle is a right triangle.

A triangle has side lengths of 6 units, 8 units, and 10 units. Is the triangle a right triangle?

- Identify the length of the longest side, 10 units. Substitute this value for c.
- The lengths of the other two sides are 6 units and 8 units. Substitute these values for a and b.
- Then see whether $a^2 + b^2 = c^2$.



$$6^2 + 8^2 \stackrel{?}{=} 10^2$$

$$36 + 64 \stackrel{?}{=} 100$$

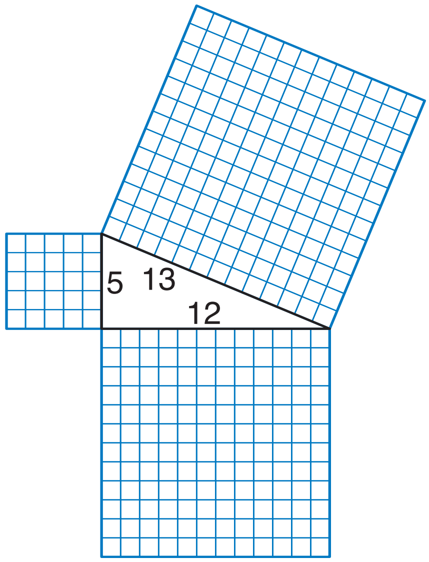
$$100 = 100$$

Since the Pythagorean Theorem is true for the lengths of the sides of this triangle, this triangle is a right triangle.

Try It

Mark cut two pieces of wood, one 5 feet long and the other 12 feet long. If the third piece he cuts is 13 feet long, could the three pieces form a right triangle?

The piece that is ________ feet long should be the hypotenuse of the triangle because it is the ________________ side. The other two pieces should be the legs of the triangle.



The square formed by the longest side measures ________ feet by ________ feet.

It has an area of ________ • ________ = ________ square feet.

The square formed by the shortest side measures ________ feet by ________ feet.

It has an area of ________ • ________ = ________ square feet.

The square formed by the remaining side measures ________ feet by ________ feet.

It has an area of ________ • ________ = ________ square feet.

Since ________ = ________ + ________ , the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs.

The three pieces of wood could form a ________________ triangle.

Objective 3

The piece that is 13 feet long should be the hypotenuse of the triangle because it is the longest side. The square formed by the longest side measures 13 feet by 13 feet. It has an area of 13 • 13 = 169 square feet. The square formed by the shortest side measures 5 feet by 5 feet. It has an area of 5 • 5 = 25 square feet. The square formed by the remaining side measures 12 feet by 12 feet. It has an area of 12 • 12 = 144 square feet. Since 169 = 25 + 144, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs. The three pieces of wood could form a right triangle.

Now practice what you've learned.

Question 22

A circle has a radius of 4 inches. The circle is dilated by reducing its radius by 60%. What is the radius of the reduced circle?

A 6.4 in.

B 2.4 in.

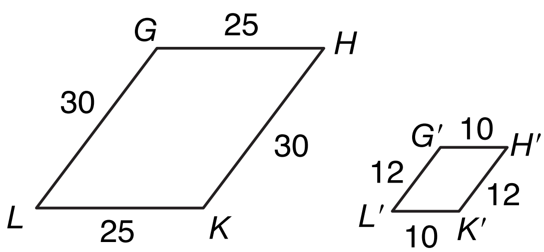
C 2 in.

D 1.6 in.

Answer Key: page 162

Question 23

The smaller parallelogram is a dilation of the larger one.



What scale factor was used to reduce the larger parallelogram?

A 0.3

B 2.5

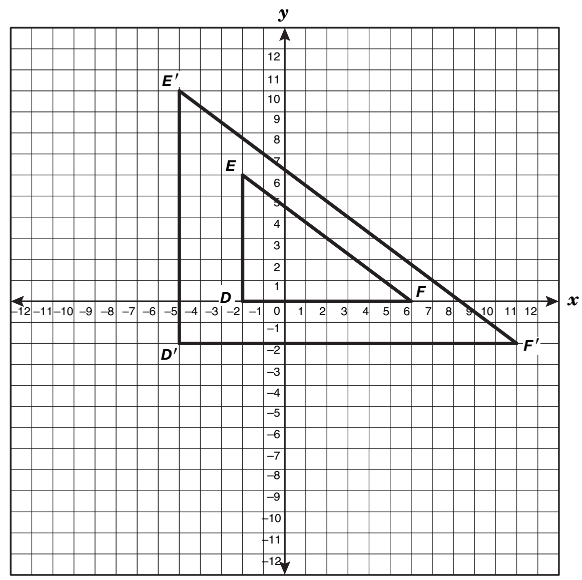
C 3.0

D 0.4

Answer Key: page 162

Question 24

The drawing shows $\triangle DEF$ and its enlargement, $\triangle D'E'F'$. By what scale factor was $\triangle DEF$ enlarged?



A $\frac{1}{4}$

B 4

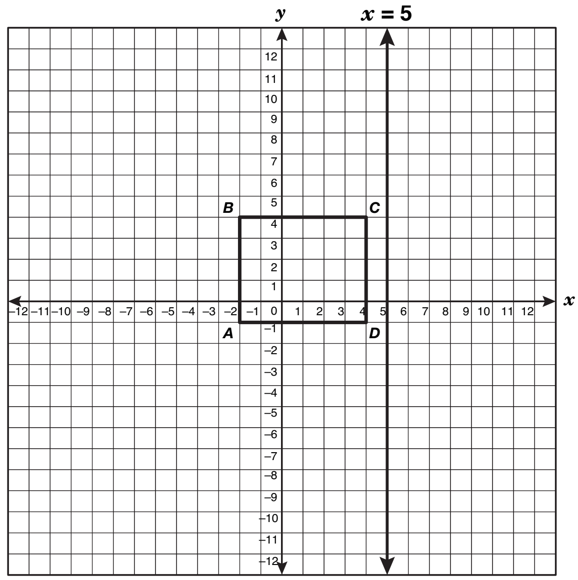
C 2

D $\frac{1}{2}$

Answer Key: page 162

Question 25

If rectangle *ABCD* is reflected across the line $x = 5$, what will be the coordinates of the image of point *B*?



A (12, 4)

B (2, 4)

C (−2, 6)

D (11, −2)

Answer Key: page 162

Question 26

Triangle *ABC* is reflected across a line of reflection to form triangle *A'B'C'*. Triangle *ABC* has coordinates *A* (1, 2), *B* (4, 5), and *C* (5, 3). Its reflected image has coordinates *A'* (1, −2), *B'* (4, −5), and *C'* (5, −3). Which line is the line of reflection for this transformation?

A *y*-axis

B *x*-axis

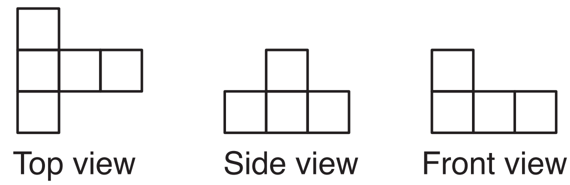
C $x = 1$

D $y = 2$

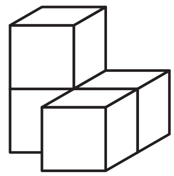
Answer Key: page 162

Question 27

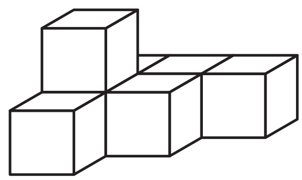
Which 3-dimensional figure has the top, side, and front views that are shown below?



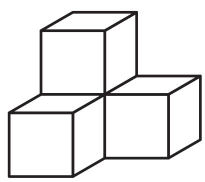
A



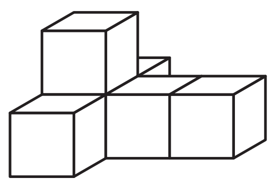
B



C



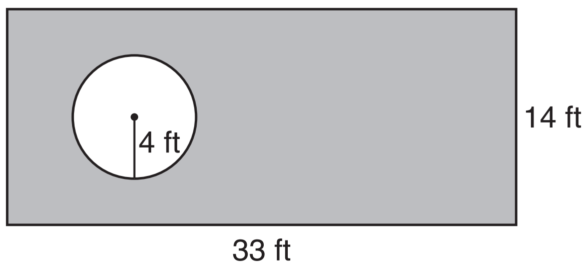
D



Answer Key: page 162

Question 28

Mr. Wythe uses a machine to polish the lobby floor in an office building. The floor is a rectangle with a circular fountain, as shown below. Mr. Wythe cannot polish the area covered by the fountain. Which of the following is closest to the number of square feet of floor Mr. Wythe polishes?



A 512.24 ft^2

B 462 ft^2

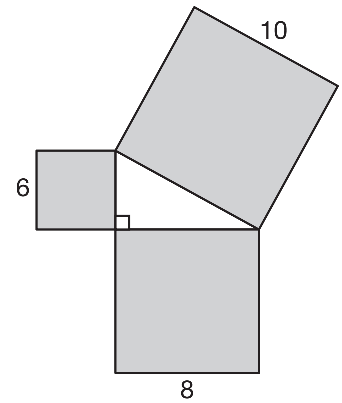
C 411.76 ft^2

D 436.88 ft^2

Answer Key: page 162

Question 29

The right triangle shown below is drawn with a square adjacent to each side.



Which equation represents the relationship between the lengths of the sides of the right triangle?

A $6^2 - 10^2 = 8^2$

B $6^2 + 8^2 = 10^2$

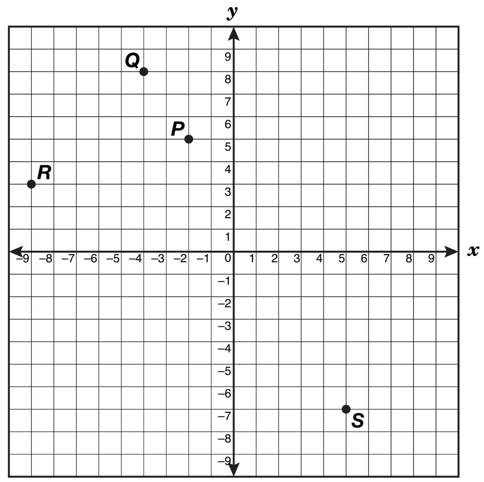
C $(6 + 8)^2 = 10^2$

D $(8 - 6)^2 = 10^2$

Answer Key: page 162

Question 30

Which of the following points on the coordinate grid below satisfies the conditions $x < -2.5$ and $y > 3$?



A Point P

B Point Q

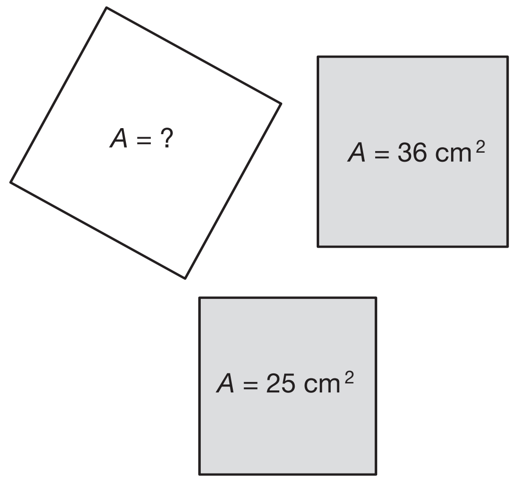
C Point R

D Point S

Answer Key: page 163

Question 31

Three squares are shown below. The areas of the shaded squares are also shown.



The three squares are to form a right triangle when joined at their vertices. One side of each of the two shaded squares will be the legs and one side of the unshaded square will be the hypotenuse. What is the area of the unshaded square?

A 11 cm ^{2}

B 50 cm ^{2}

C 61 cm ^{2}

D 49 cm ^{2}

Answer Key: page 163

Objective 4

The student will demonstrate an understanding of the concepts and uses of measurement.

For this objective you should be able to

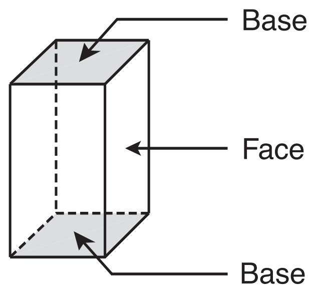
- use procedures to determine the measures of 3-dimensional figures;
- use indirect measurement to solve problems; and
- describe how changes in dimensions affect linear, area, and volume measures.

How Do You Find the Surface Area of 3-Dimensional Figures?

You can use models or formulas to find the surface area of prisms, cylinders, and other 3-dimensional figures.

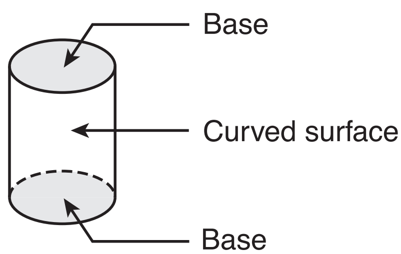
- A **prism** is a 3-dimensional figure with two bases. The bases are congruent polygons. The other faces of the prism are rectangles. The prism is named by the shape of its bases. For example, a square prism has two squares as its bases.

Square Prism

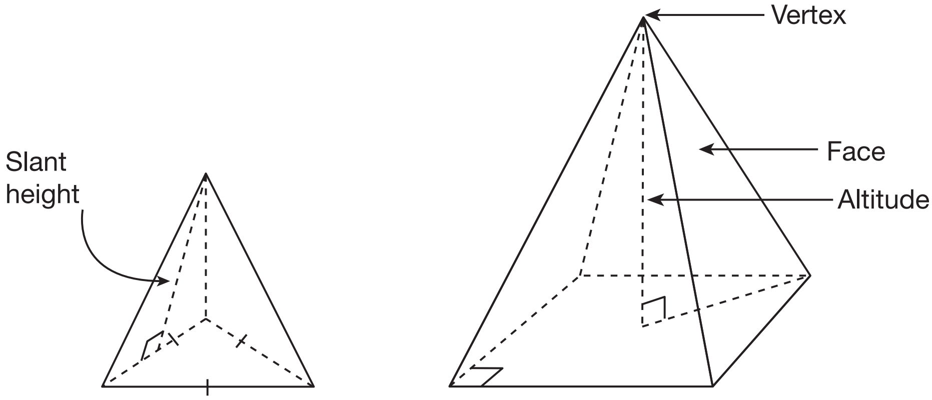


- A **cylinder** is a 3-dimensional figure with two congruent circular bases and a curved surface.

Cylinder



- A **pyramid** is a 3-dimensional figure whose base is a polygon and whose faces are triangles. A pyramid may be named by the shape of its base. A **regular pyramid** is one with a regular polygon for its base. A regular **triangular pyramid** has an equilateral triangle for its base, and a **square pyramid** has a square for its base.



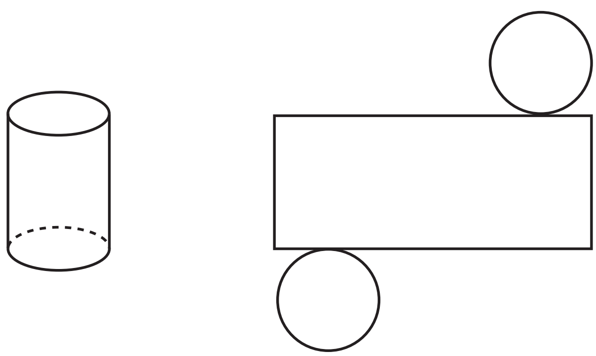
Regular Triangular Pyramid

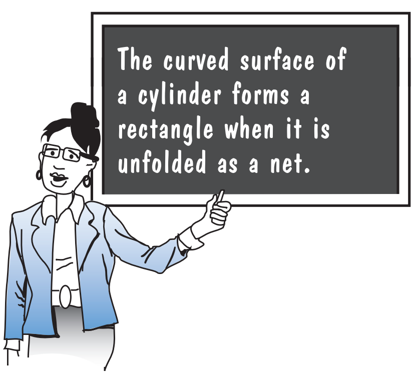
Square Pyramid

Like the area of a plane figure, the surface area of a 3-dimensional figure is measured in square units.

- The **total surface area** of a 3-dimensional figure is equal to the sum of the areas of all its surfaces.
- The **lateral surface area** of a 3-dimensional figure is equal to the sum of the areas of all its faces and curved surfaces. It does not include the areas of the figure's bases.

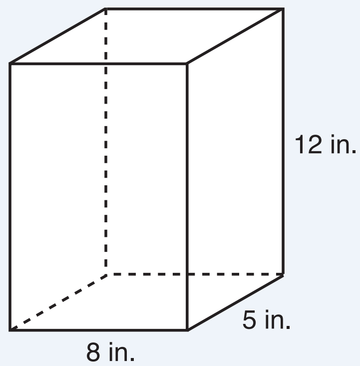
One way to compute the surface area of a 3-dimensional figure is to use a net of the figure. A **net** of a 3-dimensional figure is a 2-dimensional drawing that shows what the figure would look like if it were unfolded, with all its surfaces laid flat. Use the net to find the area of each surface.



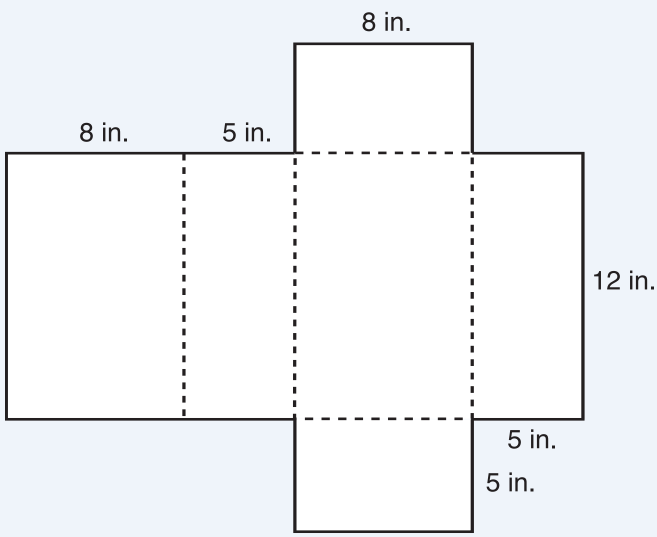


You can also find the surface area of a 3-dimensional figure by using a formula. Substitute the dimensions of the figure in the formula and compute its surface area. The formulas for the total surface area and lateral surface area of several types of 3-dimensional figures are included in the Mathematics Chart.

A gift box has dimensions of 5, 8, and 12 inches. Find its surface area in square inches.



The box, a rectangular prism, can be unfolded to make a net that shows the prism's faces. To find the surface area of a rectangular prism, find the sum of the areas of the six rectangular faces.



Find the areas of the six faces.

- There are two rectangular faces that each measure 5 by 8 inches.

  Each of these faces has an area of $5 \cdot 8 = 40 \text{ in.}^2$

  The two faces have a total area of $2 \cdot 40 = 80 \text{ in.}^2$

- There are two rectangular faces that each measure 5 by 12 inches.

  Each of these faces has an area of $5 \cdot 12 = 60 \text{ in.}^2$

  The two faces have a total area of $2 \cdot 60 = 120 \text{ in.}^2$

- There are two rectangular faces that each measure 8 by 12 inches.

Each of these faces has an area of $8 \cdot 12 = 96$ in.^{2}

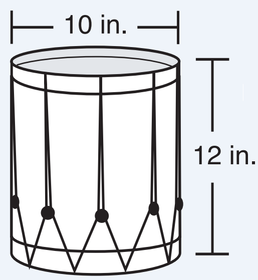
The two faces have a total area of $2 \cdot 96 = 192$ in.^{2}

The surface area of the prism is the sum of the areas of all the faces.

$$80 \text{ in.}^2 + 120 \text{ in.}^2 + 192 \text{ in.}^2 = 392 \text{ in.}^2$$

The surface area of the gift box is 392 square inches.

A drum maker manufactures a cylindrical drum with a diameter of 10 inches and a height of 12 inches. The top and bottom of the drum are made of leather, but the curved surface of the drum is made of wood. Approximately how many square inches of wood is required to form the curved surface of the drum?

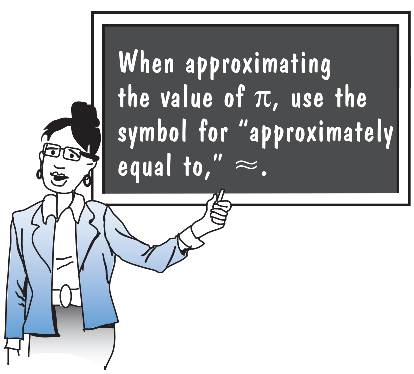


The drum is shaped like a cylinder. The area of the curved surface of the drum is its lateral surface area. Use the formula for the lateral surface area of a cylinder in the Mathematics Chart, $S = 2\pi rh$.

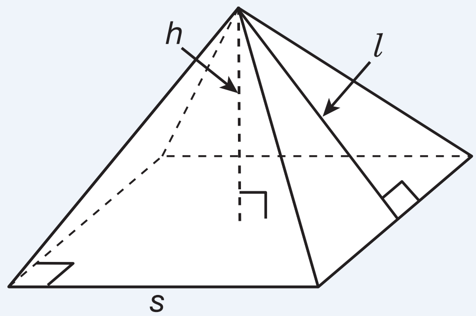
- The diameter of the drum is 10 inches, but the formula for the lateral surface area requires that you know r, the radius of the drum. The radius of a circle is $\frac{1}{2}$ the diameter. The radius of the drum is 5 inches. Substitute 5 for r in the formula.
- Substitute 12, the height of the drum, for h in the formula.
- Use 3.14 as an approximate value of π.

$$S = 2\pi rh$$
$$S \approx 2(3.14)(5)(12)$$
$$S \approx 376.8 \text{ in.}^2$$

About 377 square inches of wood is required to form the curved surface of the drum.

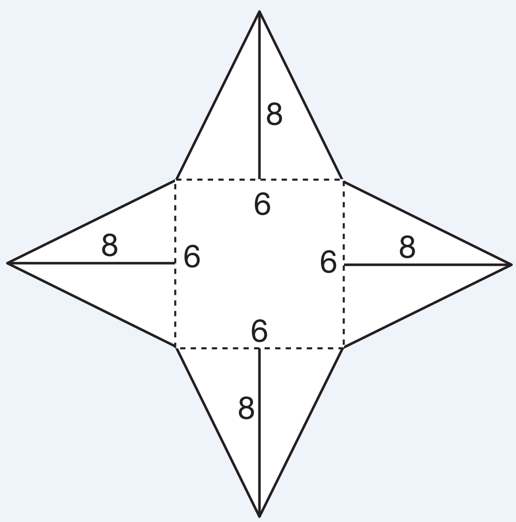


Find the lateral and total surface area of a square pyramid whose base has a side length, s, of 6 in. and whose slant height, l, is 8 in.



- Using a Net

This pyramid can be unfolded to make a net that shows the pyramid's faces. To find the total surface area of the pyramid, find the sum of the areas of its faces including the area of its square base.



The base of the pyramid is a square with a side length of 6 inches. The side length is also the base of each triangular face. The slant height (l) of the pyramid is the height of each triangular face, which is 8 inches. So, to find the sum of the areas of the four triangular faces, find the area of one face and multiply that amount by 4.

The area of one triangular face is $A = \frac{1}{2}bh$, where b is 6 inches and h is 8 inches.

$$A = \frac{1}{2}(6 \times 8) = 24 \text{ in.}^2$$

The lateral surface area of the pyramid is $4 \times 24 = 96 \text{ in.}^2$

Objective 4

Next we find the area of the square base of the pyramid: $A = s^2$.

$$A = 6^2 = 36 \text{ in.}^2$$

The total surface area of the pyramid is the sum of the lateral surface area and the area of the base.

$$\text{Total Surface Area} = 96 \text{ in.}^2 + 36 \text{ in.}^2 = 132 \text{ in.}^2$$

- Using a Formula

The total surface area can also be found using the formula $S = \frac{1}{2}Pl + B$, where $\frac{1}{2}Pl$ represents the lateral surface area and B represents the area of the base of the pyramid. In the expression $\frac{1}{2}Pl$, P represents the perimeter of the base of the pyramid and l represents the slant height (the height of its triangular faces). In this example the perimeter of the base (P) is

$$P = 6 + 6 + 6 + 6 = 24 \text{ in.}$$

The slant height (l) is 8 in. The lateral surface area is

$$S = \frac{1}{2}(24 \times 8) = 96 \text{ in.}^2$$

Since the base is a square, its area $A = s^2$ is used to find B.

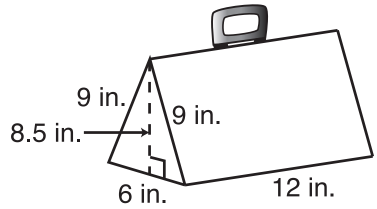
$$B = 6^2 = 36 \text{ in.}^2$$

The total surface area is found by adding the lateral surface area to the area of the base

$$S = 96 \text{ in.}^2 + 36 \text{ in.}^2 = 132 \text{ in.}^2$$

Try It

Maria is building a carrying case for a musical instrument. The case will have the shape of a triangular prism. She wants to know how much material will be needed to build it. The case will have the dimensions given in the diagram. What is the surface area of the case in square inches?



The case has the shape of a ________________________________.

The surface area of a prism is equal to the ________________ of the areas of the prism's surfaces.

There are _______ rectangular surfaces that each measure 9 by 12 inches. Each of these surfaces has an area of _______ • _______ = _______ in.^{2} The two surfaces have a total area of 2 • _______ = _______ in.^{2}

There is ________ rectangular surface that measures 6 by 12 inches. This surface has an area of _______ • _______ = _______ in.^{2}

There are _______ triangular surfaces. They each have a base of ______ inches and a height of ______ inches. The formula for the area of a triangle is

________________ .

Each of these surfaces has an area of ______ • ______ • ______ = ______ in.^{2}

The two surfaces have a total area of 2 • _______ = _______ in.^{2}

Find the sum of the areas of the prism's surfaces.

_______ + _______ + _______ = _______ in.^{2}

The surface area of the case is _______ square inches.

The case has the shape of a triangular prism. The surface area of a prism is equal to the sum of the areas of the prism's surfaces.

There are two rectangular surfaces that each measure 9 by 12 inches. Each of these surfaces has an area of 9 • 12 = 108 in.^{2} The two surfaces have a total area of 2 • 108 = 216 in.^{2}

There is one rectangular surface that measures 6 by 12 inches. This surface has an area of 6 • 12 = 72 in.^{2}

There are two triangular surfaces. They each have a base of 6 inches and a height of 8.5 inches. The formula for the area of a triangle is $A = \frac{1}{2}bh$. Each of these surfaces has an area of $\frac{1}{2}$ • 6 • 8.5 = 25.5 in.^{2} The two surfaces have a total area of 2 • 25.5 = 51 in.^{2}

Find the sum of the areas of the prism's surfaces: 216 + 72 + 51 = 339 in.^{2}

The surface area of the case is 339 square inches.

Objective 4

What Is Volume?

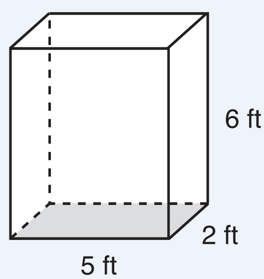
The **volume** of a 3-dimensional figure is a measure of the space it occupies. Volume is measured in cubic units.

You can find the volume of a 3-dimensional figure by using the appropriate volume formula. The formulas for computing the volume of several 3-dimensional figures are given in the Mathematics Chart.

When using a formula to find the volume of a 3-dimensional figure, follow these guidelines.

- Identify the 3-dimensional figure with which you are working. This will help you select the correct volume formula.
- You may wish to use a model to help you visualize the 3-dimensional figure and correctly assign the variables in the volume formula.
- Substitute the appropriate dimensions of the 3-dimensional figure for the corresponding variables in the volume formula.
- Compute the volume and state your answer in cubic units.

Find the volume of a rectangular prism that is 6 feet tall and has a base that measures 5 feet by 2 feet.



Do you see that . . .

- The formula for the volume of a prism is found by multiplying the area of its base, B, by its height, h.

$$V = Bh$$

- The base of this prism is a rectangle. The area of a rectangle is equal to its length times its width.

$$B = 5 \text{ ft} \cdot 2 \text{ ft}$$
$$B = 10 \text{ ft}^2$$

- Substitute 10 square feet for B and 6 feet for h in the formula for the volume of a prism.

$$V = Bh$$
$$V = 10 \text{ ft}^2 \cdot 6 \text{ ft}$$
$$V = 60 \text{ ft}^3$$

The volume of the rectangular prism is 60 cubic feet.

Try It

What is the approximate volume of a cylinder with a radius of 1.5 inches and a height of 4.25 inches?

The formula for the volume of a cylinder is ________________ .

In the formula, B represents the __________ of the base of a cylinder.

The base of a cylinder is a __________ .

Its area, B, is equal to ______________ .

Use 3.14 as an estimate of the value of __________ .

Substitute __________ inches for r, the __________ of the cylinder's base.

$$B \approx \underline{\qquad\qquad} \cdot \underline{\qquad\qquad}$$

$$B \approx \underline{\qquad\qquad} \text{ in.}^2$$

Substitute __________ square inches for B in the formula for the volume of a cylinder.

Substitute __________ inches for h, the __________ of the cylinder.

$$V = Bh$$

$$V \approx \underline{\qquad\qquad} \cdot \underline{\qquad\qquad}$$

$$V \approx \underline{\qquad\qquad} \text{ in.}^3$$

The cylinder has a volume of approximately ________ cubic inches.

The formula for the volume of a cylinder is $V = Bh$. In the formula, B represents the area of the base of a cylinder. The base of a cylinder is a circle. Its area, B, is equal to πr^2. Use 3.14 as an estimate of the value of π. Substitute 1.5 inches for r, the radius of the cylinder's base.

$$B \approx 3.14 \cdot 1.5^2$$

$$B \approx 7.07 \text{ in.}^2$$

Substitute 7.07 square inches for B in the formula for the volume of a cylinder. Substitute 4.25 inches for h, the height of the cylinder.

$$V = Bh$$

$$V \approx 7.07 \cdot 4.25$$

$$V \approx 30.05 \text{ in.}^3$$

The cylinder has a volume of approximately 30 cubic inches.

Objective 4

Héctor can buy either a rectangular box of oats that measures 8 by 1.5 by 10 inches or a cylindrical container of oats that has a radius of 2.5 inches and a height of 8 inches. Which container has the greater volume?

- Find the volume of the box, which is a rectangular prism.

  The formula for the volume of a rectangular prism is $V = Bh$. Find B, the area of the base of the prism. The base of the prism is a rectangle. Its area is equal to lw. Substitute 8 for l and 1.5 for w.

$$B = 8 \cdot 1.5$$

$$B = 12 \text{ in.}^2$$

  Substitute 12 for B and 10 for h in the formula for the volume of a prism.

$$V = Bh$$

$$V = 12 \cdot 10$$

$$V = 120 \text{ in.}^3$$

  The volume of the rectangular box is 120 cubic inches.

- Find the volume of the cylindrical container.

  The formula for the volume of a cylinder is $V = Bh$. Find B, the area of the base of the cylinder. The base of the cylinder is a circle. Its area is equal to πr^2. Substitute 2.5 for r and use 3.14 as an approximate value of π.

$$B \approx 3.14 \cdot 2.5^2$$

$$B \approx 3.14 \cdot 6.25$$

$$B \approx 19.6 \text{ in.}^2$$

  Substitute 19.6 for B and 8 for h in the formula for the volume of a cylinder.

$$V = Bh$$

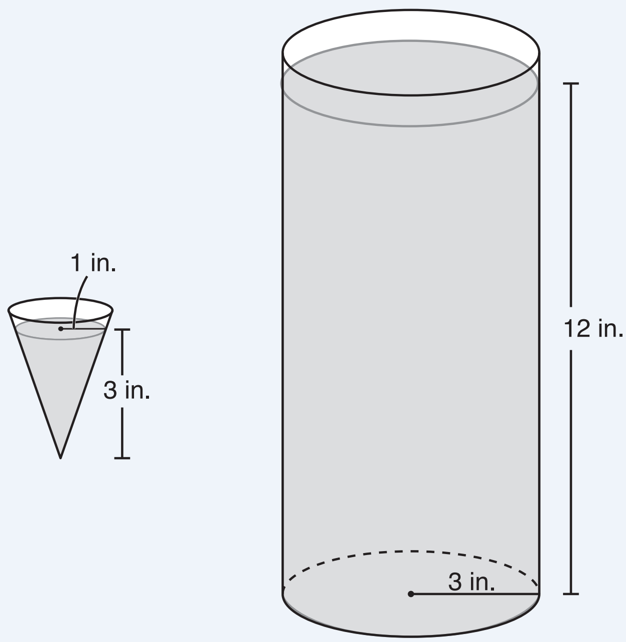
$$V \approx 19.6 \cdot 8$$

$$V \approx 156.8 \text{ in.}^3$$

  The volume of the cylindrical container is about 157 cubic inches.

Since $157 \text{ in.}^3 > 120 \text{ in.}^3$, the cylindrical container has a greater volume than the rectangular box.

A cylindrical container with a 3-inch radius is filled with juice to a height of 12 inches. How many cone-shaped paper cups can be filled with juice from the container? Assume that a full paper cup has a height of 3 inches and a radius of 1 inch.



To find the number of paper cups that can be filled, first find the volume of one full paper cup. Then find the volume of juice in the container and divide it by the volume of one full paper cup.

- Find the volume of one full paper cup, which is a cone.

  The formula for the volume of a cone is $V = \frac{1}{3}Bh$.

  Find B, the area of the base of the cone. The base of the cone is a circle. Its area is equal to πr^2. Substitute 1 for r and use 3.14 as an approximate value of π.

$$B \approx 3.14 \cdot 1^2$$

$$B \approx 3.14 \cdot 1$$

$$B \approx 3.14 \text{ in.}^2$$

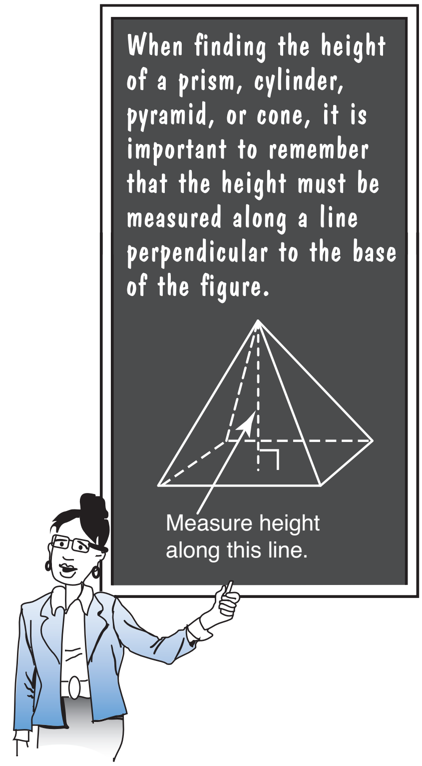
  Substitute 3.14 for B and 3 for h in the formula for the volume of a cone.

$$V = \frac{1}{3} Bh$$

$$V \approx \frac{1}{3} \cdot 3.14 \cdot 3$$

$$V \approx 3.14 \text{ in.}^3$$

  The volume of one full paper cup is about 3.14 cubic inches.



Objective 4

- Find the volume of juice in the container, which is a cylinder.

  The formula for the volume of a cylinder is $V = Bh$.
  Find B, the area of the base of the cylinder. The base of the cylinder is a circle. Its area is equal to πr^2. Substitute 3 for r and use 3.14 as an approximate value of π.

  $$B \approx 3.14 \cdot 3^2$$
  $$B \approx 3.14 \cdot 9$$
  $$B \approx 28.26 \text{ in.}^2$$

  Substitute 28.26 for B and 12 for h in the formula for the volume of a cylinder.

  $$V = Bh$$
  $$V \approx 28.26 \cdot 12$$
  $$V \approx 339.12 \text{ in.}^3$$

  The volume of juice in the container is about 339.12 cubic inches.

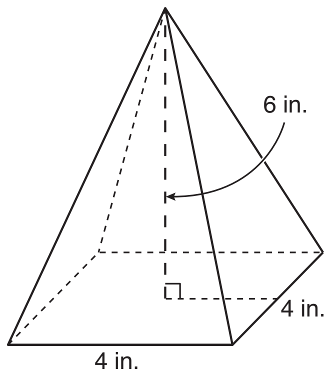
- Divide to find the number of paper cups that can be filled from the juice container.

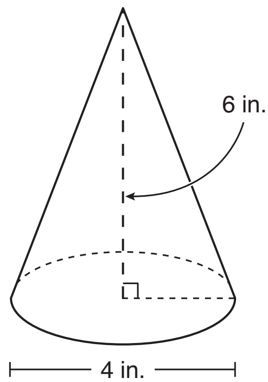
  $$339.12 \div 3.14 = 108$$

About 108 paper cups can be filled from the juice container.

Try It

Which has the greater volume, a 4-inch-square pyramid that is 6 inches tall or a cone that is 6 inches tall and has a 4-inch diameter?





The formula for the volume of a pyramid is ____________________ .

Find B, the area of the ____________________ of a pyramid. The base of the pyramid is a ____________________ . Its area is equal to __________ . Substitute __________ for s.

$B =$ __________

$B =$ __________ in.^2

Substitute __________ for B and __________ for h in the formula for the volume of a pyramid.

$V = \frac{1}{3}Bh$

$V = \frac{1}{3} \cdot$ __________ $\cdot$ __________

$V =$ __________ in.^3

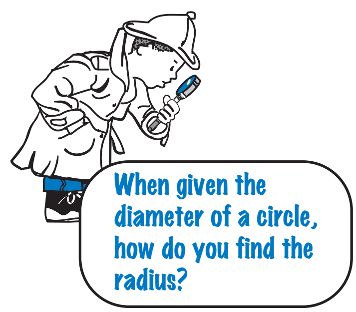
The volume of the pyramid is __________ cubic inches.

The formula for the volume of a cone is ____________________ .

Find B, the area of the ____________________ of a cone. The base of the cone is a ____________________ . Its area is equal to __________ . Substitute __________ for r and use 3.14 as an approximate value of π.

$B \approx 3.14 \cdot$ __________

$B \approx 3.14 \cdot$ __________



Objective 4

$B \approx$ _________ in.^{2}

Substitute _________ for B and _________ for h in the formula for the volume of a cone.

$$V = \frac{1}{3}Bh$$

$$V \approx \frac{1}{3} \cdot \text{_________} \cdot \text{_________}$$

$$V \approx \text{_________ in.}^3$$

The volume of the cone is about _________ cubic inches.

The ________________ has the greater volume.

The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$. Find B, the area of the base of a pyramid. The base of the pyramid is a square. Its area is equal to s^2. Substitute 4 for s.

$$B = 4^2$$

$$B = 16 \text{ in.}^2$$

Substitute 16 for B and 6 for h in the formula for the volume of a pyramid.

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3} \cdot 16 \cdot 6$$

$$V = 32 \text{ in.}^3$$

The volume of the pyramid is 32 cubic inches.

The formula for the volume of a cone is $V = \frac{1}{3}Bh$. Find B, the area of the base of a cone. The base of the cone is a circle. Its area is equal to πr^2. Substitute 2 for r and use 3.14 as an approximate value of π.

$$B \approx 3.14 \cdot 2^2$$

$$B \approx 3.14 \cdot 4$$

$$B \approx 12.56 \text{ in.}^2$$

Substitute 12.56 for B and 6 for h in the formula for the volume of a cone.

$$V = \frac{1}{3}Bh$$

$$V \approx \frac{1}{3} \cdot 12.56 \cdot 6$$

$$V \approx 25.12 \text{ in.}^3$$

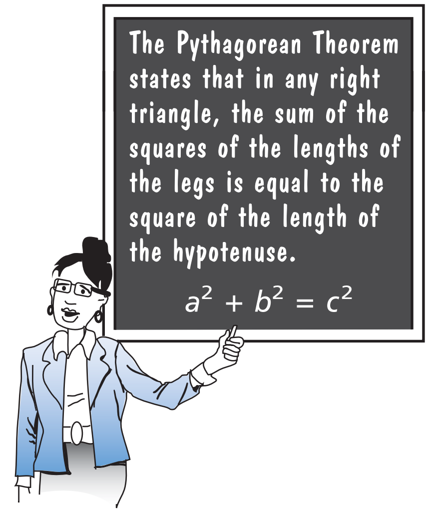
The volume of the cone is about 25.12 cubic inches.

The pyramid has the greater volume.

Objective 4

How Can You Solve Problems Using the Pythagorean Theorem?

The **Pythagorean Theorem** is a relationship among the lengths of the sides of a right triangle. The Pythagorean Theorem applies only to right triangles.



You can use the Pythagorean Theorem to determine whether a triangle is a right triangle.

- Identify which side could be the hypotenuse, c. It must be the longest side.
- See whether $a^2 + b^2 = c^2$.
- If the lengths of the sides satisfy the Pythagorean Theorem, then the triangle is a right triangle.

Any set of three whole numbers that satisfies the Pythagorean Theorem is called a **Pythagorean triple**.

- The numbers 3, 4, and 5 form a Pythagorean triple because they satisfy the equation $a^2 + b^2 = c^2$.

$$\begin{aligned} 3^2 + 4^2 &= 5^2 \\ 9 + 16 &= 25 \\ 25 &= 25 \end{aligned}$$

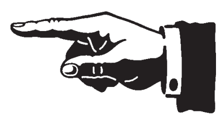
  A triangle with side lengths of 3, 4, and 5 units is a right triangle.

- Any multiple of a Pythagorean triple is also a Pythagorean triple.

  Because the numbers 3, 4, and 5 form a Pythagorean triple, it is also true that these numbers multiplied by 3 form a Pythagorean triple: 9, 12, and 15.

$$\begin{aligned} 9^2 + 12^2 &= 15^2 \\ 81 + 144 &= 225 \\ 225 &= 225 \end{aligned}$$

  A triangle with side lengths of 9, 12, and 15 units is also a right triangle.



Do you see that . . .

A triangle has one leg that is 15 meters long and another leg that is 20 meters long. For the triangle to be a right triangle, what must the length of the hypotenuse be?

The side lengths of a right triangle must satisfy the Pythagorean Theorem. Substitute 15 for a and 20 for b in the equation.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 15^2 + 20^2 &= c^2 \\ 225 + 400 &= c^2 \\ 625 &= c^2 \\ \sqrt{625} &= \sqrt{c^2} \\ c &= 25 \end{aligned}$$

The hypotenuse must be 25 meters long.

The hypotenuse of a right triangle is 17 centimeters long. One leg of the triangle is 15 centimeters long. Find the length of the other leg.

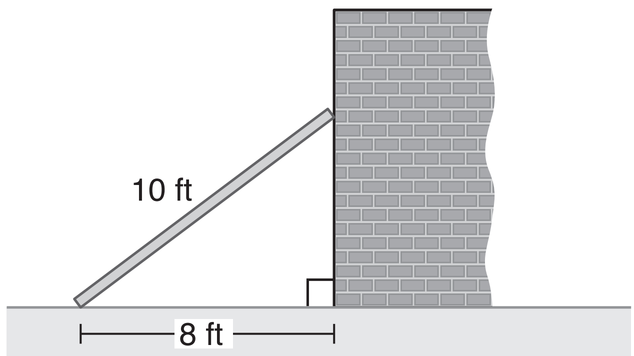
The side lengths of a right triangle must satisfy the Pythagorean Theorem. Substitute 17 for c and 15 for a in the equation.

$$a^2 + b^2 = c^2$$
$$15^2 + b^2 = 17^2$$
$$225 + b^2 = 289$$
$$b^2 = 64$$
$$b = 8$$

The other leg of the triangle is 8 centimeters long.

Try It

A 10-foot-long piece of lumber is leaning against a wall. The bottom of the piece of lumber is 8 feet from the base of a wall. How high up the wall does the piece of lumber reach?



The side lengths of a right triangle must satisfy the

____.

Substitute ________ for c and ________ for a in the equation.

$$a^2 + b^2 = c^2$$
$$\_\_\_\_\_\_ + b^2 = \_\_\_\_\_\_$$
$$\_\_\_\_\_\_ + b^2 = \_\_\_\_\_\_$$
$$b^2 = \_\_\_\_\_\_$$
$$b = \_\_\_\_\_\_$$

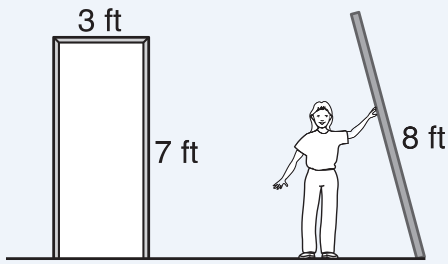
The piece of lumber reaches ________ feet up the wall.

The side lengths of a right triangle must satisfy the Pythagorean Theorem. Substitute 10 for c and 8 for a in the equation.

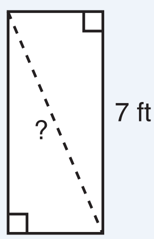
$$8^2 + b^2 = 10^2$$
$$64 + b^2 = 100$$
$$b^2 = 36$$
$$b = 6$$

The piece of lumber reaches 6 feet up the wall.

Shirley is trying to roll a circular folding table with an 8-foot diameter through a doorway that measures 7 feet by 3 feet. She plans to lean the table over and roll it through the diagonal of the doorway. Will the table fit through the doorway?



You can model this problem by drawing a picture of the diagonal of a rectangle.



The rectangle represents the doorway, and the dashed line represents the diagonal of the doorway.

The triangles formed by the width, height, and diagonal of the doorway are right triangles. The diagonal is the hypotenuse of the triangles.

Find the length of the hypotenuse and compare it to 8 feet. If the hypotenuse is greater than 8 feet, the table will fit through the doorway. If the hypotenuse is less than 8 feet, the table will not fit.

Use the Pythagorean Theorem to find the length of the diagonal. Substitute 3 and 7 for the legs, a and b.

$$a^2 + b^2 = c^2$$
$$3^2 + 7^2 = c^2$$
$$9 + 49 = c^2$$
$$58 = c^2$$
$$\sqrt{58} = c$$

Because $7^2 = 49$ and $8^2 = 64$, $\sqrt{58}$ is between 7 and 8. The diagonal of the doorway measures less than 8 feet.

Since the diameter of the table, 8 feet, is greater than the diagonal of the doorway, the table will not fit through the doorway.

Try It

Television sizes are described by the diagonal measurement across the screen. The rectangular screen of John's television set measures 12 inches by 16 inches. What is the size of his television to the nearest inch?

©Photodisc

Model the television screen by drawing a ____________________ .

The size of the television screen is the length of the ______________ .

The diagonal divides the rectangle into two right ________________ .

Use the ____________________________________ to find the length of the diagonal.

The diagonal is opposite the right angle, so it is the ______________ of the right triangle.

Represent the diagonal with c. Substitute ________ and ________ for a and b, the two legs of the triangle.

$$a^2 + b^2 = c^2$$

$$\_\_\_\_\_\_ + \_\_\_\_\_\_ = c^2$$

$$\_\_\_\_\_\_ + \_\_\_\_\_\_ = c^2$$

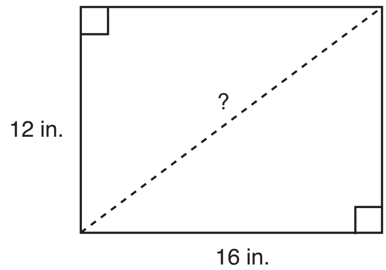
$$\_\_\_\_\_\_ = c^2$$

$$\_\_\_\_\_\_ = c$$

John has a ________-inch television set.

Model the television screen by drawing a rectangle.

The size of the television screen is the length of the diagonal. The diagonal divides the rectangle into two right triangles. Use the Pythagorean Theorem to find the length of the diagonal. The diagonal is opposite the right angle, so it is the hypotenuse of the right triangle. Represent the diagonal with c. Substitute 12 and 16 for a and b, the two legs of the triangle.



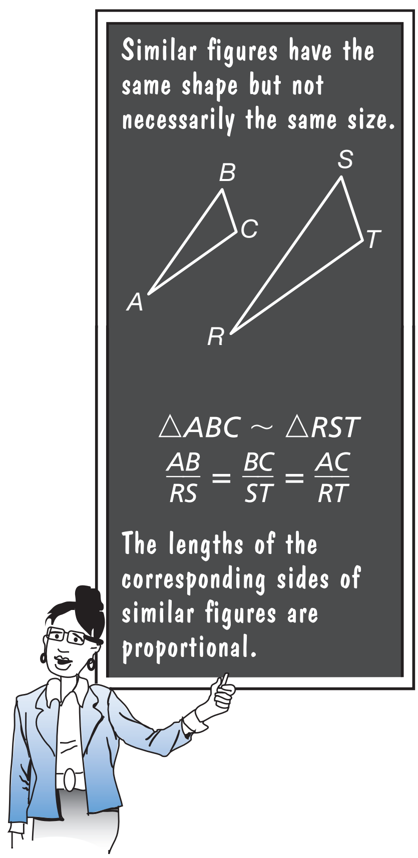
$$a^2 + b^2 = c^2$$
$$12^2 + 16^2 = c^2$$
$$144 + 256 = c^2$$
$$400 = c^2$$
$$20 = c$$

John has a 20-inch television set.

How Can You Use Proportional Relationships to Solve Problems?

You can use proportional relationships to find missing side lengths in similar figures. To solve problems that involve similar figures, follow these guidelines.

- Identify the corresponding sides of similar figures.
- Write and solve a proportion by using cross products.
- Answer the question asked.



A sign company sells advertising banners that are 20 feet long by 4 feet wide. A customer asked the company to increase a banner's size proportionally so it would be 25 feet long. How wide will the banner be when it is enlarged?

The original banner and the new banner are similar rectangles. You can use proportional relationships to find missing side lengths in similar figures.

Write a proportion. Let x represent the width of the enlarged banner.

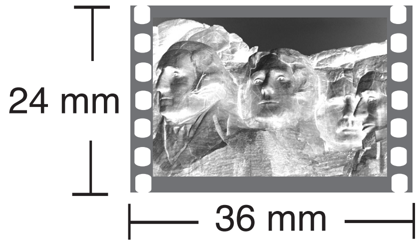
$$\frac{\text{original}}{\text{new}} \qquad \frac{20}{25} = \frac{4}{x}$$
$$20x = 25 \cdot 4$$
$$20x = 100$$
$$x = 5$$

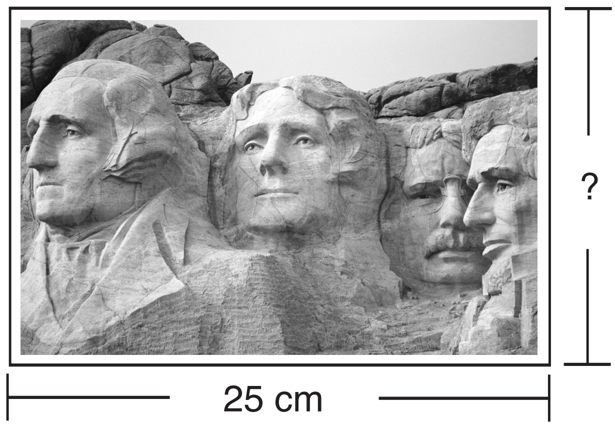
The new banner will be 5 feet wide.

Objective 4

Try It

A 35-millimeter film negative is approximately 24 millimeters tall by 36 millimeters wide. Chris wants to use the negative to make a print that is proportional in size to the negative.





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If the print will be 25 centimeters wide, how tall must it be, to the nearest centimeter?

The negative and the print are ________________ figures because the lengths of their corresponding sides are proportional.

Identify corresponding sides.

The width of the negative corresponds to the ______________ of the print.

The height of the negative corresponds to the ______________ of the print.

Let h represent the ________________ of the print.

Write a proportion.

$$\frac{\text{negative}}{\text{print}} \qquad \frac{36}{\square} = \frac{\square}{h}$$

Use cross products to solve the proportion.

________ h = ________ • ________

________ h = ________

$h \approx$ ________

The print will be about ________ centimeters tall.

The negative and the print are similar figures because the lengths of their sides are proportional. The width of the negative corresponds to the width of the print. The height of the negative corresponds to the height of the print. Let h represent the height of the print.

$$\frac{36}{25} = \frac{24}{h}$$

$$36h = 25 \cdot 24$$

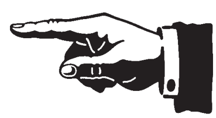
$$36h = 600$$

$$h \approx 16.67$$

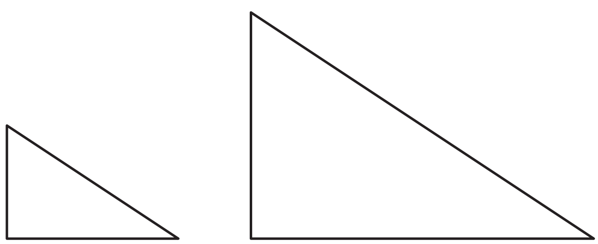
The print will be about 17 centimeters tall.

How Is the Perimeter of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. For example, the triangle on the left has been dilated by a scale factor of 2 to form the triangle on the right. The perimeter of the dilated figure will change by the same scale factor.



Do you see that . . .



If a triangle with side lengths of 3, 7, and 10 inches is dilated by a scale factor of 2, its new dimensions will be 6, 14, and 20 inches. Each dimension has been multiplied by 2. What effect will this dilation have on the triangle's perimeter?

- Find the perimeter of the original triangle.

$$P = 3 + 7 + 10 = 20 \text{ inches}$$

- Find the perimeter of the dilated triangle.

$$P = 6 + 14 + 20 = 40 \text{ inches}$$

The perimeter of the dilated triangle has also increased by a factor of 2, because the perimeter of a dilated figure changes by the same scale factor as the side lengths of the figure.

A rectangle with a perimeter of 10 inches is dilated by a scale factor of $\frac{1}{3}$. What is the perimeter of the new rectangle?

The perimeter of the original rectangle is 10 inches. The perimeter should change by the same scale factor, $\frac{1}{3}$. Multiply the original perimeter by the scale factor.

$$P = 10\left(\frac{1}{3}\right) = \frac{10}{3} = 3\frac{1}{3} \text{ in.}$$

The perimeter of the new rectangle is $3\frac{1}{3}$ inches.

Try It

A triangle with a perimeter of 128.5 feet is enlarged by a scale factor of 10.5. What is the perimeter of the new triangle?

The perimeter of the original triangle is ____________ feet.

The perimeter should change by a scale factor of ____________ .

Multiply the original perimeter by the same scale factor.

$$P = \underline{\qquad\qquad} \cdot \underline{\qquad\qquad} = \underline{\qquad\qquad}$$

The perimeter of the new triangle is ____________ feet.

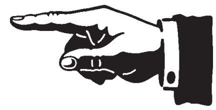
The perimeter of the original triangle is 128.5 feet. The perimeter should change by a scale factor of 10.5.

$$P = 128.5 \cdot 10.5 = 1{,}349.25$$

The perimeter of the new triangle is 1,349.25 feet.

How Is the Area of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The area of the dilated figure will change by the square of the scale factor.



Do you see that . . .

Marsha put a 5-by-9-inch picture on a photocopier and dilated it by a scale factor of $\frac{1}{3}$. How is the area of the picture affected by the reduction?

The area of the picture should change by the square of the scale factor, which is $\left(\frac{1}{3}\right)^2$, or $\frac{1}{9}$.

Check to see if this is true.

- Find the area of the original picture.

$$A = lw = 5 \cdot 9 = 45 \text{ in.}^2$$

- Find the dimensions of the dilated picture.

new length = scale factor • original length = $\left(\frac{1}{3}\right)5 = \frac{5}{3}$ in.

new width = scale factor • original width = $\left(\frac{1}{3}\right)9 = 3$ in.

- Find the area of the dilated picture.

$$\text{A} = \left(\frac{5}{3}\right)3 = 5 \text{ in.}^2$$

Since $\frac{1}{9}$ of 45 is 5, the area of the dilated picture is $\frac{1}{9}$ the area of the original picture.

A hexagon with an area of 125 cm^2 is dilated by a scale factor of 5. What is the area of the new hexagon?

The area should change by the square of the scale factor, which is $5^2 = 25$. Multiply the area of the original hexagon by the square of the scale factor.

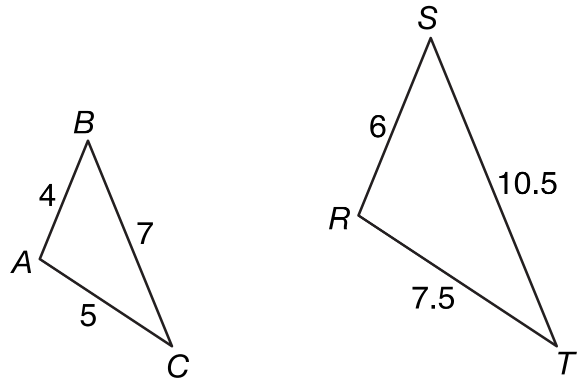
$$A = 125 \cdot 25 = 3{,}125 \text{ cm}^2$$

The area of the new hexagon is 3,125 square centimeters.

Objective 4

Try It

$\triangle ABC$ is similar to $\triangle RST$.



What is the ratio of the area of the larger triangle to the area of the smaller triangle?

Find the scale factor.

$\overline{AB}$ corresponds to ________.

$RS =$ ________ units

$AB =$ ________ units

$$\frac{RS}{AB} = \frac{6}{\square} = \frac{\square}{\square}$$

The scale factor is $\frac{\square}{\square}$.

The area of the new triangle will increase by the ____________ of the scale factor.

The square of the scale factor is $\left(\frac{\square}{\square}\right)^2 = \frac{\square^2}{\square^2} = \frac{\square}{\square}$.

The ratio of the area of the larger triangle to the area of the smaller triangle is $\frac{\square}{\square}$.

$\overline{AB}$ corresponds to $\overline{RS}$.

$RS = 6$ units

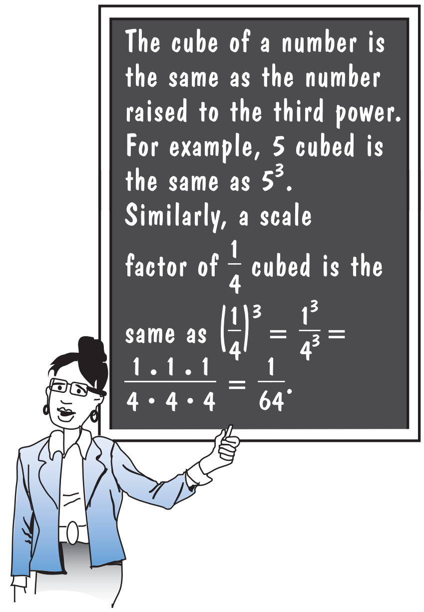
$AB = 4$ units

$$\frac{RS}{AB} = \frac{6}{4} = \frac{3}{2}$$

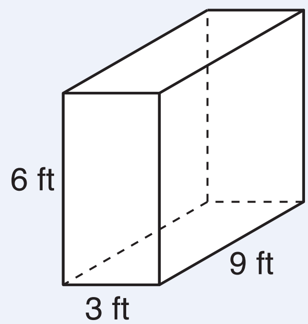
The scale factor is $\frac{3}{2}$. The area of the new triangle will increase by the square of the scale factor. The square of the scale factor is $\left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$. The ratio of the area of the larger triangle to the area of the smaller triangle is $\frac{9}{4}$.

How Is the Volume of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The volume of the dilated figure will change by the cube of the scale factor.



The dimensions of a rectangular prism are 9 by 3 by 6 feet. If the dimensions of the prism are dilated by a scale factor of $\frac{1}{3}$, how is the volume of the prism affected?



The volume of the prism should change by the cube of the scale factor, which is $\left(\frac{1}{3}\right)^3 = \frac{1^3}{3^3} = \frac{1}{27}$.

Check to see if this is true.

- Find the volume of the original prism.

$$V = Bh$$
$$V = (9 \cdot 3) \cdot 6$$
$$V = 27 \cdot 6$$
$$V = 162 \text{ ft}^3$$

- Find the dimensions of the dilated prism.

  new length = scale factor • original length = $\left(\frac{1}{3}\right)9 = 3$ ft

  new width = scale factor • original width = $\left(\frac{1}{3}\right)3 = 1$ ft

  new height = scale factor • original height = $\left(\frac{1}{3}\right)6 = 2$ ft

- Find the volume of the dilated prism.

$$V = Bh$$
$$V = (3 \cdot 1) \cdot 2$$
$$V = 3 \cdot 2$$

$$V = 6 \text{ ft}^3$$

Since $\frac{1}{27}$ of 162 is 6, the volume of the dilated prism is $\frac{1}{27}$ the volume of the original prism.

A cone with a volume of 75 cubic meters is dilated by a scale factor of 1.5. What is the volume of the new cone?

The volume should change by the cube of the scale factor.

$$(1.5)^3 = (1.5)(1.5)(1.5) = 3.375$$

Multiply the original volume by the cube of the scale factor.

$$V = 75 \cdot 3.375 = 253.125 \text{ m}^3$$

The volume of the new cone is 253.125 cubic meters.

Try It

A hexagonal prism has a volume of 80 cubic inches. The dimensions of the prism are enlarged by a scale factor of $\frac{3}{2}$ to produce a new prism. What is the volume of the enlarged hexagonal prism?

The dimensions of the prism were enlarged by a scale factor of $\frac{\square}{\square}$.

The volume of the new prism will increase by the cube of $\frac{\square}{\square}$.

$$\left(\frac{3}{2}\right)^3 = \frac{\square^3}{\square^3} = \frac{\square}{\square}$$

Multiply the original volume by the cube of the scale factor.

$$\_\_\_\_\_ \cdot \frac{\square}{\square} = \_\_\_\_\_ \text{ in.}^3$$

The volume of the enlarged prism is _______ cubic inches.

The dimensions of the prism were enlarged by a scale factor of $\frac{3}{2}$. The volume of the new prism will increase by the cube of $\frac{3}{2}$.

$$\left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$$

Multiply the original volume by the cube of the scale factor.

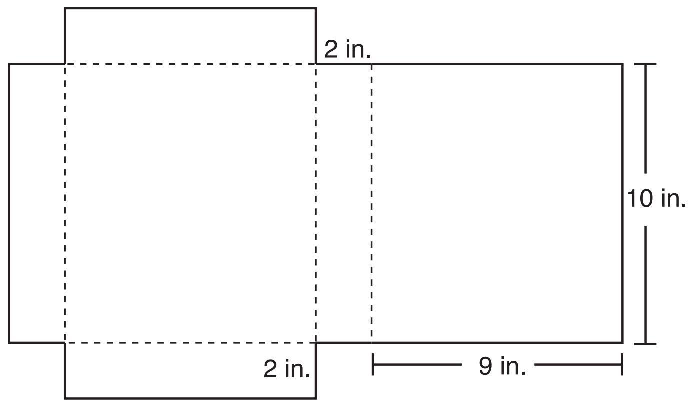
$$80 \cdot \frac{27}{8} = 270 \text{ in.}^3$$

The volume of the enlarged prism is 270 cubic inches.

Now practice what you've learned.

Question 32

What is the surface area of the rectangular prism that can be formed from the net shown at the right?



A 128 square inches

B 360 square inches

C 256 square inches

D 42 square inches

Answer Key: page 163

Question 33

Mrs. Solis bought a cement doorstop shaped like a square pyramid. The side length of the base of the doorstop is 5 in., and the slant height of the pyramid is 3 in. If she wants to paint the entire surface of the doorstop, including the base, what is the area in square inches that she must paint?

A 30 in.^{2}

B 55 in.^{2}

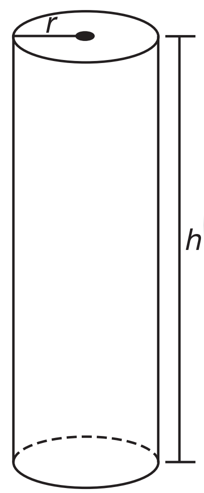
C 85 in.^{2}

D 75 in.^{2}

Answer Key: page 163

Question 34

A small candy container is shaped like a cylinder. The manufacturer wraps a label around the curved surface of the can. Use the ruler on the Mathematics Chart to measure the height and radius of the cylinder in centimeters. Which of the following is closest to the area of the label in square centimeters?



A 19 cm^2

B 6 cm^2

C 38 cm^2

D 44 cm^2

Answer Key: page 163

Question 35

Gerard is building a rectangular patio measuring 12.5 feet long by 9 feet wide. If the floor will be a cement slab 4 inches thick, how many cubic feet of cement will it take to build the slab?

A 37.5 ft^3

B 450 ft^3

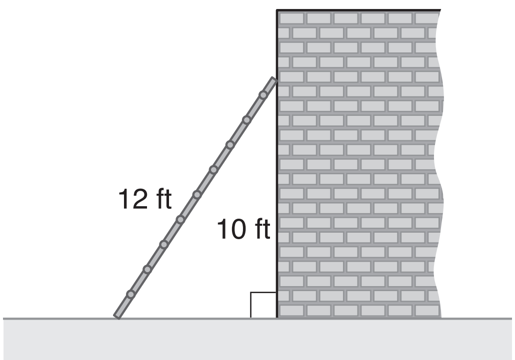
C 375 ft^3

D 4,500 ft^3

Answer Key: page 163

Question 36

A 12-foot ladder is leaning against the side of a building. The top of the ladder reaches 10 feet up the side of the building. Approximately how far is the bottom of the ladder from the base of the building?



A 2 ft

B 15.6 ft

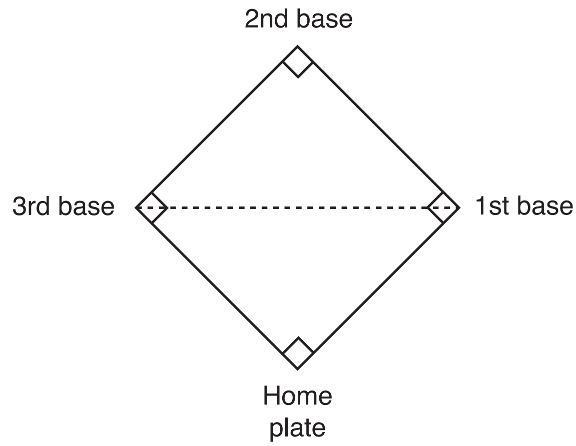
C 6.6 ft

D 1.4 ft

Answer Key: page 163

Question 37

A baseball diamond is a square with a side length of 90 feet. To the nearest foot, what is the distance between first base and third base?



A 810 ft

B 90 ft

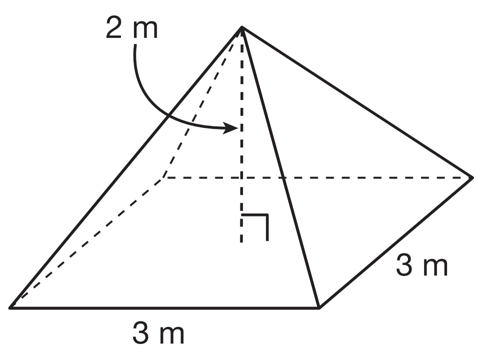
C 180 ft

D 127 ft

Answer Key: page 164

Question 38

A square pyramid is shown below.



What is the volume of this pyramid?

A 9 m^3

B 6 m^3

C 18 m^3

D 8 m^3

Answer Key: page 164

Question 39

An architect built a scale model of a house to show his client. The length of the model is 12.5 inches, and its width is 7.75 inches. If the actual house will be 64 feet long, how wide will it be to the nearest foot?

A 84 ft

B 40 ft

C 60 ft

D 103 ft

Answer Key: page 164

Question 40

A rectangle has a perimeter of 39 centimeters. The rectangle is dilated by a scale factor of $\frac{1}{2}$ to produce a new rectangle. What is the perimeter of the new rectangle?

A 19.5 cm

B 9.75 cm

C 78 cm

D 156 cm

Answer Key: page 164

Question 41

A triangle has an area of 6 square feet. The triangle is dilated by a scale factor of 5 to produce a new triangle. What is the area of the new triangle?

A $\frac{6}{5}$ ft^2

B 30 ft^2

C 150 ft^2

D 180 ft^2

Answer Key: page 164

Question 42

A cylinder has a volume of 3,600 cubic centimeters. If the cylinder is dilated by a factor of $\frac{1}{4}$, what is the volume of the new cylinder?

A 14,400 cm^3

B 900 cm^3

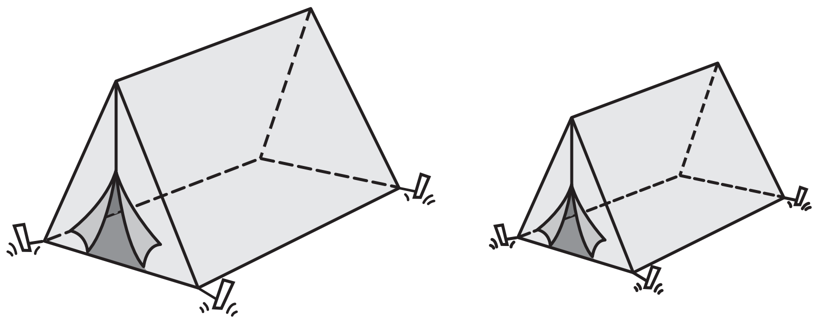
C 225 cm^3

D 56.25 cm^3

Answer Key: page 164

Question 43

A tent has dimensions that are $\frac{3}{4}$ of the dimensions of the tent next to it. If the volume of the smaller tent is 1,000 cubic feet, what is the volume of the larger tent, to the nearest cubic foot?



A 1,777 ft^3

B 2,370 ft^3

C 750 ft^3

D 1,333 ft^3

Answer Key: page 165

Objective 5

The student will demonstrate an understanding of probability and statistics.

For this objective you should be able to

- apply theoretical and experimental probability concepts to make predictions;
- use statistical procedures to describe data; and
- evaluate predictions and conclusions based on statistical data.

What Is Probability?

Probability is a measure of how likely an event is to occur. The probability of an event occurring is the ratio of the number of favorable outcomes to the number of possible outcomes. In a probability experiment, favorable outcomes are the outcomes that you are interested in.

The probability, P, of an event occurring is from 0 to 1.

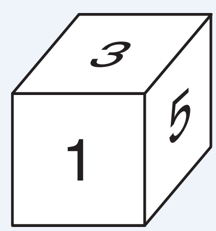
- If an event is impossible, its probability is 0.
- If an event is certain to occur, its probability is 1.

For example, the probability of drawing a white marble from a bag containing 3 red marbles and 1 white marble is the ratio of the number of favorable outcomes, 1 white marble, to the number of possible outcomes, 4 marbles. The probability of drawing a white marble is $\frac{1}{4}$. This is often written as $P(\text{white}) = \frac{1}{4}$.

Do you see that . . .

Drawing a certain kind of marble from a bag of marbles is a **simple event**. An event that is made up of a sequence of simple events is called a **compound event**. For example, drawing a white marble first and drawing a red marble second is a compound event.

What is the probability of rolling a 1 or a 2 on a number cube numbered 1 through 6?



The sample space for this experiment is {1, 2, 3, 4, 5, 6}. There are 6 possible outcomes.

There are 2 favorable outcomes for this experiment: rolling a 1 or a 2.

The probability of rolling a 1 or a 2 is the ratio of the number of favorable outcomes to the number of possible outcomes: 2 out of 6. This could also be expressed as $\frac{2}{6}$ or $\frac{1}{3}$; it is not always necessary to simplify fractions in probability.

The probability of rolling a 1 or a 2 on a number cube is $\frac{1}{3}$. This can also be written as $P(1 \text{ or } 2) = \frac{1}{3}$.

Objective 5

What Is the Difference Between Dependent and Independent Events?

In a compound event, if the outcome of the first event affects the possible outcomes of the second event, the events are called **dependent events**. If the outcome of the first event does not affect the possible outcomes of the second event, the events are called **independent events**.

A bag contains 6 blue marbles, 3 red marbles, and 1 green marble. Lilah draws one marble from the bag, records its color, and then puts it back in the bag. Then she draws another marble and records its color. What is the probability of drawing 2 blue marbles?

This is an example of a compound event made up of two independent events. The same number and color of marbles are in the bag for each draw. The outcome of the first marble drawn does not affect the possible outcomes of the second marble drawn. Therefore, the events are independent.

A bag contains 6 blue marbles, 3 red marbles, and 1 green marble. One marble is drawn from the bag, and its color is recorded. Another marble is drawn, and its color is also recorded. What is the probability of drawing 2 blue marbles if the first marble is not returned to the bag before the second marble is drawn?

This is an example of a compound event made up of dependent events. On the first draw there are 10 marbles in the bag, but on the second draw there are only 9 marbles in the bag. The outcome of the first marble drawn affects the possible outcomes of the second marble drawn. Therefore, the events are dependent.

How Do You Find the Probability of Compound Events?

When finding the probability of a compound event, first determine whether the simple events included are independent or dependent.

- If the simple events are independent, then multiply the probabilities of the simple events that make up the compound event.

If $P(A)$ represents the probability of event A and $P(B)$ represents the probability of event B, then the probability of the compound event (A and B) can be represented algebraically.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

For example, the probability of tossing a head on a coin toss is $P(H) = \frac{1}{2}$. The probability of tossing heads twice on two coin tosses is

$$P(H \text{ and } H) = P(H) \cdot P(H) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Objective 5

A probability experiment consists of rolling a number cube numbered 1 through 6 and then spinning a spinner with two equally likely outcomes, red or blue. Find the probability of rolling a 4 on the number cube and spinning blue on the spinner.

- The probability of rolling a 4 on the number cube is

$$P(4) = \frac{1}{6}$$

- The probability of spinning blue on the spinner is

$$P(\text{blue}) = \frac{1}{2}$$

- Since these events are independent, the probability of rolling a 4 on the number cube and spinning blue on the spinner is

$$\begin{aligned} P(4 \text{ and blue}) &= P(4) \cdot P(\text{blue}) \\ &= \frac{1}{6} \cdot \frac{1}{2} \\ &= \frac{1}{12} \end{aligned}$$

Another way to find the probability of this compound event is to look at the sample space for the experiment and identify the favorable outcomes. This method works only if the outcomes are all equally likely. The following table lists all the possible outcomes. The one favorable outcome is shaded.

Sample Space

Number Cube	Spinner
1	red
1	blue
2	red
2	blue
3	red
3	blue
4	red
4	**blue**
5	red
5	blue
6	red
6	blue

There are 12 possible outcomes; only 1 of them is favorable. The probability of rolling a 4 on the number cube and spinning blue on the spinner is $\frac{1}{12}$.

This matches the result obtained from using the rule for the probability of a compound event.

$$P(4 \text{ and blue}) = \frac{1}{12}$$

Finding the probability for dependent events is slightly more complicated, since the probability for the second event is affected by what happens in the first event. Let us consider another experiment with a bag of marbles.

A bag contains 3 marbles; 2 of the marbles are red, and 1 marble is white. One marble is drawn at random, its color is noted, and it is not returned to the bag. A second marble is then drawn at random, and its color is noted. What is the probability of drawing a red marble first and a white marble second?

Since 2 of the 3 marbles are red, the probability of drawing a red marble first, $P(\text{Red})$, is $\frac{2}{3}$. Since we are not replacing the first marble drawn, there are now 2 marbles remaining in the bag. There is still only one white marble, so the probability of drawing a white marble, $P(\text{White})$, is $\frac{1}{2}$. So the probability of drawing a red marble and then a white marble, without putting either one back, can be found by multiplying these probabilities together.

$$P(\text{Red}) \cdot P(\text{White}) = \frac{2}{3} \cdot \frac{1}{2} = \frac{2}{6}$$

A bag contains 5 marbles: 3 blue marbles and 2 green marbles. We will draw one marble from the bag, note its color, and not put it back in the bag. Then we will draw a second marble. What is the probability that both marbles we draw will be blue?

Find the probability of getting a blue marble on the first draw. The probability of drawing one of the 3 blue marbles out of the 5 marbles in the bag is $\frac{3}{5}$.

$$P(\text{Blue}_{\text{first}}) = \frac{3}{5}$$

Now find the probability of getting a blue marble on the second draw, given that the first marble was blue. Out of the 4 marbles remaining in the bag, we now have 2 blue marbles left. Therefore, the probability of drawing a blue marble now is $\frac{2}{4}$.

$$P(\text{Blue}_{\text{second}}) = \frac{2}{4}$$

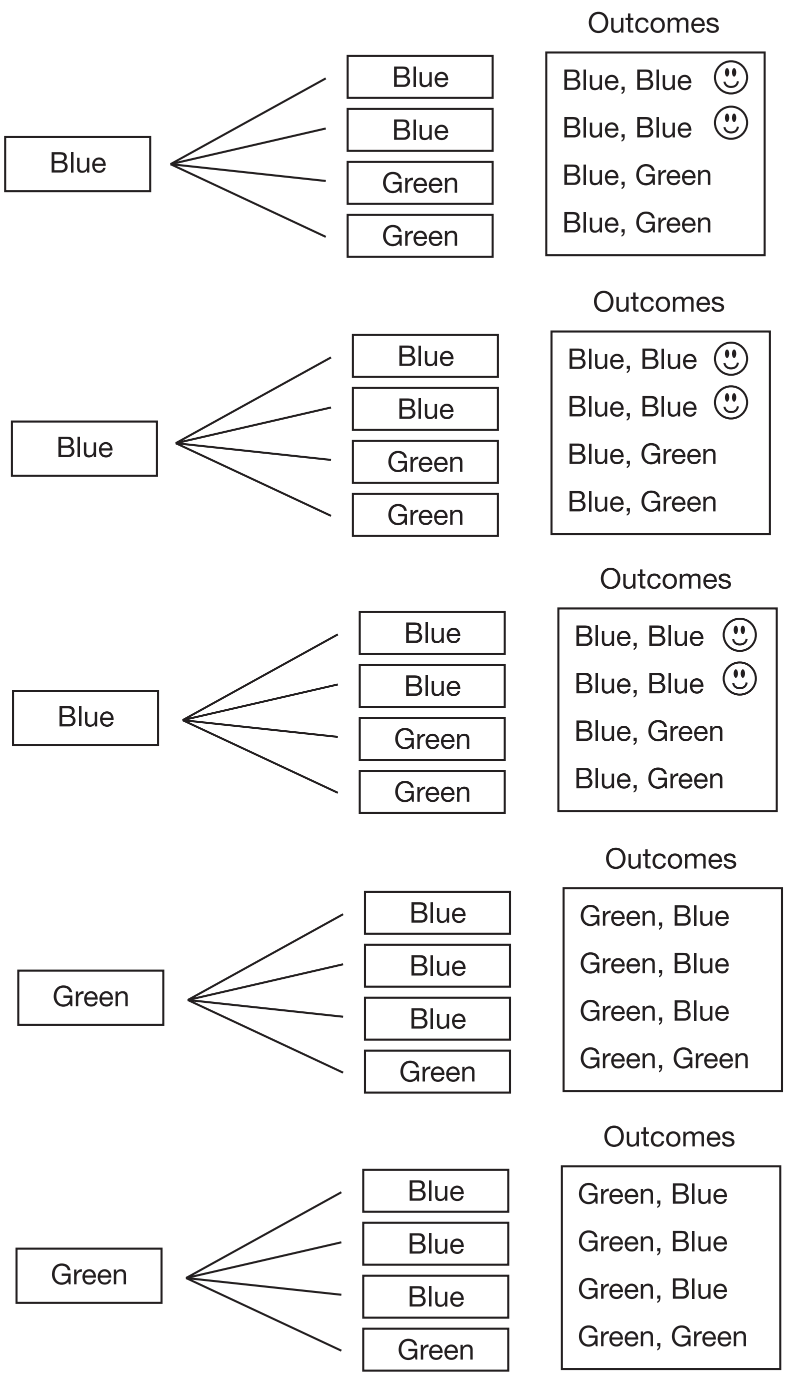
Objective 5

And so we can find the probability of drawing 2 blue marbles.

$$P(\text{Blue}_{\text{first}}) \cdot P(\text{Blue}_{\text{second}}) = \frac{3}{5} \cdot \frac{2}{4}$$
$$= \frac{6}{20}$$

The probability of drawing 2 blue marbles when the first one is not put back in the bag is $\frac{6}{20}$.

Another way to find the probability of this dependent event is to look at a tree diagram for the experiment and identify the favorable outcomes.



There are 20 possible outcomes. Of those, 6 are favorable. The probability of drawing 2 blue marbles without putting the first one back is $\frac{6}{20}$. This matches our result from finding the probabilities of the dependent events and multiplying them together.

Objective 5

Try It

A bag contains 10 tiles numbered 1 through 10. Another bag contains 5 tiles with the letters *A*, *E*, *U*, *X*, and *Z* written on them. One tile is drawn from each bag. What is the probability of drawing both an even number and a vowel?

The outcome from the first event does not affect the possible outcomes of the second event. These events are ______________________ events. The probability of drawing an even number from the first bag is the ratio of the number of ______________________ outcomes to the number of ______________________ outcomes. There are ______ favorable outcomes. There are a total of ______ possible outcomes.

$$P(\text{even number}) = \frac{\square}{\square}$$

The probability of drawing a vowel from the second bag is the ratio of the number of ______________________ outcomes to the number of ______________________ outcomes. There are ______ favorable outcomes. There are a total of ______ possible outcomes.

$$P(\text{vowel}) = \frac{\square}{\square}$$

The probability of drawing both an even number and a vowel is

$$P(\text{even number and vowel}) = \frac{\square}{\square} \cdot \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$$

These events are independent events. The probability of drawing an even number from the first bag is the ratio of the number of favorable outcomes to the number of possible outcomes. There are 5 favorable outcomes. There are a total of 10 possible outcomes.

$$P(\text{even number}) = \frac{5}{10}$$

The probability of drawing a vowel from the second bag is the ratio of the number of favorable outcomes to the number of possible outcomes. There are 3 favorable outcomes. There are a total of 5 possible outcomes.

$$P(\text{vowel}) = \frac{3}{5}$$

$$P(\text{even number and vowel}) = \frac{5}{10} \cdot \frac{3}{5} = \frac{15}{50} = \frac{3}{10}$$

Objective 5

What Is the Difference Between Theoretical and Experimental Probability?

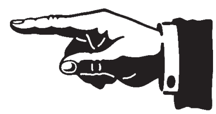
The **theoretical probability** of an event occurring is the ratio comparing the number of ways the favorable outcome should occur to the number of possible outcomes. If you toss a coin, theoretically it should land on heads $\frac{1}{2}$ of the time.

$$P(H) = \frac{1}{2} = 0.5$$

The **experimental probability** of an event occurring is the ratio of the actual number of times the favorable outcome occurs in a series of repeated trials of an experiment to the total number of trials in the experiment. If you toss a coin 100 times, it is possible that the coin will land on heads 48 times and tails 52 times. The experimental probability of the coin landing on heads in this situation is $\frac{48}{100}$.

$$P(H) = \frac{48}{100} = 0.48$$

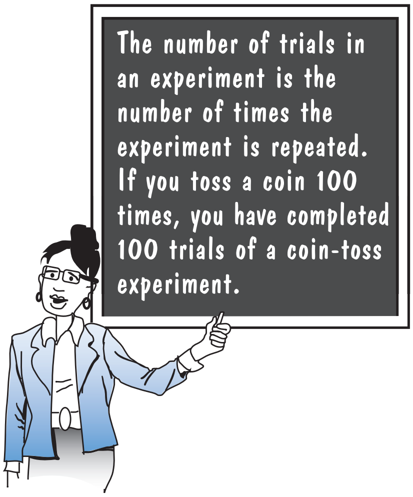
The two types of probabilities, theoretical and experimental, are not always equal. In this case, the theoretical probability is 0.5, but the experimental probability is 0.48.



Do you see that . . .

For a given situation, the experimental probability is usually close to, but slightly different from, the theoretical probability. The greater the number of trials, the closer the experimental probability should be to the theoretical probability.

Objective 5



A cube has the letters *A* through *F* written on its faces, with one letter on each face. Find the experimental and theoretical probabilities of rolling the letter *B*.

- The theoretical probability of rolling the letter *B* is $\frac{1}{6}$. There is 1 face with a *B* on it out of the 6 faces on the cube.
- The experimental probability is determined by conducting repeated trials of rolling the cube and recording how many times the letter *B* is actually rolled.

  Suppose you roll the cube 30 times and get the results listed below. How many times did you actually roll the letter *B*?

Rolls of a Cube

Outcome	Frequency
A	5
B	**4**
C	5
D	6
E	4
F	6
Total	30

In this experiment the letter *B* was rolled 4 times out of 30 rolls of the cube.

The experimental probability of rolling the letter *B* is the number of times the letter *B* was actually rolled compared to the total number of trials: $\frac{4}{30}$, or $\frac{2}{15}$.

For this experiment the two probabilities, theoretical and experimental, are not equal. The theoretical probability was $\frac{1}{6}$, but the experimental probability was $\frac{2}{15}$.

Try It

A bag contains 10 marbles: 5 red, 3 blue, and 2 green. Wendy draws a marble from the bag, records the color, and then puts the marble back in the bag. She repeats this for a total of 100 trials. The table below summarizes the results of her experiment.

Draws of a Marble

Outcome	Frequency
Red	48
Blue	27
Green	25
Total	100

How does the experimental probability of drawing a blue marble compare to the theoretical probability of drawing a blue marble?

The experimental probability of drawing a blue marble is the number of times Wendy actually drew a __________ marble compared to the total number of trials.

Experimental probability: $P(\text{blue}) = \frac{\square}{\square} = 0.27$.

The theoretical probability of drawing a blue marble from the bag is the number of ______________ outcomes compared to the number of ______________ outcomes. There are ______ blue marbles out of a total of ______ marbles in the bag.

Theoretical probability: $P(\text{blue}) = \frac{\square}{\square} = 0.30$.

Since $0.27 < 0.30$, the experimental probability of drawing a blue marble is slightly ______________ than the theoretical probability of drawing a blue marble.

The experimental probability of drawing a blue marble is the number of times Wendy actually drew a blue marble compared to the total number of trials. Experimental probability: $P(\text{blue}) = \frac{27}{100} = 0.27$. The theoretical probability of drawing a blue marble from the bag is the number of favorable outcomes compared to the number of possible outcomes. There are 3 blue marbles out of a total of 10 marbles in the bag. Theoretical probability: $P(\text{blue}) = \frac{3}{10} = 0.30$. Since $0.27 < 0.30$, the experimental probability of drawing a blue marble is slightly less than the theoretical probability of drawing a blue marble.

Objective 5

How Can You Use Probability to Make Predictions and Decisions?

You can use theoretical and experimental probabilities to make predictions. If you know the probability of an event occurring and you know the total number of trials, then you can use a proportion to predict the likely number of favorable outcomes.

- Write a ratio that represents the probability of an event occurring.
- Write a ratio that compares the number of favorable outcomes to the total number of trials.
- Write a proportion by setting the two ratios equal to each other.
- Solve the proportion by using cross products.

A student plays basketball in a neighborhood league. During practice for a citywide tournament, this student successfully completes 65 out of 100 attempted free throws. If this student attempts at least 80 free throws at the tournament, what is the best prediction of the number of successful free throws he will make?

This is an example of using an experimental probability to make a prediction.

- Find the experimental probability of the student making a successful free throw. During practice the student successfully completed 65 free throws out of a total of 100 attempted. The experimental probability of this student successfully completing a free throw is $\frac{65}{100}$, or $\frac{13}{20}$.
- During his games at the tournament, this student attempts 80 free throws. The predicted number of successful free throws, x, can be found by solving the proportion $\frac{13}{20} = \frac{x}{80}$.

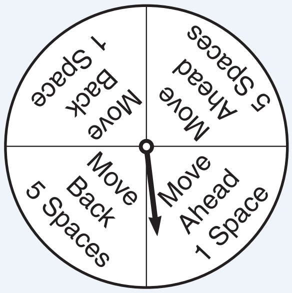
$$\frac{13}{20} = \frac{x}{80}$$

$$1040 = 20x$$

$$x = 52$$

Based on results in practice, the student can be expected to successfully make 52 of the next 80 attempted free throws.

A game uses a spinner to determine the players' next moves.



The spinner is divided into four congruent sectors. If the spinner is spun 60 times, what is the best prediction of the number of times it will land on the sector marked "Move back 5 spaces"?

This is an example of using a theoretical probability to make a prediction.

- Find the theoretical probability of spinning "Move back 5 spaces."

  The spinner is divided into 4 congruent sectors.

  The probability of landing on "Move back 5 spaces" is $\frac{1}{4}$.

- The spinner will be spun 60 times.

  Write a proportion.

$$\frac{1}{4} = \frac{x}{60}$$

- Solve the proportion for x, the predicted number of times the spinner will land on "Move back 5 spaces."

$$\frac{1}{4} = \frac{x}{60}$$

$$4x = 60$$

$$x = 15$$

The spinner can be expected to land on "Move back 5 spaces" 15 out of the next 60 times it is spun.

Try It

A newspaper surveyed 100 residents in a city at random and found that 63 of them planned to vote on election day. If there are 14,500 residents in the city, what is the best estimate of the number of residents who will vote on election day?

Find the ________________ probability of a resident voting on election day.

Of the residents surveyed, ______ plan to vote on election day.

The newspaper surveyed ______ residents.

The ratio of residents who plan to vote to the total number of residents surveyed is $\frac{\square}{\square}$.

The experimental probability of a resident voting is $\frac{\square}{\square}$.

Use this probability to predict the number of residents who will vote.

There are ______________ residents in the city.

Write the proportion. $\frac{\square}{100} = \frac{x}{\square}$

$$\underline{\qquad\qquad} = \underline{\qquad}\,x$$

$$\underline{\qquad\qquad} = x$$

An estimate of the number of residents in the city who should be expected to vote on election day is ________________ .

Find the experimental probability of a resident voting on election day. Of the residents surveyed, 63 plan to vote on election day. The newspaper surveyed 100 residents. The ratio of residents who plan to vote to the total number of residents surveyed is $\frac{63}{100}$. The experimental probability of a resident voting is $\frac{63}{100}$.

There are 14,500 residents in the city.

$$\frac{63}{100} = \frac{x}{14{,}500}$$

$$913{,}500 = 100x$$

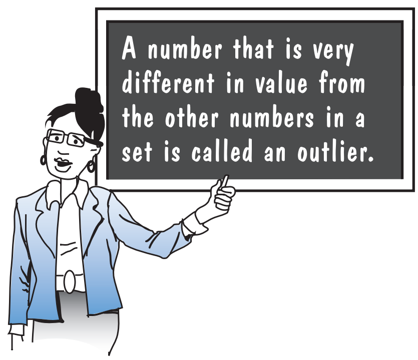
$$9{,}135 = x$$

An estimate of the number of residents in the city who should be expected to vote on election day is 9,135.

How Do You Use Mode, Median, Mean, and Range to Describe Data?

There are many ways to describe the characteristics of a set of data. The mode, median, and mean are all called **measures of central tendency**. These measures of central tendency and range are described in the table below.

Mode	The **mode** of a set of data describes which value occurs most frequently. If two or more numbers occur the same number of times and occur more often than all the other numbers in the set, those numbers are all modes for the data set. If each number in the set occurs the same number of times, the set of data has no mode.	Use the mode to show which value in a set of data occurs most often. For the set {1, 1, 2, 3, 5, 6, 10}, the mode is 1 because it occurs most frequently.
Median	The **median** of a set of data describes what value is in the middle if the set is ordered from greatest to least or from least to greatest. If there are an even number of values, the median is the average of the two middle values. Half of the values are greater than the median, and half of the values are less than the median. The median is a good measure of central tendency to use when a set of data has an outlier.	Use the median to show which number in a set of data is in the middle when the numbers are listed in order. For the set {1, 1, 2, 3, 5, 6, 10}, the median is 3 because it is in the middle when the numbers are listed in order.
Mean	The **mean** of a set of data describes their average. To find the mean, add all of the numbers and then divide by the number of items in the set. The mean of a set of data can be greatly affected if one of the numbers is an outlier. The mean is a good measure of central tendency to use when a set of data does not have any outliers.	Use the mean to show the numerical average of a set of data. For the set {1, 1, 2, 3, 5, 6, 10}, the mean is the sum, 28, divided by the number of items, 7. The mean is $28 \div 7 = 4$.
Range	The **range** of a set of data describes how big a spread there is from the largest value in the set to the smallest value.	Use the range to show how much the numbers vary. For the set {1, 1, 2, 3, 5, 6, 10}, the range is $10 - 1 = 9$.



To decide which of these measures to use to describe a set of data, look at the numbers and ask yourself, *What am I trying to show about the data?*

Sometimes you need to describe how a change in data affects one or more measures of the data set.

Mr. Jonas owns an ice-cream store. The table below shows his sales for several months, to the nearest five dollars.

Ice-Cream Sales

Month	Sales
April	$610
May	$560
June	$660
July	$625

In August the store was closed for 2 weeks, and the sales for the month were only $275. Which measure of central tendency for these 5 months would change the most because of the store's low sales in August?

- The mean of a set of data is the average of the set's values. The value $275 is very different in size from the other values, so it should have a significant impact on the mean.

  The mean for April through July:

  $$(610 + 560 + 660 + 625) \div 4 = 2{,}455 \div 4 = \$613.75$$

  The mean for April through August:

  $$(610 + 560 + 660 + 625 + 275) \div 5 = 2{,}730 \div 5 = \$546.00$$

  The difference in the means is 613.75 – 546.00 = $67.75.

- The median of a set of data is the middle value when the values are listed in order. Even though $275 is very different from the other values, it should not have a significant impact on the median.

  The median for April through July:

  $$(610 + 625) \div 2 = \$617.50$$

  The median for April through August is $610.00.

  The difference between the medians is 617.50 – 610.00 = $7.50.

- The mode of a set of data is the value occurring most frequently. No values are repeated in either of these sets, so neither set has a mode.

The measure of central tendency that changed the most because of the store's low sales in August was the mean.

Try It

Mrs. Jenkins recorded the scores for all the eighth-grade students who took a test. The table below summarizes the measures of the data.

Test Scores

Measure	Value (percent)
Mean	85
Median	80
Mode	83
Range	37

If 55% is the lowest score on the test, which measure of data would be best for finding the highest score?

The ______________________________ of a set of data is the average of the scores. It does not help you find the greatest or least score.

The ______________________________ of a set of data is the middle score when the scores are listed in order. It does not help you find the highest score because it tells you the ______________________________ score rather than the greatest or least score.

The ______________________________ of a set of data is the value that occurs most ______________________________. It does not help you find the highest score because it tells you the most __________________________________ score rather than the greatest or least score.

The _________________________ of a set of data is the difference between the highest and _____________________ scores. If _______ % is the lowest score, then the highest score is _______ + _______ = _______%.

Use the ______________________________ of the values to find the highest score.

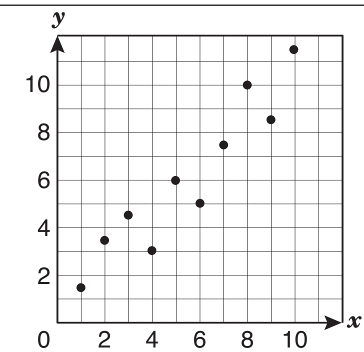
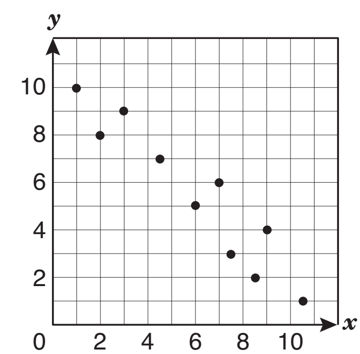
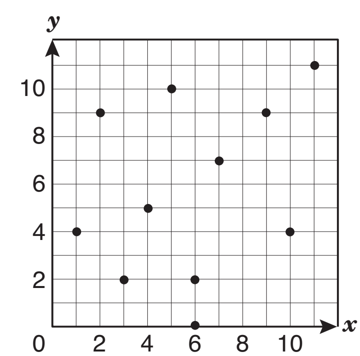
The mean of a set of data is the average of the scores. The median of a set of data is the middle score when the scores are listed in order. It does not help you find the highest score because it tells you the middle score rather than the greatest or least score. The mode of a set of data is the value that occurs most frequently. It does not help you find the highest score because it tells you the most frequent score rather than the greatest or least score. The range of a set of data is the difference between the highest and lowest scores. If 55% is the lowest score, then the highest score is 55 + 37 = 92%. Use the range of the values to find the highest score.

Objective 5

How Do You Use a Scatterplot to Draw Conclusions and Make Predictions?

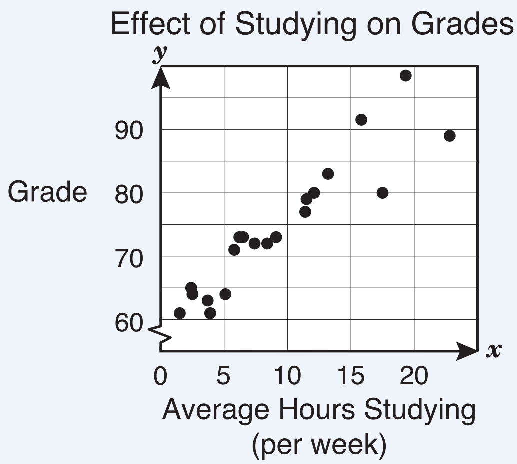
A **scatterplot** is a type of graph that can be used to show whether there is a relationship between two quantities.

To make predictions using a scatterplot, look for correlations in the data or a pattern in the data points. Patterns in a scatterplot are usually called trends. The trend may not be true for every point, but look for the overall pattern the data seem to fit.

As you move from left to right on the graph, if the data points …	as shown in this scatterplot …	they show this type of correlation:
move up		positive correlation.
move down		negative correlation.
show no pattern		no correlation.

The following scatterplot shows the relationship between the average number of hours spent studying per week and the grade for the last semester for 20 students.

What trend do these data show?



Look for a pattern.

The average number of hours spent studying per week increases as you move from left to right on the x-axis.

Do the corresponding y-coordinates for the points on the graph increase or decrease?

In general, they appear to increase. This is not true for every point, but the data seem to fit into the overall pattern.

When one value increases as the other increases, the pattern is a positive trend.

These data show a positive trend between the average number of hours spent studying per week and the grade earned.

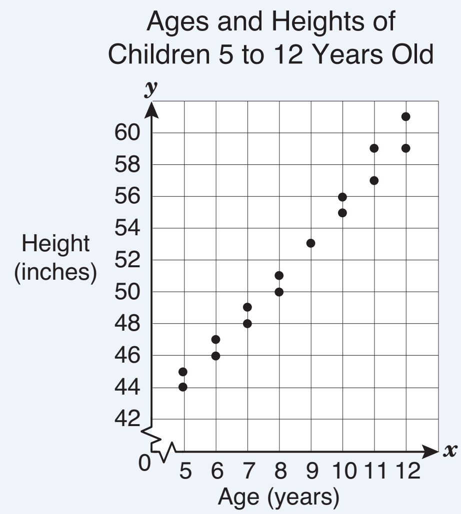
Objective 5

Suppose you had a set of data that listed the heights and ages of a group of people. Would a scatterplot of those data show a positive trend between height and age?

Not necessarily. The results would depend on the sample of people chosen.

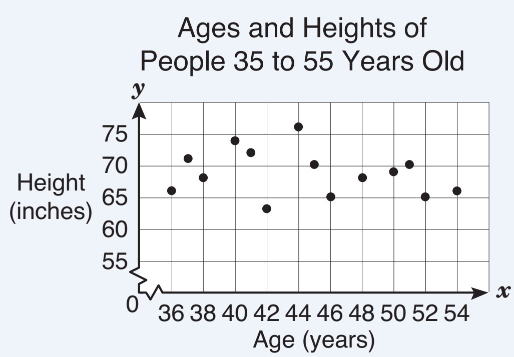
- The scatterplot would show a positive trend if people's heights increased with their ages. The older they were, the taller they would be.

  This relationship would be likely if the people in the sample data were young and still growing.



- The scatterplot would show no trend if people's heights neither increased nor decreased in any pattern with age. If one person from the group were older than another, there would be no way to predict who was taller or shorter.

  This relationship would be likely if the people in the sample data were all full-grown adults.

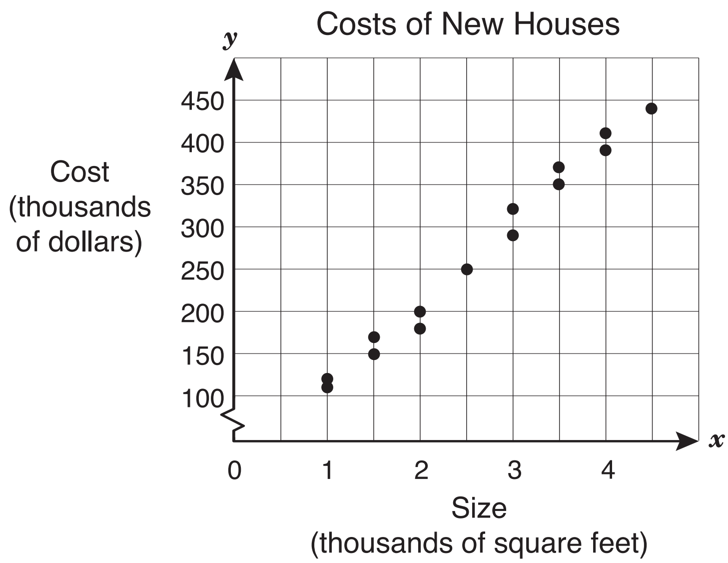


- The scatterplot would show a negative trend if people's heights decreased while their ages increased.

  This type of relationship is unlikely in this case, but it is quite possible in a context other than one involving age and height.

Try It

The scatterplot below shows the costs of new houses compared to their size measured in square feet.



Based on the information in the scatterplot, what conclusion can you draw about the relationship between the cost of a house and its size in square feet?

As you move from left to right on the *x*-axis, the *y*-coordinates of the points generally ____________________________.

This means that there is a ____________________________ trend in the data.

The graph shows that the cost of a new house ____________________ as its size increases.

As you move from left to right on the *x*-axis, the *y*-coordinates of the points generally increase. This means that there is a positive trend in the data. The graph shows that the cost of a new house increases as its size increases.

How Do You Use Graphs to Represent Data?

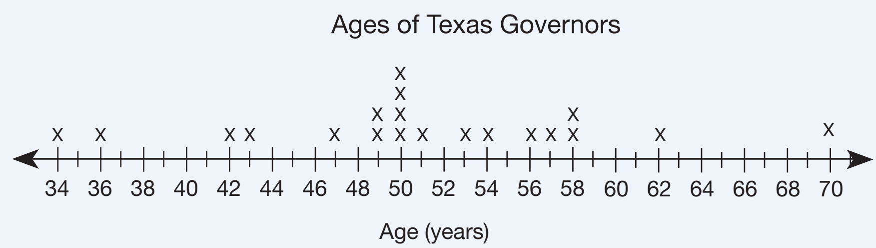
There are many ways to represent data graphically. Line plots, histograms, circle graphs, box and whisker plots, and Venn diagrams are five types of graphs used to display data. Graphical representations of data often make it easier to see relationships in the data. However, in order to draw valid conclusions, you must read and interpret the data from the graph accurately.

A **line plot** represents a set of data by showing how often a piece of data appears in that set. It consists of a number line that includes the values of the data set. An X is placed above the corresponding value each time that value appears in the data set.

A student made a list of how old the last 20 governors of Texas were when they took office.

50, 49, 58, 70, 43, 62, 50, 57, 51, 47,
42, 54, 53, 49, 36, 58, 56, 34, 50, 50

The student then made a line plot from these data. Notice that there are four people who were 50 years old when they became governor and that there are four Xs above the number 50 on the number line. Similarly, only one person was 70 years old at the time of becoming governor, and there is only one X above the number 70 on the number line.



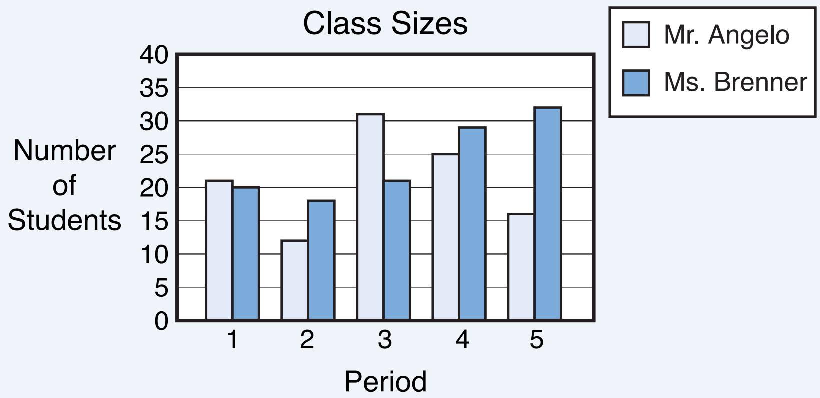
You can use this plot to make the following observations.

- The ages vary from 34 to 70.
- The most common age is 50.
- The ages cluster from 49 to 58.

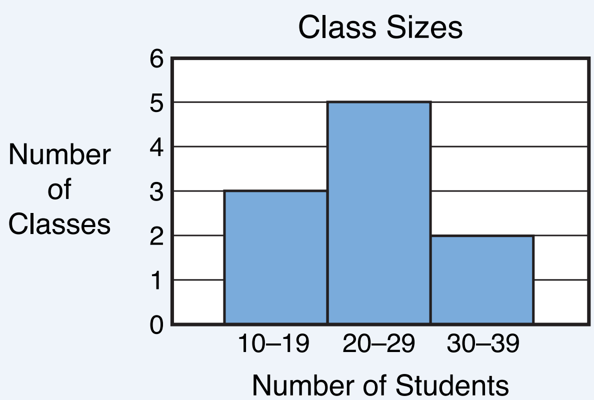
A **histogram** is a special kind of bar graph. A histogram shows the number of data points that fall within specific intervals of values.

Mr. Angelo and Ms. Brenner both teach math classes at Longmont Middle School. The graphs below show the number of students in their classes, but the two graphs do so in different ways.

- The first graph is a double bar graph. It compares the number of students in each teacher's classes, period by period.



- The second graph shows their class sizes using a histogram. It shows how many classes in different size ranges they both teach.

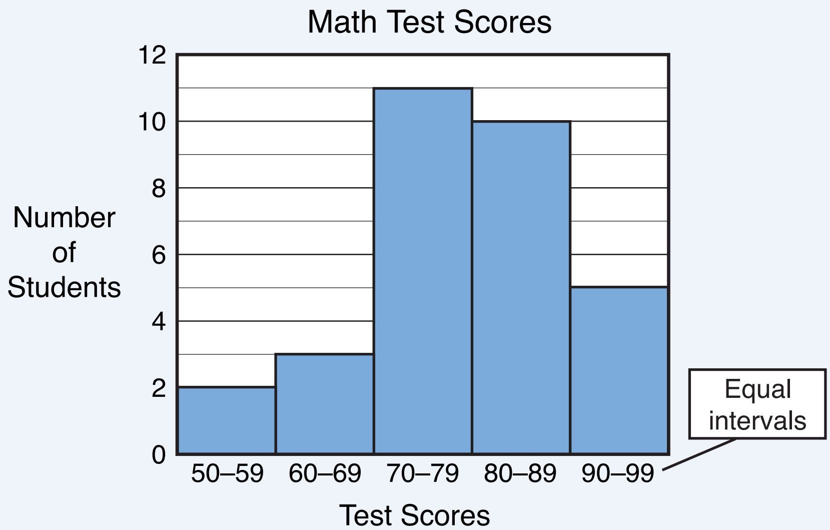


Objective 5

Do you see that . . .

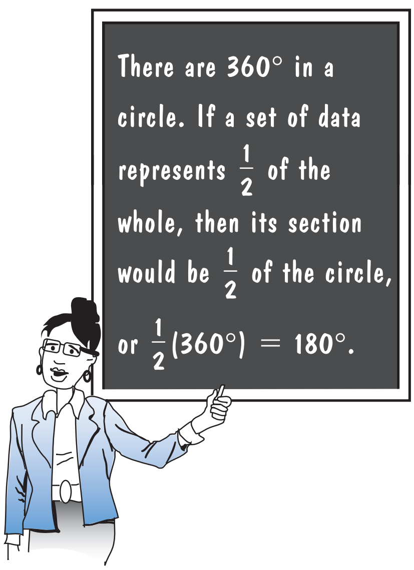
In a histogram the data's range should be divided into equal intervals. If the intervals are not equal, the graph could be misleading and result in invalid conclusions.

This histogram shows how students scored on a math test last week.



What conclusions can you draw about student scores from the graph?

- The greatest number of students scored between 70 and 79 on the test.
- The smallest number of students scored between 50 and 59 on the test.
- Fewer students scored 90–99 than 80–89 on the test.



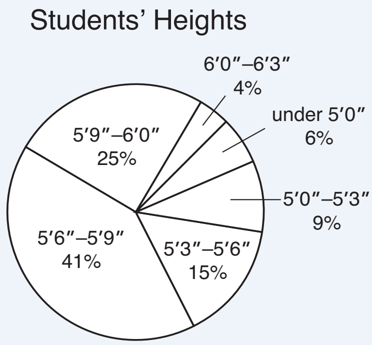
A **circle graph** compares the numbers in a set of data by showing the relative sizes of the parts that make up a whole. The circle represents the whole, which is made up of all the data elements. Each section of the circle represents part of the whole.

The circle graph below compares the height of 100 students in a school. If 25 out of 100 students are between 5′9″ and 6′ tall, then $\frac{25}{100}$ = 25% are between 5′9″ and 6′ tall.
The same fraction of the circle was used to represent that height.

$$0.25(360°) = \frac{1}{4}(360°) = 90°$$

Students' Heights

Height	Number of Students
Under 5′0″	6
5′0″–under 5′3″	9
5′3″–under 5′6″	15
5′6″–under 5′9″	41
5′9″–under 6′0″	25
6′0″–under 6′3″	4
Total	100



The greatest number of students are between 5′6″ and 5′9″ tall, and the greatest part of the circle is used to represent that height.

This circle graph compares the number of different types of TV shows that were televised one week.

What conclusions can you draw about the different types of TV shows televised that week?

- The circle graph shows that the most common type of TV show televised was comedy. The section representing comedy has the greatest area.
- The circle graph shows that news and documentary programs make up approximately $\frac{1}{4}$ of the total TV shows. The sections representing news and documentaries together form an angle that is approximately $\frac{1}{4}$ of 360°, or 90°.

Try It

On a walk Matthew counted the number of different kinds of trees that he saw. Matthew saw 40 oak trees, 5 dogwood trees, 20 maple trees, 10 cedar trees, and 5 pine trees. If he wants to represent these data in a circle graph, what size angle should he use to form the section labeled "cedar trees"?

First find the fraction of the total number of trees that were cedar trees.

Matthew saw _______ cedar trees.

Matthew saw a total of _______ trees.

The fraction of the trees that were cedar trees is $\frac{\square}{\square}$, or $\frac{\square}{\square}$.

Then find the size of the angle that should be used to form the section for cedar trees.

The sum of all the angles in a circle graph is _______ °.
The part of the circle that should be used to represent cedar trees

should be the _______ as the fraction of cedar trees that Matthew saw. The angle for cedar trees should be

$$\frac{\square}{\square} \cdot \_\_\_\_\_\_\_^\circ = \_\_\_\_\_\_\_^\circ.$$

A _______ ° angle should be used to form the section labeled "cedar trees."

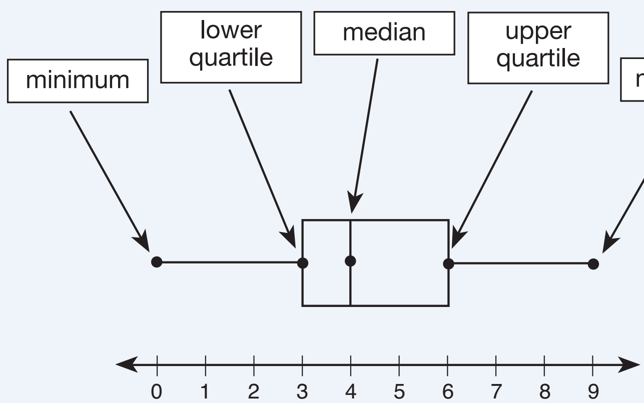
Matthew saw 10 cedar trees. Matthew saw a total of 80 trees. The fraction of the trees that were cedar trees is $\frac{10}{80}$, or $\frac{1}{8}$. The sum of all the angles in a circle graph is 360°. The part of the circle that should be used to represent cedar trees should be the same as the fraction of cedar trees that Matthew saw. The angle for cedar trees should be $\frac{1}{8} \cdot 360^\circ = 45^\circ$.
A 45° angle should be used to form the section labeled "cedar trees."

A **box and whisker plot** can be used to describe the distribution of a data set (that is, how near or far the data points are from each other). It also yields specific information about certain values related to the data set.

The table below represents the number of goals scored by students at soccer practice each day over the course of one month.

Number of Goals	Frequency
0	1
1	1
2	2
3	3
4	6
5	4
6	3
7	2
8	2
9	1

The box and whisker plot below represents the data set shown in the table above.



The **lower quartile** can be thought of as the "middle value of the first half" of the data set. Likewise, the **upper quartile** is the "middle value of the second half" of the data set. In this sense, they are very much like medians.

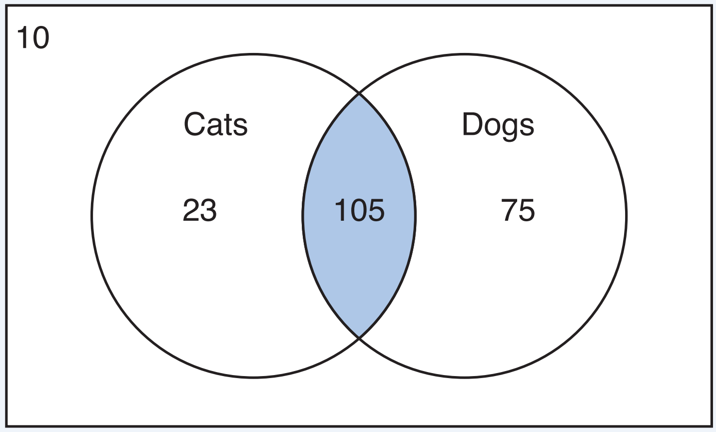
What data can be interpreted from this graph?

The fewest number of goals in one day was 0, and the most was 9. The median number of goals was 4. The lower quartile was 3, and the upper quartile was 6.

Objective 5

A **Venn diagram** is used to show how many pieces of data have a certain property in common. The data in each section of a Venn diagram show how many values fall within each category.

This Venn diagram shows the number of students in Jim's class who have cats, dogs, both, or neither as pets.



- The diagram includes a rectangle and two overlapping circles. The rectangle represents the universal set that is made up of all the students in Jim's class.
- The 10 shown in the rectangle but not in the circles represents the 10 students who do not have a cat or a dog.
- The part where the two circles overlap shows the number of students who have both cats and dogs as pets.
- The two circles show that 23 students have only cats and 75 students have only dogs.

You could use the Venn diagram to find that a total of $23 + 105 = 128$ students in all have cats as pets.

How Do You Know Whether a Sample Is Representative of a Larger Group?

There are times when you want to know something about a large group of people but it is impractical to ask every member of the group. Instead, you ask a smaller group, a sample, and apply your conclusions to the entire population. This process is known as **sampling**. Sampling is frequently used in science experiments, social studies, and surveys.

From a survey you can draw conclusions about a total population based on information you have about a few of its members. Those conclusions are valid only if the sample is representative of the total population.

A sample will be representative of a total population if these guidelines are followed.

- The group of people who are surveyed should be selected at random. This is called taking a **random sample**. The sample should not be selected in a way that might bias the results.

Suppose you want to know the favorite restaurant of the residents of a certain town. To find out, you stand outside a particular restaurant in that town and ask the people entering, *What is your favorite restaurant?* Would this method of selecting people generate a random sample?

No. This method of sampling is biased because it surveys people who probably prefer that restaurant. You need to select the sample more randomly. You could, for example, open the town's phone book and call every hundredth person. Using a list of names drawn randomly from the phone book would ensure that your survey was not biased.

- The group surveyed must be varied enough to be representative of the total population.

Suppose you want to know the favorite teacher of all the students in a school. Would it be valid to ask only students in Spanish classes who their favorite teacher is?

No. This method of sampling is biased because the opinions might not be representative of the opinions of the total student population. Nor would it be valid to ask only freshmen, who would probably name only teachers who teach freshmen. Your sample should model the total population with respect to gender, race, age, grades earned, etc. Your sample needs to have a mix of students proportional to the mix in the total student population.

- The size of the group surveyed must be large enough to be representative of the total population.

If only 5 people in a city of 100,000 are in a survey sample, would the conclusions reached be valid for the entire city's population?

No. The survey group of 5 is not a large enough sample to be representative of all the people in the city. A sample size can never be too large, but it can be too small. There is no simple formula for just how large a sample must be, but common sense will usually tell you whether a sample is too small.

Try It

Wellington Middle School has 965 students. Ms. Philips, the principal, wanted to know her students' preference for a new school mascot. To find out, she went to an eighth-grade English class one day and distributed a survey. Of the 35 students who completed the survey, 30 said they preferred a bulldog for the school mascot. Based on this survey, Ms. Philips concluded that most of the school's students wanted a bulldog for the new mascot. Why might her conclusion not be valid?

The sample of students surveyed was not representative of the school's population because the sample was not a

____________________________ sample. Only eighth-grade students were surveyed.

Only _______ out of 965 students were surveyed. The sample was

not ____________________________ to be representative.

The sample of students surveyed was not representative of the school's population because the sample was not a random sample. Only 35 out of 965 students were surveyed. The sample was not large enough to be representative.

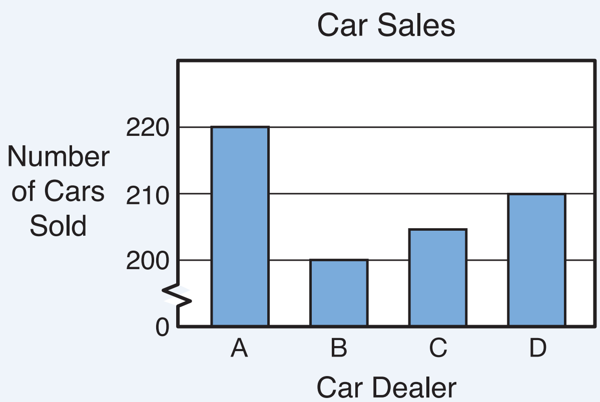
How Do You Know Whether Conclusions Drawn from Graphical or Numerical Information Are Valid?

To know whether conclusions drawn from graphical or numerical information are valid, you must determine whether you have read the graph accurately and analyzed the numerical information correctly.

Here are some guidelines to follow to ensure that the conclusions you draw from graphs and data are valid.

- Be certain that any pattern you find in the data is truly reflected by the data.
- Compare information presented in a table or graph accurately.
- Make sure the units used in a table or graph are consistent with the conclusions drawn.
- Be logical. Do not draw conclusions beyond those represented by the data.

The graph shows the total number of cars sold last month by four different car dealers.



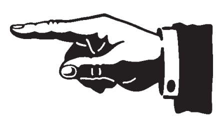
Based on the graph, is it valid to infer that Car Dealer A sold more than twice as many cars last month as Car Dealer B?

- The bar for Dealer A appears to be more than twice as long as the bar for Dealer B. This might lead you to conclude that Dealer A sold more than twice as many cars as Dealer B.
- The vertical axis representing the number of cars sold is broken to show that part of the bars have been omitted.
- Read the values from the graph for the number of cars sold.

  Dealer A sold 220 cars. Dealer B sold 200 cars.
- Compare the numerical values, not the lengths of the bars.

  Dealer A sold 20 more cars than Dealer B.

Dealer A sold more cars than Dealer B, but not twice as many. Dealer B sold 200 cars. Twice as many cars would be 400 cars. Dealer A did not sell 400 cars.

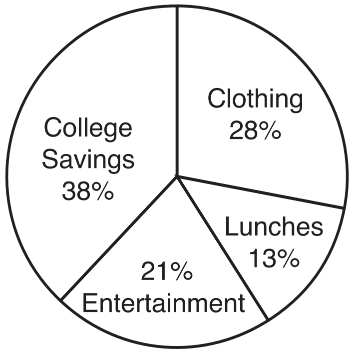


Do you see that . . .

Try It

Jorge has a part-time job after school. The graph shows how he spends his earnings in a typical month.

Jorge's Monthly Spending



If Jorge earned $265 last month, does the graph support the conclusion that he spent a little more than $90 on lunches and entertainment last month?

Find the percent spent on lunches and entertainment combined.

Of Jorge's earnings, _______% is spent on lunches, and _______% on entertainment.

The combined total of his spending on these two items is _______%.

Jorge earned $_______ last month, and _______ % of $_______

is $_______ .

Jorge spent $_______ on lunches and entertainment last month.

The graph does support the conclusion that he spent a little more than $90 on lunches and entertainment last month.

Of Jorge's earnings, 13% is spent on lunches, and 21% on entertainment. The combined total of his spending on these two items is 34%. Jorge earned $265 last month, and 34% of $265 is $90.10. Jorge spent $90.10 on lunches and entertainment last month.

Now practice what you've learned.

Question 44

A spinner for a board game has 3 red sectors, 2 blue sectors, 2 green sectors, and 1 white sector. The sectors are all congruent. What is the probability of spinning red on the first spin and green on the second spin?

A $\frac{9}{64}$

B $\frac{3}{32}$

C $\frac{1}{16}$

D $\frac{6}{56}$

Answer Key: page 165

Question 45

Erica is playing a game using tiles with the following letters written on them: 4 tiles with E, 3 tiles with A, 3 tiles with T, and 5 tiles with R. Erica draws the first tile and records the letter. She does not replace the tile. She draws a second tile and records the letter. What is the probability that Erica draws two vowels?

A $\frac{14}{15}$

B $\frac{7}{15}$

C $\frac{49}{225}$

D $\frac{1}{5}$

Answer Key: page 165

Question 46

Of the 30 eighth-grade students surveyed, 5 said they would vote for Nancy as their representative on the student council. If the results of this survey are used to predict the election outcome and 332 eighth-grade students vote, which answer is closest to the number of votes Nancy should get?

A 199

B 53

C 55

D 553

Answer Key: page 165

Question 47

Mark has participated in 8 track meets so far this season. His running times for the 440-meter race have been 70, 63, 68, 65, 69, 61, 66, and 64 seconds. To qualify for the state finals, Mark must run each of his next two 440-meter races in less than 65 seconds. Based only on his times for the first 8 meets, what is the probability that Mark will run each of his next two races in less than 65 seconds?

A $\frac{3}{8}$

B $\frac{9}{64}$

C $\frac{7}{16}$

D $\frac{3}{4}$

Answer Key: page 165

Question 48

The students in a science lab followed the directions in the lab manual. They combined two chemicals and then measured the mass of the mixture. The students recorded their data and then determined the mean, median, mode, and range. The table below summarizes the measures of data for the experiment.

Chemical Mixture

Measure	Mass (grams)
Mean	2.96
Median	2.97
Mode	3.01
Range	0.17

Which measure of the data could best be used to find the greatest mass the students recorded if the smallest mass recorded was 2.87 grams?

A Mean

B Median

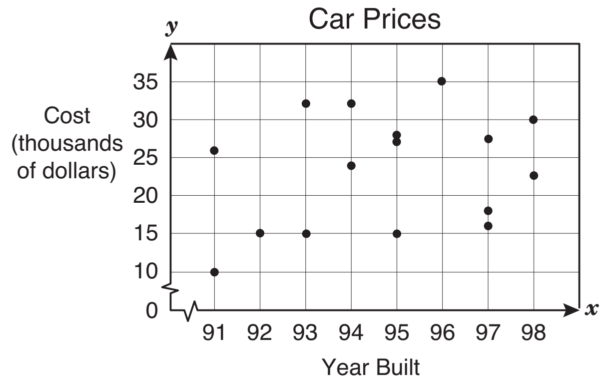
C Mode

D Range

Answer Key: page 166

Question 49

A car dealer put a sticker in the window of each used car in his lot to show the sale price of the vehicle and the year in which it was built. The scatterplot below shows the data.



Which description best represents the relationship in the recorded data?

A No trend

B Positive trend

C Negative trend

D Not here

Answer Key: page 166

Question 50

The school cafeteria served sandwiches for lunch. The table shows the types of sandwiches selected by students.

Sandwich Choices

Sandwich	Number of Students
Grilled cheese	30
Tuna salad	40
Roast beef	75
Turkey	35

If the data are displayed in a circle graph, what size angle should form the section of the circle for tuna salad?

A 90°

B 40°

C 180°

D 80°

Answer Key: page 166

Question 51

Last week Janis surveyed customers leaving Food Superstore. Of the 500 people surveyed, 447 said that Food Superstore was their favorite grocery store. From these survey results Janis concluded that Food Superstore was the favorite grocery store among all the people in her town. Which is the best explanation for why her conclusion might not be valid?

A The sample may not have been representative of all the people in Janis's town.

B She asked every customer coming out of the store rather than asking every fifth customer who left the store.

C The survey she used did not ask how old the customers were.

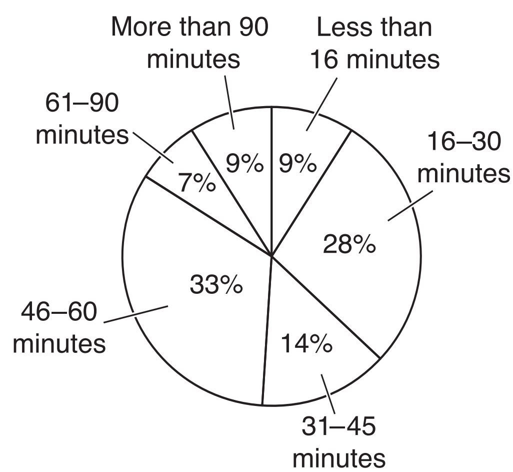
D The sample size was too small.

Answer Key: page 166

Question 52

The circle graph below categorizes customers at Max's Market according to the amount of time they spend shopping there in a single visit.

Time Spent Shopping



Which of the following statements is best supported by the data represented in the graph?

A More than half the customers spend at least 46 minutes shopping.

B Exactly one-fourth of the customers spend from 16 to 30 minutes shopping.

C Out of every 200 customers, 28 will spend from 31 to 45 minutes shopping.

D There are more customers who spend at least an hour shopping than there are customers who spend less than an hour shopping.

Answer Key: page 166

Objective 5

Question 53

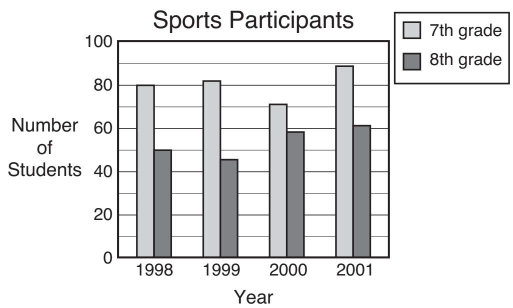
This table shows the number of 7th and 8th graders who played sports during the past 4 years.

Sports Participants

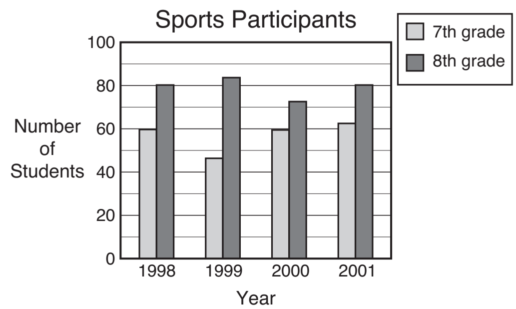
Year	7th Grade	8th Grade
1998	50	80
1999	46	83
2000	59	72
2001	62	90

Which bar graph best represents the data in the table?

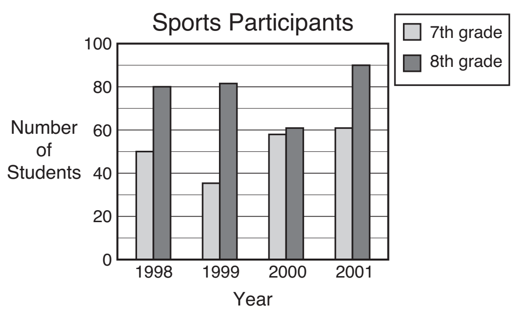
A



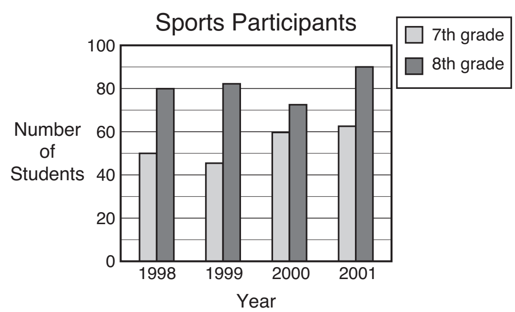
C



B



D



Answer Key: page 166

Question 54

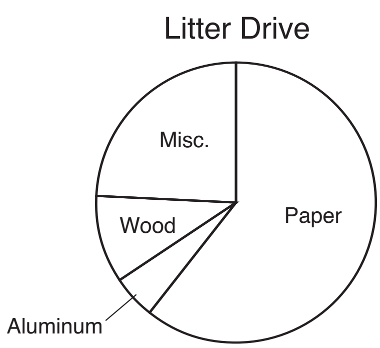
The eighth-grade class at Barryville Middle School conducted a litter drive as a community-service project. The table below shows the number of pounds of different types of litter they collected.

Litter Drive

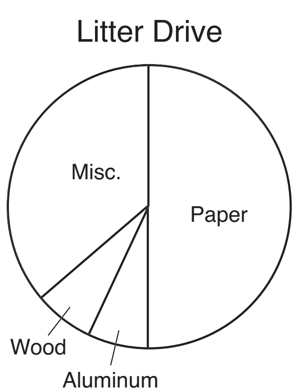
Type of Litter	Number of Pounds
Paper	305
Aluminum	25
Wood	51
Miscellaneous	126

Which graph best represents these data?

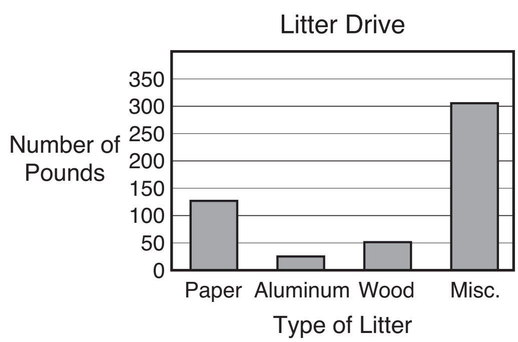
A



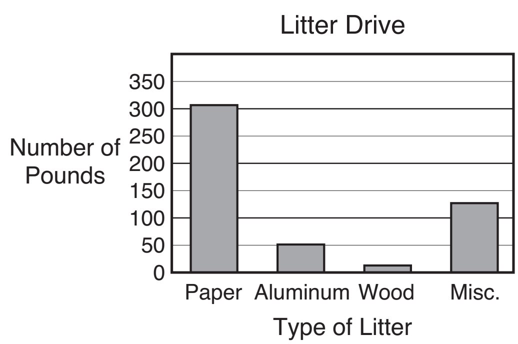
B



C



D



Answer Key: page 166

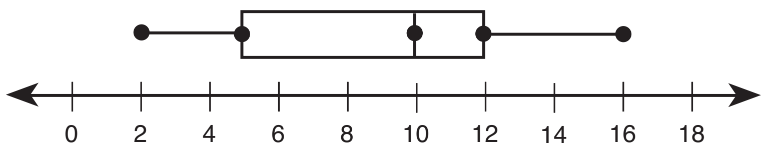
Question 55

A set of data is shown below.

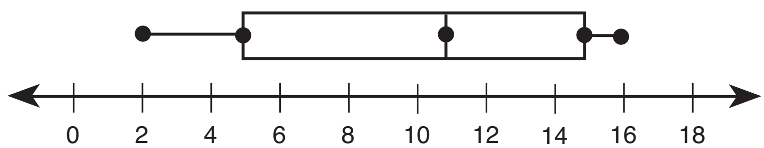
10, 16, 13, 2, 9, 11, 12, 13, 14, 6, 14

Which of the following box and whisker plots best represents these data?

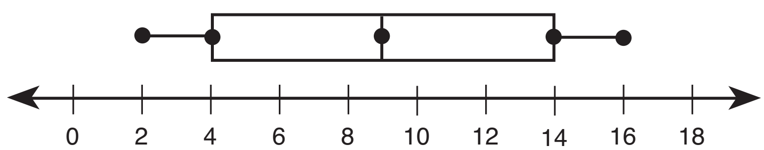
A



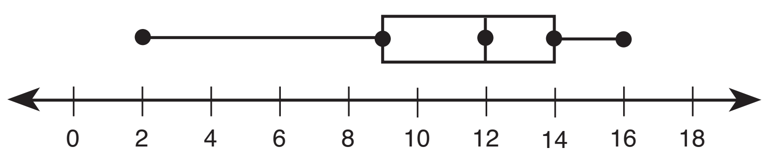
B



C



D



Answer Key: page 166

Objective 6

The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

For this objective you should be able to

- apply mathematics to everyday experiences and activities;
- communicate about mathematics; and
- use logical reasoning.

How Can You Use Mathematics to Solve Everyday Problems?

Many situations in everyday life involve mathematics. For example, you might want to compute the likelihood of your favorite team winning its next game based on its win-loss record, or you might need to estimate the area of a room based on its dimensions. Solving everyday problems often requires the use of mathematics.

Solving problems involves more than just numerical computation; logical reasoning and careful planning also play important roles. Here are some steps to follow when solving problems:

- Understand the problem. Organize the information you are given and identify exactly what you must find. You may need information that is not given in the problem, such as a formula. You may be given information that is not needed in order to solve the problem.
- Make a plan. After you have organized the information, decide how to use this information to find an answer. Think about the math concepts that apply to the situation. Identify the order in which you will find new information and the formulas or equations you will use to find it.
- Carry out the plan. After you have chosen a problem-solving strategy, use the strategy to work toward a solution to the problem. Go step-by-step through your plan, writing down important information at each step.
- Check to see whether your answer is reasonable. Check to see whether your answer makes sense. Does it answer the question asked? Is it stated in the correct units? Is it reasonable? You can estimate the solution and then compare the estimate to your calculated answer. They should be approximately equal.

Objective 6

Andy bought a $50 portable radio on sale for 15% off the list price. When he went to pay for his purchase, the clerk reminded him that he must also pay sales tax. If he paid for the radio with a $50 bill, how much change should he have received?

What additional information is needed to find the amount of change Andy should have received?

Understand the problem.

- What information do you already have?

  The list price of the radio was $50.

  The list price was reduced by 15%.

  There was sales tax on the purchase.

  Andy paid for the radio with a $50 bill.

- What do you want to know?

  How much change should Andy have received from the $50 bill?

Make a plan.

- Find the amount by which the sale reduced the list price.
- Subtract the reduction from the list price to find the sale price.
- Find the sales tax on the sale price.
- Add the sales tax to the sale price to find the total purchase price.
- Subtract the total purchase price from $50 to find the amount of change Andy should have received.

Carry out the plan.

- You can complete the first two steps of this problem.

  Write a proportion to find 15% of $50.

$$\frac{15}{100} = \frac{x}{50}$$

$$100x = 750$$

$$x = 7.5$$

  The list price was reduced by $7.50.

$$\$50.00 - \$7.50 = \$42.50$$

  The sale price is $42.50.

- You cannot find the sales tax, because you do not know the sales tax rate.

The missing piece of information is the sales tax rate.

Try It

A bag contains a number of colored marbles. Jay draws a marble from the bag, records its color, and returns the marble to the bag. He repeats this process 100 times. The table below shows the experimental results of the 100 trials.

Experimental Results

Color	Frequency
Red	35
Blue	20
White	45

Based on the experimental results, how many more red marbles than blue marbles can Jay expect to draw in 250 trials?

The experimental probability of drawing a red marble is $\frac{\square}{\square}$.

Write a proportion that can be used to predict r, the number of times Jay can expect to draw a red marble in 250 trials.

$$\frac{35}{100} = \frac{r}{\square}$$

$$100r = 35 \cdot \underline{\qquad}$$

$$100r = \underline{\qquad}$$

$$r = \underline{\qquad}$$

Jay can expect to draw a red marble about ________ times in 250 trials.

The experimental probability of drawing a blue marble is $\frac{\square}{\square}$.

Write a proportion that can be used to predict b, the number of times Jay can expect to draw a blue marble in 250 trials.

$$\frac{20}{100} = \frac{b}{\square}$$

$$100b = 20 \cdot \underline{\qquad}$$

$$100b = \underline{\qquad}$$

$$b = \underline{\qquad}$$

Jay can expect to draw a blue marble about ________ times in 250 trials.

Objective 6

To find the difference between the number of red marbles and the number of blue marbles Jay can expect to draw, subtract.

________ – ________ = ________

In 250 trials Jay can expect to draw about ________ more red marbles than blue marbles.

The experimental probability of drawing a red marble is $\frac{35}{100}$.

$$\frac{35}{100} = \frac{r}{250}$$

$$100r = 35 \cdot 250$$

$$100r = 8{,}750$$

$$r = 87.5$$

Jay can expect to draw a red marble about 88 times in 250 trials.

The experimental probability of drawing a blue marble is $\frac{20}{100}$.

$$\frac{20}{100} = \frac{b}{250}$$

$$100b = 20 \cdot 250$$

$$100b = 5{,}000$$

$$b = 50$$

Jay can expect to draw a blue marble about 50 times in 250 trials.

$$88 - 50 = 38$$

In 250 trials Jay can expect to draw about 38 more red marbles than blue marbles.

What Is a Problem-Solving Strategy?

A **problem-solving strategy** is a plan for solving a problem. Different strategies work better for different types of problems. Sometimes you can use more than one strategy to solve a problem. As you practice solving problems, you will discover which strategies you prefer and which work best in various situations.

Some problem-solving strategies include

- drawing a picture;
- looking for a pattern;
- guessing and checking;
- acting it out;
- making a table;
- working a simpler problem; and
- working backward.

One way to solve a problem is to draw a picture. Drawing a picture can help you organize the information you need to solve the problem.

Each tire on Phil's bike has a diameter of 27 inches. The post office is 2.5 miles from Phil's house. If Phil rides his bike directly from his house to the post office, how many complete rotations will each of his bike tires make?

- What information do you already have?

  The diameter of each tire is 27 inches.

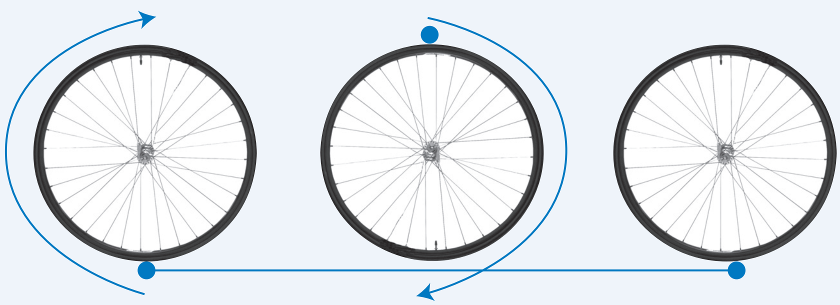
  The post office is 2.5 miles from Phil's house.

- What do you want to know?

  The distance each tire travels in each rotation

  The number of times each tire rotates in 2.5 miles

- To figure out how far each tire travels in one rotation, you could draw a picture.



  In one rotation the tire will have traveled the same distance on the ground as the circumference of the circle.

- To find the number of times the tire rotates, divide the number of feet in 2.5 miles by the number of feet covered in each rotation of the tire.

Objective 6

- Find the circumference of the tire.

$$C = \pi d$$

Use 3.14 as an approximate value of π. The diameter of the tire is 27 inches.

$$C \approx 3.14 \cdot 27 \approx 84.78 \text{ in.}$$

The circumference of the tire is about 84.78 inches. In one rotation the tire will travel about 84.78 inches on the ground.

- Divide by 12 to convert the circumference from inches to feet.

$$84.78 \div 12 = 7.065 \text{ ft}$$

The tire travels about 7.065 feet per rotation.

- Find the number of feet in 2.5 miles. There are 5,280 feet in 1 mile. Multiply by 5,280 to convert from miles to feet.

$$2.5 \cdot 5{,}280 = 13{,}200 \text{ ft}$$

There are 13,200 feet in 2.5 miles.

- To find the number of times the tire will make a complete rotation in 2.5 miles, divide 13,200 feet by 7.065 feet per rotation.

$$13{,}200 \div 7.065 \approx 1{,}868 \text{ rotations}$$

Each of Phil's bike tires makes approximately 1,868 complete rotations as he rides the 2.5 miles from his house to the post office.

Try It

A right triangle with leg lengths of 9 and 12 units is dilated by a scale factor of 2.5 to produce a new right triangle. What is the perimeter of the new triangle? You may want to draw a picture to help you solve this problem.

The perimeter should change by the ____________________.
First find the hypotenuse of the original triangle. The numbers 9 and 12 form the first two terms of a Pythagorean triple. Multiply the Pythagorean triple 3, 4, and 5 by ______ to get the new triple ______, ______, and ______.

The hypotenuse of the original triangle is ______.

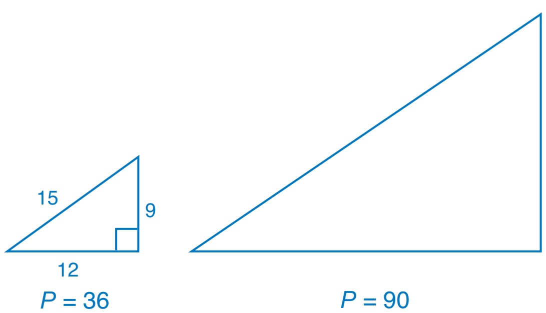
Find the perimeter of the original triangle.

$$P = \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_ = \_\_\_\_\_\_ \text{ units}$$

Find the perimeter of the dilated triangle. Multiply the original perimeter by the ____________________.

$$\_\_\_\_\_\_ \cdot 2.5 = \_\_\_\_\_\_$$

The perimeter of the dilated triangle is ______ units.



The perimeter should change by the scale factor. Multiply the Pythagorean triple 3, 4, and 5 by 3 to get the new triple 9, 12, and 15. The hypotenuse of the original triangle is 15.

$$P = 9 + 12 + 15 = 36 \text{ units}$$

Multiply the original perimeter by the scale factor.

$$36 \cdot 2.5 = 90$$

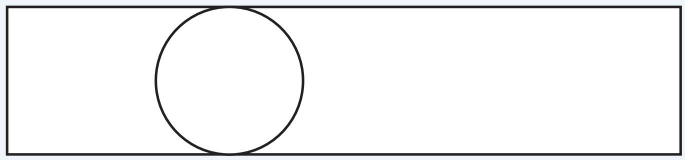
The perimeter of the dilated triangle is 90 units.

Objective 6

How Do You Change Words into Math Language and Symbols?

It is important to be able to rewrite a problem using mathematical language and symbols. The words used in the problem will give you clues about what operations to use. In some problems it may be necessary to use algebraic symbols to represent quantities and then use equations to express the relationships between the quantities. In other problems you may need to represent the given information using a table or a graph.

In the figure below, the length of the rectangle is 5 centimeters more than twice its width. The radius of the circle is 1 centimeter.



Write a ratio that compares the area of the rectangle to the area of the circle.

To find the area of the rectangle, you must know its length and width.

- Find the width of the rectangle.

  The width of the rectangle is equal to the diameter of the circle. If the radius of the circle is 1 centimeter, its diameter is 2 centimeters. Therefore, the width of the rectangle is 2 centimeters.

- Find the length of the rectangle.

  The length is 5 more than twice the width.

  $l = 2w + 5$ Substitute 2 for w in the expression.

  $l = 2(2) + 5$

  $l = 9$ cm

  The length of the rectangle is 9 centimeters.

- Find the area of the rectangle.

  The formula for the area of a rectangle is $A = lw$. Substitute 9 for l and 2 for w.

  $A = lw$

  $A = 9 \cdot 2 = 18 \text{ cm}^2$

  The area of the rectangle is 18 square centimeters.

- Find the area of the circle.

  The formula for the area of a circle is $A = \pi r^2$. Use 3.14 as an approximate value of π and substitute 1 for r.

$$A = \pi r^2$$
$$A \approx 3.14 \cdot 1^2$$
$$A \approx 3.14 \cdot 1$$
$$A \approx 3.14\,\text{cm}^2$$

  The area of the circle is approximately 3.14 square centimeters.

- Write the ratio of the area of the rectangle to the area of the circle.

$$\frac{\text{rectangle}}{\text{circle}} \qquad \frac{18}{3.14}$$

The ratio of the area of the rectangle to the area of the circle is $\frac{18}{3.14}$.

Try It

Mr. and Mrs. Braun go to a restaurant. Mr. Braun's meal costs \$15.40, and his wife's meal costs \$12.35. Before giving them their check, the server adds 7% tax to it. Describe the steps you would take to find the amount of change the Brauns should receive if they pay the check with two \$20 bills.

To solve this problem, you must first _______________ to find the total cost of the Brauns' meals before tax.

Then _______________ the total cost of the meals by __________ to find the tax on the check.

Next _______________ the tax to the cost of the meals to find the total amount of the check.

Finally, _______________ the total amount of the check from

\$________ to find their change.

To solve this problem, you must first add to find the total cost of the Brauns' meals before tax. Then multiply the total cost of the meals by 0.07 to find the tax on the check. Next add the tax to the cost of the meals to find the total amount of the check. Finally, subtract the total amount of the check from \$40 to find their change.

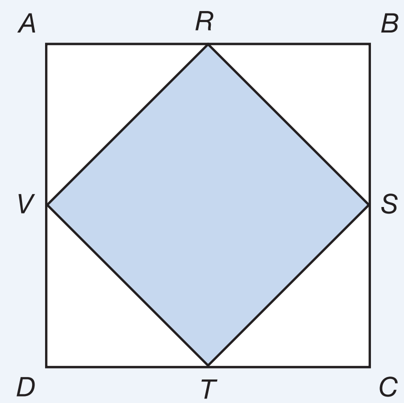
Objective 6

How Can You Use Logical Reasoning as a Problem-Solving Tool?

Logical reasoning is thinking of something in a way that makes sense. Thinking about mathematics problems involves logical reasoning. You can use logical reasoning to find patterns in a set of data. You can then use those patterns to draw conclusions from the data that can be used to solve problems.

Finding patterns involves identifying characteristics that objects or numbers have in common. You can look for a pattern in different ways. A sequence of geometric objects may have some property in common. For example, they may all be quadrilaterals, or they may all have right angles.

The midpoints of the sides of square *ABCD* are connected to form square *RSTV*.



The table below shows how the area of square *RSTV* will vary as the length of $\overline{AB}$ varies.

Length of $\overline{AB}$	Area of Square *RSTV*
1	0.5
2	2.0
4	8.0
6	18.0

If the pattern continued, what would be the area of square *RSTV* if $\overline{AB}$ measured 10 units?

To find the pattern, use the table to compare the lengths of $\overline{AB}$ with the areas of square *RSTV*. There is not an obvious pattern.

Since the problem asks you to find the area of square *RSTV*, use the length of $\overline{AB}$ to calculate the area of square *ABCD*. Then compare the area of square *ABCD* to the area of square *RSTV*.

Length of $\overline{AB}$	Area of Square *ABCD*	Area of Square *RSTV*
1	1	0.5
2	4	2.0
4	16	8.0
6	36	18.0

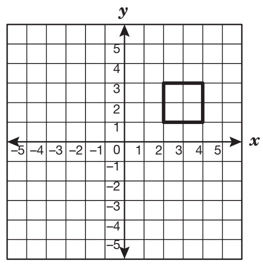
With the new data in the table, it is easier to see the pattern. The area of square *RSTV* is $\frac{1}{2}$ the area of square *ABCD*.

When $\overline{AB}$ is 10 units long, the area of square *ABCD* is 100 square units. Therefore, the area of square *RSTV* would be $\frac{1}{2} \cdot 100 = 50$ square units.

Objective 6

Try It

The graphs below have a repeating pattern.

1


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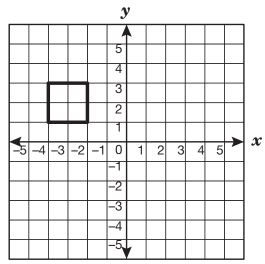

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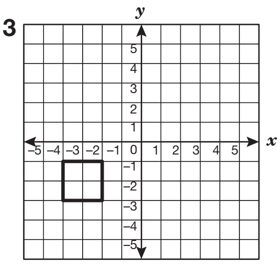

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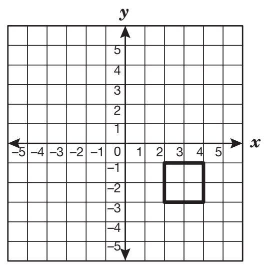

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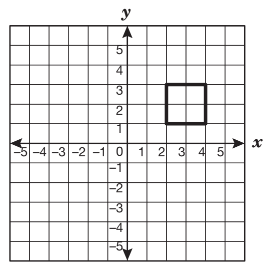


Draw the eighth graph in this pattern.

In the second graph, the original square has been reflected across the ________-axis.

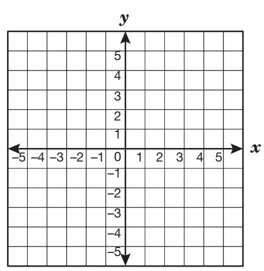
In the third graph, the previous square has been reflected across the ________-axis.

In the fourth graph, the previous square has been reflected across the ________-axis.

In the fifth graph, the previous square has been reflected across the ________-axis.

The pattern is a series of ________________________.

Continue the pattern for three more graphs to find the ________ graph.

6


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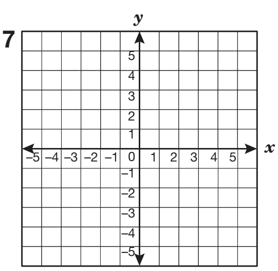

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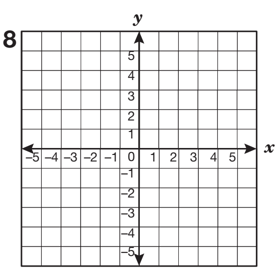
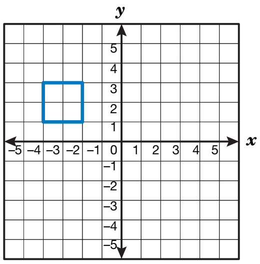


In the second graph, the original square has been reflected across the *y*-axis. In the third graph, the previous square has been reflected across the *x*-axis. In the fourth graph, the previous square has been reflected across the *y*-axis. In the fifth graph, the previous square has been reflected across the *x*-axis. The pattern is a series of reflections. Continue the pattern for three more graphs to find the eighth graph.

6



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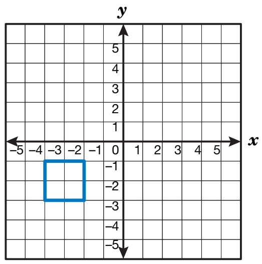

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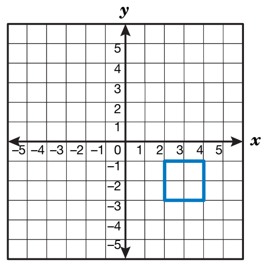


The solution to a problem can be justified by identifying the mathematical properties or relationships that produced the answer. You should have a reason for drawing conclusions, and you should be able to explain that reason.

In math class Karl is working with the equation of the line $x + 2y = 6$. What is wrong with Karl's argument below, in which he attempts to justify that the point $(4, -2)$ is a point on the line $x + 2y = 6$?

Karl's Work

Step #1: To show that the point $(4, -2)$ is a point on the line, substitute its coordinates in the equation and see whether they make the equation true.

Step #2: To check the point $(4, -2)$, substitute -2 for x and 4 for y.

Step #3:
$$x + 2y = 6$$
$$-2 + 2(4) \stackrel{?}{=} 6$$
$$-2 + 8 \stackrel{?}{=} 6$$
$$6 = 6$$

Step #4: Since the equation is true, the point $(4, -2)$ is a point on the line $x + 2y = 6$.

Objective 6

Why is this conclusion not valid?

Is Karl's arithmetic with integers correct?

Yes, the calculation $-2 + 2(4) = -2 + 8 = 6$ is correct.

Is Karl's reasoning correct?

Yes, a point is on a graph if its coordinates satisfy the graph's equation.

Did Karl substitute the coordinates of the point correctly?

No, he switched the x-coordinate and y-coordinate for the point $(4, -2)$. The x-coordinate is the first number in the ordered pair $(4, -2)$. The x-coordinate is 4. The y-coordinate is the second number in the ordered pair $(4, -2)$. The y-coordinate is -2.

Karl substituted the wrong values for the x-coordinate and y-coordinate.

Now practice what you've learned.

Question 56

Which of these situations requires calculating the area of a figure?

A Consuelo wants to mail a package shaped like a rectangular prism. The post office requires that the sum of the three dimensions of a package be smaller than 42 inches. She needs to know whether her package can be mailed.

B Mrs. Snow wants to cover a section of her living room floor with tiles. She has two choices for the shape of the section, a circle with a 6-foot radius or a square with a side of 10 feet. She needs to know which shape will require the greater number of tiles.

C Mr. Carr is drawing the boundary lines of a soccer field with a machine that requires 1 ounce of chalk powder for every 175 feet of line it draws. He needs to know how much chalk powder the machine will use.

D Dave wants to store a quantity of sugar. He has two storage containers from which to choose, one shaped like a cylinder and the other shaped like a rectangular prism. He needs to find which of the containers will hold more sugar.

Answer Key: page 166

Question 57

Tom is twice Celia's age. Jack is two-thirds of Mary's age. Celia is 15 years old. Mary is 6 years younger than Tom. How much older is Jack than Celia?

A 1 year

B 11 years

C 9 years

D 14 years

Answer Key: page 167

Question 58

Phil wants to buy a pair of shoes originally priced at $34.80. The shoes are now on sale for 25% off the original price. There is a 6% sales tax on the sale price. If Phil has only $25, about how much more money does he need in order to buy the shoes?

A $2.17

B $59.19

C $33.50

D $2.67

Answer Key: page 167

Question 59

Mr. Wilson plans to cover his 15-by-6-foot rectangular patio with square tiles that measure 9 inches on each side. If the tiles are sold in boxes of 12, how many boxes of tiles will he need to buy to cover the patio?

A 2

B 13

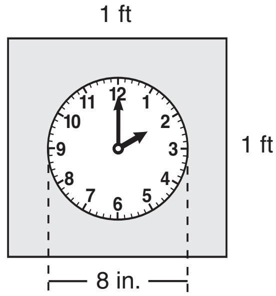
C 14

D 160

Answer Key: page 167

Question 60

A circular clock with a diameter of 8 inches is mounted in a 1-foot-square frame.



Which expression can be used to find the area of the shaded region in square inches?

A $1^2 - 8^2\pi$

B $12^2 - 4^2\pi$

C $8^2\pi - 12^2$

D $1^2 - 4^2\pi$

Answer Key: page 167

Question 61

A ladder is leaning against the side of a building. The building forms one side of a right triangle. The ground beneath the ladder forms another side of the triangle. The ladder forms the third side. Which statement about the triangle must be true?

A The longest leg of the triangle is the leg formed by the ground beneath the ladder.

B The height of the triangle is equal to the combined squares of the lengths of the legs.

C The sum of the lengths of the legs is equal to the length of the hypotenuse.

D The sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Answer Key: page 167

Question 62

A company that makes jewelry boxes wants to compare the cost of manufacturing two sizes of boxes. One is a cubical box that will be covered with material that costs \$0.12 per square foot. The other is a cubical box with dimensions that are twice as large as the smaller box. It will be covered with material that costs \$0.04 per square foot. Which of the following statements is true?

A It will cost the same amount to cover both boxes.

B The smaller jewelry box will cost more to cover than the larger box.

C The larger jewelry box will cost more to cover than the smaller box.

D It is not possible to tell which box costs more to cover.

Answer Key: page 167

Question 63

Jossi wrote down two different sequences of numbers. The first five terms of each sequence are shown below.

$$4,\ 8,\ 12,\ 16,\ 20,\ \ldots$$
$$1,\ \frac{3}{2},\ 2,\ \frac{5}{2},\ 3,\ \ldots$$

What is the sum of the ninth terms of each sequence?

A 41

B 32

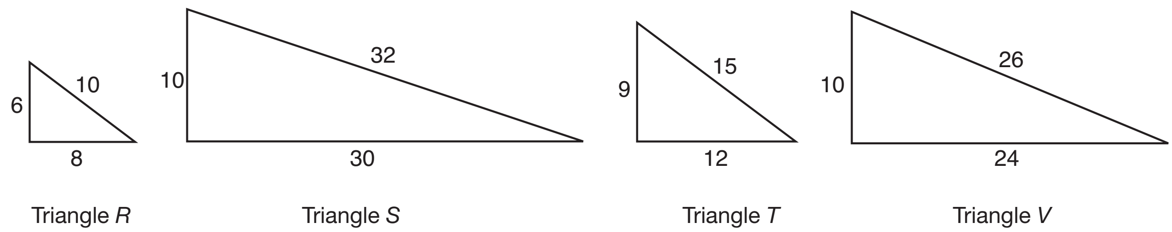
C 50

D 31

Answer Key: page 168

Question 64

Which triangle is NOT a right triangle?



A Triangle *R*

B Triangle *S*

C Triangle *T*

D Triangle *V*

Answer Key: page 168

Question 65

At Pablo's Pizza Pie, medium cheese pizzas cost $12.50, large cheese pizzas cost $15, and a 3-liter bottle of soda costs $2.50. Kevin purchased one bottle of soda and 5 pizzas for his party. If he spent exactly $70, what combination of medium and large pizzas did he purchase?

A 4 medium and 1 large

B 2 medium and 3 large

C 5 medium and 0 large

D 3 medium and 2 large

Answer Key: page 168

Mathematics Answer Key

Objective 1

Question 1 (page 25)

B Correct. To determine whether $\frac{22}{25}$ is greater than or equal to 82%, write both numbers in the same form. To convert $\frac{22}{25}$ to a decimal, divide 22 by 25.

$$22 \div 25 = 0.88$$

To convert 82% to a decimal, move the decimal point two places to the left and delete the percent sign.

$$82\% = 0.82$$

Compare 0.88 to 0.82. Since $0.88 > 0.82$, then $\frac{22}{25} > 82\%$. The fraction $\frac{22}{25}$ represents a quiz score high enough to ensure that Brian passes.

Question 2 (page 25)

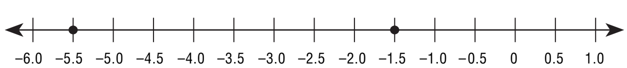
A Correct. Since positive numbers are greater than negative numbers, we will begin by ordering the positive rational numbers. Rational numbers can be compared easily if they are all in decimal form. Since 0.29 is in decimal form, we will put $\frac{3}{5}$ into decimal form by dividing 3 by 5.

$$3 \div 5 = 0.6 \text{ or } \frac{3}{5} = 0.6$$

To convert 45% to decimal form, we move the decimal two places to the left and drop the percent sign.

$$45\% = 0.45$$

From greatest to least, the positive numbers are 0.6, 0.45, 0.29. Of the negative numbers, –1.5 is in decimal form and $-5\frac{1}{2}$ may be expressed as –5.5. We will order the negative numbers by plotting them both on a number line.



Since –1.5 is to the right of –5.5, then $-1.5 > -5.5$. With the numbers written in their original form, the correct answer is $\frac{3}{5}$, 45%, .29, –1.5, $-5\frac{1}{2}$.

Question 3 (page 25)

D Correct. To find the tax on the meal, first convert 4% to a decimal.

$$4\% = 0.04$$

Multiply the cost of the meal by 0.04 to find the total tax.

$$42.25 \cdot 0.04$$

Divide the total tax by the number of friends, 5, to find the amount of tax that each of them paid.

$$(42.25 \cdot 0.04) \div 5$$

Question 4 (page 25)

C Correct. To approximate the value of $\sqrt{29}$, find two consecutive integers such that the first integer squared is less than 29 and the second integer squared is greater than 29. Since $5^2 < 29$ and $6^2 > 29$, the approximate value of $\sqrt{29}$ is between 5 and 6.

Question 5 (page 25)

B Correct. The number 384,000 is greater than 10, so the decimal point must be moved to the left to make the factor greater than or equal to 1 but less than 10. If the decimal point is moved five places to the left, the number will be 3.84. Since the decimal point was moved to the left, the exponent must be positive. The number 384,000 is written as 3.84×10^5 in scientific notation.

Question 6 (page 26)

C Correct. Multiply the number of pounds of beans by the price per pound to represent the cost of the beans.

$$1.19 \cdot 0.88$$

Divide \$1.48 by 2 to find the cost of 1 loaf of bread.

$$1.48 \div 2$$

Add to find t, the total cost of the beans and the bread.

$$t = (1.19 \cdot 0.88) + (1.48 \div 2)$$

Question 7 (page 26)

The correct answer is 329.90. The company charges \$230.93 to rent a car for 7 days. To find the rate per day, divide \$230.93 by 7.

$$230.93 \div 7 = 32.99$$

The rate per day is \$32.99. To find the cost for 10 days, multiply the daily rental rate by 10.

$$32.99 \cdot 10 = 329.90$$

	3	2	9	.	9	0
0	0	0	0		0	●
1	1	1	1		1	1
2	2	●	2		2	2
3	●	3	3		3	3
4	4	4	4		4	4
5	5	5	5		5	5
6	6	6	6		6	6
7	7	7	7		7	7
8	8	8	8		8	8
9	9	9	●		●	9

Question 8 (page 26)

C Correct. Carol made 1,215 minutes of local calls. The first 1,000 minutes cost \$29.50. The remaining 215 minutes cost \$0.04 per minute. Multiply to find the cost of the remaining local minutes.

$$\$0.04 \bullet 215 = \$8.60$$

Add to find the total cost for local calls.

$$\$29.50 + \$8.60 = \$38.10$$

Question 9 (page 26)

B Correct. Mario estimated the sum by adding the dollar part of each check and dropping the cents. The actual total will include the sum of the dollars and cents. Adding the cents will increase the actual total. The actual total is greater than the estimate because the values Mario added were all less than the actual amounts.

Question 10 (page 27)

C Correct. The formula for the area of a circle is $A = \pi r^2$. To see which answer is reasonable, estimate the area of the piece of glass.

Use $\frac{1}{2}$ of 8.1, or about 4 centimeters, as an estimate of the radius of the piece of glass. Substitute the values for π and r in the formula.

$$A = \pi r^2$$
$$A \approx 3.14 \bullet 4^2$$
$$A \approx 3.14 \bullet 16$$
$$A \approx 50.24$$

The area of the circle is approximately 50.24 cm^2, so 50 cm^2 is a reasonable value.

Question 11 (page 27)

D Correct. Use a proportion to compare the dollars earned, d, to the number of hours worked, 9.

$$\frac{\text{money}}{\text{hours}} \qquad \frac{78}{6} = \frac{d}{9}$$

Use cross products to solve the proportion.

$$6d = 78 \bullet 9$$

Objective 2

Question 12 (page 46)

B Correct. Find two ratios, each of which compares the number of words typed to the number of minutes required. Use a proportion to show the relationship between the ratios. The first ratio is $\frac{280 \text{ words}}{8 \text{ minutes}}$. The number of words that Sam can type in half an hour is represented by n. The second ratio is n words to $\frac{1}{2}$ hour. But ratios that are being compared must be expressed in the same units. Convert $\frac{1}{2}$ hour to 30 minutes. The second ratio is then $\frac{n}{30}$. Write a proportion. The proportion $\frac{280}{8} = \frac{n}{30}$ could be used to find n, the number of words Sam can type in half an hour.

Question 13 (page 46)

A Incorrect. The first ratio compares the number of newspapers delivered to the number of minutes required: $\frac{60}{45}$. The ratio in choice A compares 30 newspapers to $\frac{1}{2}$ hour. For ratios to be compared, they must be expressed in the same units; $\frac{1}{2}$ hour equals 30 minutes. The ratio in choice A compares 30 newspapers to 30 minutes: $\frac{30}{30}$. The two ratios represent the same rate of delivering newspapers if they are proportional. Check to see whether their cross products are equal.

$$\frac{60}{45} \stackrel{?}{=} \frac{30}{30}$$

$$1{,}800 \neq 1{,}350$$

Since the cross products are not equal, the two ratios are not proportional. A rate of 30 papers in $\frac{1}{2}$ hour is not equivalent to 60 papers in 45 minutes.

B Incorrect. The first ratio compares the number of newspapers delivered to the number of minutes required: $\frac{60}{45}$. The ratio in choice B compares

75 newspapers to 1 hour. For ratios to be compared, they must be expressed in the same units; 1 hour equals 60 minutes. The ratio in choice B compares 75 newspapers to 60 minutes: $\frac{75}{60}$. The two ratios represent the same rate of delivering newspapers if they are proportional. Check to see whether their cross products are equal.

$$\frac{60}{45} \stackrel{?}{=} \frac{75}{60}$$

$$3{,}600 \neq 3{,}375$$

Since the cross products are not equal, the two ratios are not proportional. A rate of 75 papers in 1 hour is not equivalent to 60 papers in 45 minutes.

C Correct. The first ratio compares the number of newspapers delivered to the number of minutes required: $\frac{60}{45}$. The ratio in choice C compares 120 newspapers to $1\frac{1}{2}$ hours. For rates to be compared, they must be expressed in the same units; $1\frac{1}{2}$ hours equals 90 minutes. The ratio in choice C compares 120 newspapers to 90 minutes: $\frac{120}{90}$. The two ratios represent the same rate of delivering newspapers if they are proportional. Check to see whether their cross products are equal.

$$\frac{60}{45} \stackrel{?}{=} \frac{120}{90}$$

$$5{,}400 = 5{,}400$$

Since the cross products are equal, the two ratios are proportional. A rate of 120 papers in $1\frac{1}{2}$ hours is equivalent to 60 papers in 45 minutes.

D Incorrect. The first ratio compares the number of newspapers delivered to the number of minutes required: $\frac{60}{45}$. The ratio in choice D compares 100 newspapers to 1 hour. For rates to be compared, they must be expressed in the same units; 1 hour equals 60 minutes. The ratio in choice D compares 100 newspapers to 60 minutes: $\frac{100}{60}$. The two ratios represent the same rate of delivering newspapers if they are proportional. Check to see whether their cross products are equal.

$$\frac{60}{45} \stackrel{?}{=} \frac{100}{60}$$

$$3{,}600 \neq 4{,}500$$

Since the cross products are not equal, the two ratios are not proportional. A rate of 100 papers in 1 hour is not equivalent to 60 papers in 45 minutes.

Question 14 (page 46)

D Correct. Write two ratios that compare the number of shoes boxed to the number of minutes required. The first ratio is 5 pairs of shoes every 3 minutes, $\frac{5}{3}$. Let x represent the number of shoes that can be placed in shoe boxes in 8 hours. The second ratio should compare x to the number of minutes in 8 hours.

There are $8 \cdot 60 = 480$ minutes in 8 hours. Write a proportion.

$$\frac{5}{3} = \frac{x}{480}$$

Then set the cross products equal to each other to solve the proportion.

$$3x = 5 \cdot 480$$
$$3x = 2{,}400$$
$$x = 800$$

During 8 hours of work, 800 pairs of shoes can be placed in shoe boxes.

Question 15 (page 46)

C Correct. Use a proportion to solve the problem. The ratio of students who ride the bus expressed as a percent is 78%, or 78 out of 100, which is equivalent to $\frac{78}{100}$. The ratio of students who ride the bus can also be expressed in terms of n, the total number of students in the school. That ratio is $\frac{975}{n}$. Write a proportion.

$$\frac{975}{n} = \frac{78}{100}$$

To solve, use cross products.

$$78n = 975 \cdot 100$$
$$78n = 97{,}500$$
$$n = 1{,}250$$

There are 1,250 students attending Cantor Middle School.

Question 16 (page 46)

B Correct. First find the additional fee per piece of lumber. Look for a pattern in the table. If 100 pieces of lumber cost an additional \$24, then each piece costs an additional \$0.24 to deliver.

$$24 \div 100 = 0.24$$

Check this unit fee for 20 pieces of lumber.

$$20 \cdot 0.24 = 4.80$$

This value agrees with the value in the table for 20 pieces of lumber. Check the other values. They all agree with a unit fee of \$0.24. There is a \$25 fee for any delivery. Use these facts to write an equation. The unit fee, \$0.24, times the number of pieces of lumber, n, plus the \$25 fee should equal the total charge, c. The equation $c = 0.24n + 25$ could be used to find the total cost for a delivery of n pieces of lumber.

Question 17 (page 47)

A Correct. In the graph, the lengths of the bars represent the number of gallons of milk drunk on a given day. The vertical scale of the graph is 0 to 50 gallons, with each mark standing for 5 gallons. Three of the bars do not end exactly on a mark. They are slightly above or below the mark. Estimate to find the values these bars represent: Monday (32 gallons), Thursday (27 gallons), and Sunday (38 gallons). The values in choice A match the estimates of the data represented in the graph.

Question 18 (page 48)

The correct answer is 1072.50. Look for a pattern in the table. The interest earned in 2 years is \$195.00.

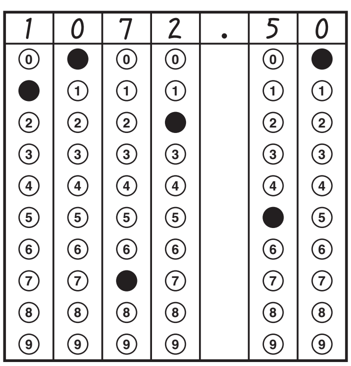
$$\$195 \div 2 = \$97.50 \text{ per year}$$

Check to see if this rate satisfies the remaining values in the table.

$$3 \cdot \$97.50 = \$292.50$$
$$4 \cdot \$97.50 = \$390.00$$
$$5 \cdot \$97.50 = \$487.50$$

Use this rate to find the interest earned in 11 years.

$$11 \cdot \$97.50 = \$1072.50$$



Question 19 (page 48)

B Correct. Write an equation that can be used to find the total annual cost of belonging to the swim club. Let n represent the number of times the pool is visited. If it costs \$2 every time a member visits the pool, then $2n$ represents the pool charges for the year. The annual membership charge is \$25. The total annual cost, c, is then $c = 2n + 25$. Find n when c is 365 by substituting values into the equation.

$$c = 2n + 25$$
$$365 = 2n + 25 \quad \text{Substitute.}$$
$$340 = 2n \quad \text{Subtract 25 from each side.}$$
$$170 = n \quad \text{Divide each side by 2.}$$

Joanne visited the pool 170 times last year.

Question 20 (page 48)

A Correct. Only the rule $3n - 1$ produces the correct value for each term.

Position	$3n - 1$	Value of Term
1	$3(1) - 1$ $= 3 - 1$ $= 2$	2
2	$3(2) - 1$ $= 6 - 1$ $= 5$	5
3	$3(3) - 1$ $= 9 - 1$ $= 8$	8
4	$3(4) - 1$ $= 12 - 1$ $= 11$	11
5	$3(5) - 1$ $= 15 - 1$ $= 14$	14

Question 21 (page 49)

D Correct. Only the sequence $\frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \ldots$ has the correct value for each term.

Position	$\frac{n+1}{3}$	Value of Term
1	$\frac{1+1}{3} = \frac{2}{3}$	$\frac{2}{3}$
2	$\frac{2+1}{3} = \frac{3}{3}$	$\frac{3}{3}$
3	$\frac{3+1}{3} = \frac{4}{3}$	$\frac{4}{3}$
4	$\frac{4+1}{3} = \frac{5}{3}$	$\frac{5}{3}$
5	$\frac{5+1}{3} = \frac{6}{3}$	$\frac{6}{3}$

Objective 3

Question 22 (page 69)

D Correct. The radius is reduced by 60%. The original radius is 4 inches.

60% of 4 inches = 0.60 • 4 = 2.4 inches

The radius is reduced by 2.4 inches. The new radius is 4 inches − 2.4 inches = 1.6 inches.

Question 23 (page 69)

D Correct. The ratios of the corresponding sides are $\frac{12}{30}$ and $\frac{10}{25}$. Both fractions are equal to $\frac{2}{5}$ or 0.4, the scale factor that was used to reduce the larger parallelogram.

Question 24 (page 69)

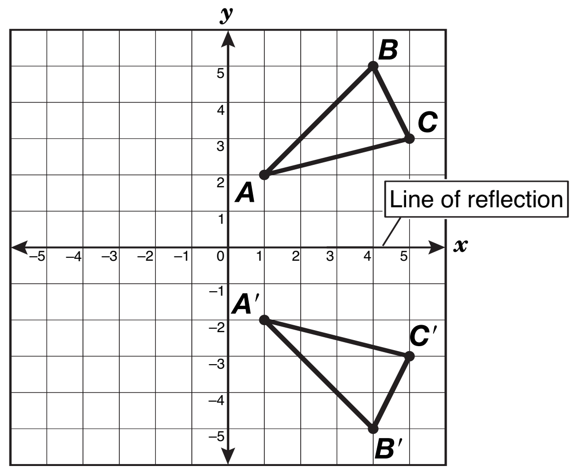
C Correct. The length of side DE is 6 units. The length of side $D'E'$ is 12 units. The ratio of these corresponding sides is $\frac{D'E'}{DE} = \frac{12}{6} = 2$. The lengths of sides DF and $D'F'$ are 8 units and 16 units, respectively. The ratio of these corresponding sides is $\frac{D'F'}{DF} = \frac{16}{8} = 2$. Triangle DEF was enlarged by a scale factor of 2.

Question 25 (page 70)

A Correct. Point B has the coordinates $(-2, 4)$. The x-coordinate of point B, -2, is 7 units to the left of the line of reflection, $x = 5$. The reflection of point B must be 7 units to the right of the line of reflection. The x-coordinate of the reflection of point B is $7 + 5 = 12$. The y-coordinate of the point is unchanged by the reflection. The y-coordinate is still 4. The coordinates of the reflection of point B are $(12, 4)$.

Question 26 (page 70)

B Correct. The x-axis is the line of reflection for this transformation.



Question 27 (page 70)

D Correct. When viewed from the top, this figure has a row of 3 blocks perpendicular to a column of three blocks in the middle. From the side you can see one row of 3 blocks with another block in the center of a second row. From the front you can see one row of 3 blocks with another block on the left of a second row. Only the figure in choice D has these three views.

Question 28 (page 71)

C Correct. The rectangular floor has an area of 33 feet • 14 feet = 462 ft^2. The circular fountain takes up an area of $\pi \cdot 4^2 \approx 3.14 \cdot 16 \approx 50.24$ ft^2. Subtract the area of the fountain from the area of the entire floor. The area that Mr. Wythe polishes is 462 ft^2 − 50.24 ft^2 ≈ 411.76 ft^2.

Question 29 (page 71)

B Correct. According to the Pythagorean Theorem on the Mathematics Chart, in any right triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$. In this case the legs are 6 and 8 and the hypotenuse is 10. The correct relationship between the sides of this right triangle is $6^2 + 8^2 = 10^2$.

Question 30 (page 72)

B Correct. Point Q has the coordinates $(-4, 8)$. The x-coordinate of point Q is –4, and $-4 < -2.5$. The y-coordinate of point Q is 8, and $8 > 3$. The coordinates of point Q satisfy both conditions, $x < -2.5$ and $y > 3$.

Question 31 (page 72)

C Correct. The Mathematics Chart shows the formula for the area of a square to be $A = s^2$. This means that the area of a square equals the square of the length, s, of any of its congruent sides. In this case the squares of the lengths of the sides of the shaded squares would be equal to their respective areas, 36 cm^2 and 25 cm^2. The square of the length of the side of the unshaded square would equal its area. The Pythagorean Theorem on the Mathematics Chart states that for any right triangle with legs a and b and a hypotenuse c, $a^2 + b^2 = c^2$. In order for one of its sides to be the hypotenuse of a right triangle, the area of the unshaded square would have to equal the sum of the areas of the two shaded squares whose sides are the legs: 36 cm^2 + 25 cm^2 = 61 cm^2.

Objective 4

Question 32 (page 100)

C Correct. The surface area of a 3-dimensional figure is the sum of the areas of its surfaces. The surfaces of the prism are all rectangles.
To find the area of each rectangle, multiply its length by its width.
There are two rectangles with dimensions of 2 by 9 in. Each has an area of 18 in.^{2} Their combined area is 36 in.^{2}
There are two rectangles with dimensions of 2 by 10 in. Each has an area of 20 in.^{2} Their combined area is 40 in.^{2}
There are two rectangles with dimensions of 9 by 10 in. Each has an area of 90 in.^{2} Their combined area is 180 in.^{2}

$$S = 36 \text{ in.}^2 + 40 \text{ in.}^2 + 180 \text{ in.}^2 = 256 \text{ in.}^2$$

The surface area of the prism is 256 square inches.

Question 33 (page 100)

B Correct. The formula for total surface area of a pyramid is $S = \frac{1}{2}Pl + B$, where P is the perimeter of the base of the pyramid, l is its slant height, and B is the area of its base. The perimeter of the base is $5 + 5 + 5 + 5 = 20$ in.

The slant height is given in the question as 3 in. $\frac{1}{2}Pl = \frac{1}{2}(20 \times 3) = 30$ in. The area of a square with a side length of 5 in. is $5^2 = 25$ in.^{2}

The total surface area of the pyramid is 30 in.^{2} + 25 in.^{2} = 55 in.^{2}

Question 34 (page 100)

C Correct. The area of the label is the lateral surface area of the can.
The formula for the lateral surface area of a cylinder is $S = 2\pi rh$. Measure the height and radius of the cylinder in centimeters.
Radius = 1 cm
Height = 6 cm
Substitute 1 for r, 6 for h, and 3.14 as an approximate value for π.

$$S = 2\pi rh$$
$$S \approx 2(3.14)(1)(6)$$
$$S \approx 37.68$$

The lateral surface area of the cylinder is approximately 38 square centimeters.

Question 35 (page 101)

A Correct. The amount of cement needed is equal to the volume of the slab, which will be a rectangular prism.
The formula for the volume of a rectangular prism is $V = Bh$.
First convert 4 inches to feet, since the question asks for the number of cubic feet of cement needed.

$$\frac{4 \text{ inches}}{x \text{ feet}} = \frac{12 \text{ inches}}{1 \text{ foot}}$$
$$12x = 4$$
$$x = \frac{1}{3}$$

Calculate B, the area of the base.

$$12.5 \cdot 9 = 112.5 \text{ ft}^2$$

Find the volume of the slab.

$$V = Bh$$
$$V = 112.5 \cdot \frac{1}{3}$$
$$V = 37.5 \text{ ft}^3$$

It will take 37.5 cubic feet of cement to build the slab.

Question 36 (page 101)

C Correct. Since the ladder and the wall of the building form a right triangle, use the Pythagorean Theorem to find the missing side.

Let b represent the distance from the bottom of the ladder to the base of the building, which is the missing leg of the right triangle. The length of the ladder is the hypotenuse of the right triangle. Replace c with 12. The known leg of the triangle, a, is 10.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 10^2 + b^2 &= 12^2 \\ 100 + b^2 &= 144 \\ b^2 &= 144 - 100 \\ b^2 &= 44 \\ \sqrt{b^2} &= \sqrt{44} \\ b &= \sqrt{44} \end{aligned}$$

Since $6^2 = 36$ and $7^2 = 49$, $\sqrt{44}$ is between 6 and 7. Only answer choice C is a value between 6 and 7 feet.

Question 37 (page 101)

D Correct. The triangle formed by home plate, first base, and third base is a right triangle. The segment between first base and third base forms the hypotenuse.

Use the Pythagorean Theorem to find the length of the missing side.

Replace a with 90 and b with 90.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 90^2 + 90^2 \\ c^2 &= 8{,}100 + 8{,}100 \\ c^2 &= 16{,}200 \\ \sqrt{c^2} &= \sqrt{16{,}200} \\ c &= \sqrt{16{,}200} \end{aligned}$$

Since $120^2 = 14{,}400$ and $130^2 = 16{,}900$, $\sqrt{16{,}200}$ is between 120 and 130. Only answer choice D is a value between 120 and 130 feet.

Question 38 (page 101)

B Correct. The Mathematics Chart on page 9 of this guide lists the volume of a pyramid as $V = \frac{1}{3}Bh$, where B represents the area of the base and h represents the height of the pyramid. The base is a square with a side length of 3 meters. The formula given for the area of a square is $A = s^2$, where s is the side length of the square. In this case the area of the base of the pyramid is $A = 3^2$, or $A = 9$. The height of the pyramid is given as 2 meters. The volume is $\frac{1}{3} \cdot 9 \cdot 2$, or $V = 6 \text{ m}^3$.

Question 39 (page 102)

B Correct. The model and the house can be represented with two rectangles. The rectangles are similar figures, so the measurements of their corresponding sides are proportional.

Let x represent the width of the house.

The ratio of the length of the house to the length of the model is $\frac{64 \text{ ft}}{12.5 \text{ in.}}$.

The ratio of the width of the house to the width of the model is $\frac{x \text{ ft}}{7.75 \text{ in.}}$.

Write a proportion.

$$\begin{aligned} \frac{x}{7.75} &= \frac{64}{12.5} \\ 12.5x &= 496 \\ x &= 39.68 \end{aligned}$$

To the nearest foot, the width of the actual house will be 40 feet.

Question 40 (page 102)

A Correct. The perimeter of the new rectangle will decrease by the same scale factor.

The scale factor is $\frac{1}{2}$. To find the perimeter of the new rectangle, multiply the scale factor by the perimeter of the original rectangle. The perimeter of the new rectangle is $39 \cdot \frac{1}{2} = 19.5$ centimeters.

Question 41 (page 102)

C Correct. The area of the new triangle will increase by the square of the scale factor.

The scale factor is 5. The square of the scale factor is $5^2 = 25$.

To find the area of the new triangle, multiply the area of the original triangle by the square of the scale factor.

The area of the new triangle is $6 \cdot 25 = 150 \text{ ft}^2$.

Question 42 (page 102)

D Correct. The volume of the new cylinder will decrease by the cube of the scale factor.

The scale factor is $\frac{1}{4}$. The cube of the scale factor is $\left(\frac{1}{4}\right)^3$.

$$\left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3} = \frac{1}{64}$$

To find the volume of the new cylinder, multiply the volume of the original cylinder by the cube of the scale factor.

The volume of the new cylinder is
$3{,}600 \cdot \frac{1}{64} = 56.25$ cubic centimeters.

Question 43 (page 103)

B Correct. The dimensions of the smaller tent are $\frac{3}{4}$ the dimensions of the larger tent. The smaller tent is a dilation of the larger tent, with a scale factor of $\frac{3}{4}$.

The cube of the scale factor is $\left(\frac{3}{4}\right)^3$.

$$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$$

The ratio of the volumes of the tents will be proportional to the cube of the scale factor.

Let x represent the volume of the larger tent.

The ratio of the volume of the smaller tent to the volume of the larger tent is $\frac{1{,}000}{x}$.

Write a proportion.

$$\frac{27}{64} = \frac{1{,}000}{x}$$
$$27x = 64{,}000$$
$$x = 2{,}370.37$$
$$x \approx 2{,}370 \text{ ft}^3$$

The volume of the larger tent is approximately 2,370 cubic feet.

Objective 5

Question 44 (page 135)

B Correct. The outcome of the first spin has no effect on the outcome of the second spin. Therefore, this is a compound event made up of two independent events. There are 3 red sectors on the spinner, out of a total of 8 sectors. Calculate the probability that the spinner will land on a red sector on the first spin.

$$P(\text{red}) = \frac{3}{8}$$

There are 2 green sectors on the spinner. Calculate the probability that the spinner will land on a green sector on the second spin.

$$P(\text{green}) = \frac{2}{8}$$

To find the probability of spinning red on the first spin and green on the second spin, multiply the probabilities of the two events.

$$P(\text{red and green}) = P(\text{red}) \cdot P(\text{green}) =$$
$$\frac{3}{8} \cdot \frac{2}{8} = \frac{6}{64} = \frac{3}{32}$$

Question 45 (page 135)

D Correct. The number of tiles in the bag will be different for the second draw than for the first draw because Erica does not replace the tile after the first draw. Therefore, this is a compound event made up of two dependent events.

For the first draw, there are 7 vowels out of a total of 15 tiles in the bag.

$$P(\text{vowel}_{\text{first}}) = \frac{7}{15}$$

If a vowel is drawn on the first draw, there will be only 6 vowels and a total of 14 tiles in the bag for the second draw.

$$P(\text{vowel}_{\text{second}}) = \frac{6}{14}$$

To find the probability that Erica will draw two vowels, multiply the probabilities of the two events.

$$P(\text{vowel}_{\text{first}} \text{ and vowel}_{\text{second}}) =$$
$$P(\text{vowel}_{\text{first}}) \cdot P(\text{vowel}_{\text{second}}) =$$
$$\frac{7}{15} \cdot \frac{6}{14} = \frac{42}{210} = \frac{1}{5}$$

Question 46 (page 135)

C Correct. Use a proportion to solve the problem. The ratio of students voting for Nancy in the survey should equal the ratio of students voting for Nancy in the election.

$$\frac{5}{30} = \frac{x}{332}$$
$$30x = 1{,}660$$
$$x \approx 55.33$$

Nancy should get about 55 votes in the election.

Question 47 (page 135)

B Correct. Use Mark's experimental probability of running the 440-meter race in less than 65 seconds to predict his future performance. Mark has run 8 races. Of those, he ran 3 in less than 65 seconds. His experimental probability of running the 440-meter race in less than 65 seconds is $\frac{3}{8}$. Running his next two races in less than 65 seconds is a compound event. Running the first race in less than 65 seconds is the first event, and running the second race in less than 65 seconds is the second event. To find the probability of Mark's running the first and second races in less than 65 seconds each, multiply the probabilities of the two events.

P(1st under 65 and 2nd under 65) =
P(1st under 65) • P(2nd under 65) =
$\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$

The theoretical probability that Mark will run both of his next two races in less than 65 seconds each is $\frac{9}{64}$.

Question 48 (page 136)

D Correct. The range of a set of numbers tells how large the spread is between the greatest and the smallest values in the set. Use the range to find the greatest value when you are given the smallest value.

Question 49 (page 136)

A Correct. As you move from left to right on the coordinate grid, the years increase from 1991 to 1998. As the years increase, the cost of the vehicles neither increases nor decreases in any pattern. This means that there is no trend in the data.

Question 50 (page 137)

D Correct. A total of 180 sandwiches were served. Of those, 40 were tuna salad sandwiches. The fraction of the total number of sandwiches served that were tuna salad is $\frac{40}{180}$, or $\frac{2}{9}$. The sum of all the angles in a circle graph is 360°. The angle that should form the section for tuna salad is $\frac{2}{9} \cdot 360° = 80°$. An 80° angle should form the section for tuna salad.

Question 51 (page 137)

A Correct. Janis surveyed only people who were shopping at Food Superstore. The people shopping at Food Superstore are more likely to choose Food Superstore as their favorite grocery store. This was a biased sample, not representative of all the people in Janis's town.

Question 52 (page 137)

C Correct. 14% of customers spent 31 to 45 minutes shopping. So if we take 14% of 200 customers, that gives us 28 customers shopping for 31 to 45 minutes. $0.14 \cdot 200 = 28$

Question 53 (page 138)

D Correct. For each year, the height of the left bar should be equal to the number of seventh-grade participants that year, and the height of the right bar should be equal to the number of eighth-grade participants that year. Only the heights of the bars in the graph in choice D match the data in the table.

Question 54 (page 139)

A Correct. This circle graph best represents the data. The total number of pounds of litter collected was 507 pounds. Divide each weight by 507 to find the percent each represents.

Barryville Litter Drive

Type of Litter	Pounds	Percent
Paper	305	≈60%
Aluminum	25	≈5%
Wood	51	≈10%
Misc.	126	≈25%
Total	507	100%

Only the graph in choice A appears to match these percentages.

Question 55 (page 140)

D Correct. First, order the data from least to greatest as follows: 2, 6, 9, 10, 11, 12, 13, 13, 14, 14, 16. In this data set the median is 12, the minimum is 2, the maximum is 16, the lower quartile is 9, and the upper quartile is 14. Answer choice D is the only box and whisker plot that correctly represents these data.

Objective 6

Question 56 (page 155)

A Incorrect. To determine whether the package can be mailed, she needs to add the three dimensions together, not find the area of the package.

B Correct. To find which shape would require the greater number of tiles, she needs to compare the area of the circle to the area of the square.

C Incorrect. To determine the amount of chalk powder the machine will use, he needs to calculate the perimeter of the soccer field, not its area.

D Incorrect. To find which container will hold more sugar, he needs to find the volumes of the containers, not their areas.

Question 57 (page 155)

A Correct. Work backward to find a solution. Celia is 15 years old. Tom is twice Celia's age, so he is $2 \cdot 15 = 30$ years old. Mary is 6 years younger than Tom, so she is $30 - 6 = 24$ years old. Jack is two-thirds of Mary's age, so he is $\frac{2}{3} \cdot 24 = 16$ years old. If Jack is 16 years old and Celia is 15 years old, then Jack is 1 year older than Celia.

Question 58 (page 155)

D Correct. To find the sale price, subtract the amount of the discount from the original price.

Find 25% of \$34.80.

$$25\% = 0.25$$
$$34.80 \cdot 0.25 = 8.70$$

The discount is \$8.70.

The sale price is

$$\$34.80 - \$8.70 = \$26.10.$$

To find the total price, add the amount of sales tax to the sale price.

Find 6% of \$26.10.

$$6\% = 0.06$$
$$26.10 \cdot 0.06 = 1.566$$

The amount of tax Phil will pay is \$1.57.

The total cost of the shoes is

$$\$26.10 + \$1.57 = \$27.67.$$

If Phil has only \$25, then he needs \$27.67 − \$25.00 = \$2.67 more to buy the shoes.

Question 59 (page 155)

C Correct. Find the area of the patio.

$$A = lw$$
$$A = 15 \cdot 6 = 90 \text{ ft}^2$$

Find the area of a tile in square feet, not square inches.

The tiles are 9 inches, or $\frac{9}{12} = \frac{3}{4} = 0.75$ feet on a side.

$$A = 0.75 \cdot 0.75 = 0.5625 \text{ ft}^2$$

Write a proportion to find the number of tiles needed.

$$\frac{1}{0.5625} = \frac{x}{90}$$
$$0.5625x = 90$$
$$x = 160$$

Mr. Wilson will need to buy 160 tiles. They are sold in boxes of 12. Divide to find the number of boxes of tiles he will need to buy.

$$160 \div 12 \approx 13.33$$

Mr. Wilson will need to buy 14 boxes of tiles.

Question 60 (page 156)

B Correct. The formula for the area of a square is $A = s^2$. Find the area of the square in square inches. Since 1 foot = 12 inches, the area is 12^2 square inches.

The circle has a diameter of 8 inches. The radius is half of the diameter, or 4 inches. The formula for the area of a circle is $A = \pi r^2$. Substitute 4 for r, the radius of the circle, to find the area of the circle.

The area of the circle is $4^2\pi$ square inches.

Subtract the area of the circle from the area of the square to find the area of the shaded part.

$12^2 - 4^2\pi$.

Question 61 (page 156)

D Correct. The ladder forms a right triangle with the building and the ground. In a right triangle, $a^2 + b^2 = c^2$, where a and b are the legs of the triangle and c is its hypotenuse. Choice D states the Pythagorean Theorem in words.

Question 62 (page 156)

C Correct. To solve this problem, you do not need to know the surface areas of the two boxes, just the ratio of their areas. If the dimensions of a figure are dilated by a scale factor, then the surface area of the dilated figure will change by the square of that scale factor. If the dimensions of the box are doubled (times 2), then the surface area of the larger box will be 2^2, or 4 times the surface area of the smaller box. Their areas are in the ratio of $\frac{1}{4}$. Use this ratio to compare the cost of covering the boxes.

$$\frac{\text{total cost to cover small box}}{\text{total cost to cover large box}} = \frac{1}{4} \cdot \frac{\$0.12}{\$0.04} = \frac{\$0.12}{\$0.16}$$

If the smaller box had an area of 1 ft^2, then it would cost \$0.12 to cover. If the smaller box had an area of 1 ft^2, then the larger box would have an area of 4 ft^2. It would cost 4(\$0.04), or \$0.16, to cover.

If the smaller box had an area of 2 ft^2, then it would cost 2($0.12) = $0.24 to cover. If the smaller box had an area of 2 ft^2, then the larger box would have an area of 8 ft^2. It would cost 8($0.04), or $0.32, to cover.

Therefore, it will cost more to cover the larger box.

Question 63 (page 156)

A Correct. Each term of the top sequence is the term's position in the sequence times 4. The ninth term is $9 \times 4 = 36$. Each term of the bottom sequence is found by adding 1 to the position of the term and dividing that sum by 2. The ninth term of the bottom sequence is $(9 + 1) \div 2 = 5$. The sum of the two ninth terms is $36 + 5 = 41$.

Question 64 (page 157)

B Correct. All the triangles except triangle S are right triangles. The lengths of their sides satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$. The lengths of the sides of triangle S do not satisfy the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$10^2 + 30^2 \stackrel{?}{=} 32^2$$
$$100 + 900 \stackrel{?}{=} 1{,}024$$
$$1{,}000 \neq 1{,}024$$

Triangle S does not belong in this group.

Question 65 (page 157)

D Correct. To find the amount Kevin spent on the pizzas, subtract the cost of the soda from the total amount spent.

$$70 - 2.50 = 67.50$$

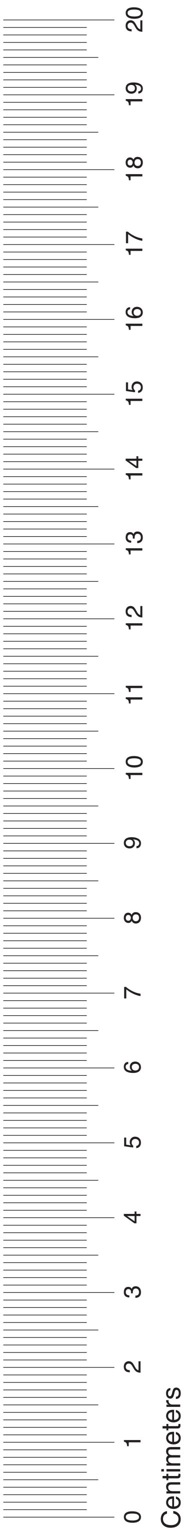
Kevin spent $67.50 on pizza. One way to solve this problem would be to make a table that shows the cost of the different combinations of medium and large pizzas that total 5 pizzas all together.

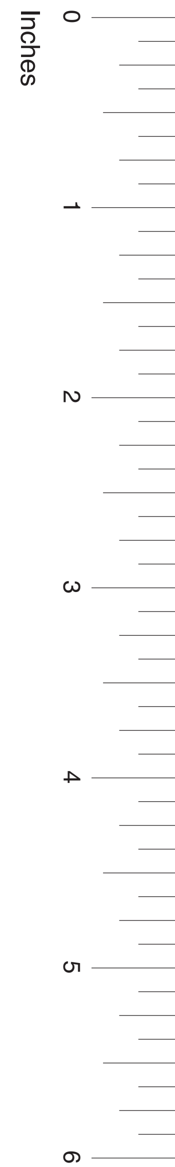
Number of Pizzas			Cost
Medium	Large	Total	
0	5	5	0 • 12.50 + 5 • 15 = $75
1	4	5	1 • 12.50 + 4 • 15 = $72.50
2	3	5	2 • 12.50 + 3 • 15 = $70.00
3	2	5	3 • 12.50 + 2 • 15 = $67.50
4	1	5	4 • 12.50 + 1 • 15 = $65.00
5	0	5	5 • 12.50 + 0 • 15 = $62.50

The only combination of five pizzas that totals $67.50 is 3 medium and 2 large pizzas.

TAKS Texas Assessment of Knowledge and Skills

Grade 8 Mathematics Chart





LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

Continued on the next side

Grade 8 Mathematics Chart

Perimeter	square	$P = 4s$
	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	square	$A = s^2$
	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	circle	$A = \pi r^2$

P represents the Perimeter of the Base of a three-dimensional figure.

B represents the Area of the Base of a three-dimensional figure.

Surface Area	cube (total)	$S = 6s^2$
	prism (lateral)	$S = Ph$
	prism (total)	$S = Ph + 2B$
	pyramid (lateral)	$S = \frac{1}{2}Pl$
	pyramid (total)	$S = \frac{1}{2}Pl + B$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
Volume	prism	$V = Bh$
	cylinder	$V = Bh$
	pyramid	$V = \frac{1}{3}Bh$
	cone	$V = \frac{1}{3}Bh$
	sphere	$V = \frac{4}{3}\pi r^3$
Pi	π	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Simple Interest Formula		$I = prt$